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FINITE ELEMENT MODELING OF MASONRY STRUCTURES: A SIMPLIFIED MODELING PROCESS WITH FRAME AND SHELL ELEMENTS ON ŞEHZADE MEHMET MOSQUE

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JUNE 2024

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(M.Sc. Thesis)

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ABSTRACT

This thesis proposes a finite element modeling (FEM) methodology tailored for historical masonry structures by utilizing frame and shell elements to construct a finite element model with structural elements that are representing a very close behavior to their real state. Method uses a macro-mechanical perspective that simplifies the modeling process and enhances the clarity of the numerical model's outputs. Traditional finite element models often rely on software to create a full mesh of the structure out of solid elements, which often results in a complex mesh that can be challenging to interpret. This result potentially hinders the decision-making in restoration thus directly affecting the historical building's preservation. The proposed method allows to create hand constructed finite element models that have far less complexity with adequate accuracy. This streamlines the model-interpreting process and enhances the collaboration among the disciplines that contribute to the preservation process. A case study has been carried out on Şehzade Mehmet Mosque to demonstrate the proposed method's practical effectiveness, yielding clear and interpretable results that were accessible to every discipline in the process.

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YIĞMA YAPILARIN SONLU ELEMANLAR TEKNİĞİ İLE MODELLENMESİ: ŞEHZADE MEHMET CAMİİ ÜZERİNDE ÇUBUK VE KABUK ELEMANLARLA BASİTLEŞTİRİLMİŞ BİR MODELLEME SÜRECİ

(Yüksek Lisans Tezi)

Doğukan KARTAL

GAZİ ÜNİVERSİTESİ
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Haziran 2024

ÖZET

Bu tez, gerçek durumlarına çok yakın bir davranışı temsil eden yapısal elemanlarla bir sonlu eleman modeli oluşturmak için çerçeve ve kabuk elemanları kullanarak tarihi yığma yapılar için uyarlanmış bir sonlu eleman modelleme (FEM) metodolojisi önermektedir. Yöntem, modelleme sürecini basitleştiren ve sayısal modelin çıktılarının netliğini artıran makro-mekanik bir bakış açısı kullanmaktadır. Geleneksel sonlu eleman modelleri genellikle ağ oluşturmak için büyük oranda yazılımlara bel bağlar ve bu durum genellikle çıktılarının yorumlanması zor katı elemanlardan oluşan karmaşık bir ağ ile sonuçlanır. Bu sonuç, restorasyonda karar verme sürecini potansiyel olarak engelleyerek tarihi binanın korunmasını doğrudan etkilemektedir. Önerilen yöntem, yeterli doğrulukla çok daha az karmaşıklığa sahip elle oluşturulmuş sonlu eleman modellerinin oluşturulmasına olanak sağlamaktadır. Bu, model yorumlama sürecini kolaylaştırır ve koruma sürecine katkıda bulunan disiplinler arasındaki işbirliğini geliştirir. Önerilen yöntemin pratikteki etkinliğini göstermek için Şehzade Mehmet Camii üzerinde bir vaka çalışması gerçekleştirilmiş ve süreçteki her disiplinin erişebileceği net ve yorumlanabilir sonuçlar elde edilmiştir.

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I want to dedicate this thesis to all four of my grandparents that were ready to go anywhere with me on this long journey.

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SYMBOLS AND ABBREVIATIONS

The symbols and abbreviations used in this study are presented below, along with their explanations.

Symbols	Explanations
A	Area
Ca	Calsium
C	Carbon
cm	Centimeter
E	Young's Modulus
F	Force
H	Hydrogen
k	Stiffness
kN	Kilonewton
L	Length
MPa	Megapascal
m	Meter
M	Moment
$\mathbf{m^2}$	Square meters
$\mathbf{m^3}$	Cubic meters
$\mathbf{mm^2}$	Square millimeters
N	Newton
O	Oxygen
R	Reaction
T	Ton
W	Weight
$\boldsymbol{\varepsilon}$	Strain
$\boldsymbol{\upsilon}$	Poisson's Ratio
$\boldsymbol{\Delta}$	Differential
$\boldsymbol{\sigma}$	Normal Stress

Abbreviations	Explanations
A.D.	Anno Domini (After Christ)
B.C.	Before Christ
FE	Finite Element
FEM	Finite Element Model or Finite Element Modeling

1. INTRODUCTION

The meaning of history as a word is "the whole series of events in the past that relate to the development of a country, subject, or a person" [1]. Historical, an adjective that can be considered a by-product of history means, "relating to events or people in the past, or the study of history" [2]. Any object that is bestowed with the title of being "historic" differs greatly from the other objects of its kind. From that moment they are treated as witnesses of events that built history and time gates that connect past to present, thus becoming invaluable. The status of being historical also means they should be protected and passed on to the next generations. This also means over time they are going to witness more events and become more important, which increases the priority of protection.

The historical character can be exercised by many different types of objects such as books, clothing, and weapons. In this very extensive catalog of historical objects, one type stands out more than others, historical structures. Historical structures can contain historical data in many forms. While they are witnessing events by default, they also contain traces of the past in them. Plan, design, ornamentation, or arrangement of space of a historical structure can give hints about architectural philosophy of their time while its materials and building technique represent the state of structural engineering.

When the historical building stock in the world is analyzed, it is seen that a large part of it consists of masonry constructions. This is because masonry, the skill of building with brick and stone, was a technology that was *mainstream* in construction for a long time and is still being used today, although not as frequently as before. The masonry workmanship in structures can be traced as far back as the Tower of Jericho which was built in 8300 B.C. in the Tower of Jericho (See Figure 1.3), while the foundations of the construction technologies we are using today were laid as late as the middle of the 19th century. Considering this, it can be said that masonry construction had a time of 10000 years to be categorized as historic while structures built with contemporary technologies have not even had a time of 300 years yet, which explains the overwhelming quantity difference between masonry historical structures and the others. While this does not mean that historic structures built with modern technologies are less valuable than historic masonry structures, it does emphasize the importance of protecting historic masonry structures from external threats [3-5].

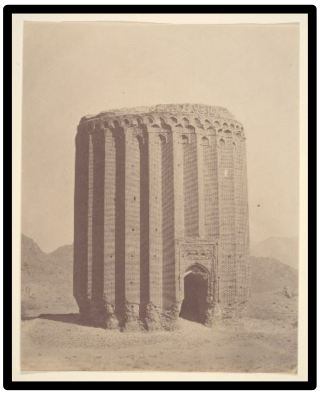

Figure 1.1. Tughrul Tower [6]

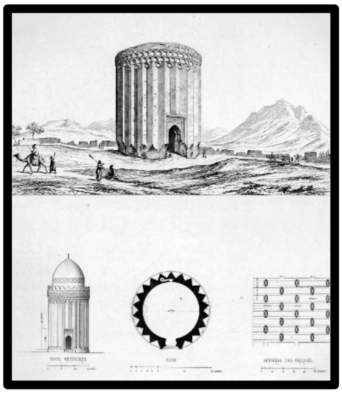

Figure 1.2. Plan and section of the Tughrul Tower [7]

Figure 1.3. Tower of Jericho [8]

Brief history of masonry

Masonry, “the art and craft of building and fabricating in stone, clay, brick or concrete block” was conceived in early times when men tried to replicate their natural caves artificially with piles of stone. When it was 400 B.C. some of the most extraordinary creations of mankind were already constructed with masonry, the pyramids [9].

When it comes to the material of masonry structures, it was more like “use what you have”. Egypt was located near the River Nile. The River Nile had hills located along it and these hills were rich in limestone, alabaster, granite, sandstone, and basalt so stone blocks made from these materials were used. For civilizations in western Asia however, the conditions were different. They had insufficient stone resources, but they had rich clay deposits. They gave shape to the clay and dried it under the sun, giving birth to the material “brick” (See Figure 1.5).

Figure 1.4. Limestone blocks from the Pyramid of Khafre [10]

Figure 1.5. Mud bricks [11]

Until the Romans invented “concrete”, stone and brick were the primary materials of masonry for a long time. Discovering the true power of cement by using pozzolanic tuff, and volcanic ash, and mixing it with lime, water, and fragments of bricks and stones, creating an epochal binding material, along with the usage of concrete created a more economical and faster construction process.

Taking advantage of the concrete and pozzolanic cement, Romans invented the arch form, a form that bypassed the lack of tensile strength in masonry material by keeping the whole structure in compression, therefore breaking a lot of limitations for the craft of masonry. After this, an architectural revolution was triggered, and Romans started to build big arched bridges and aqueducts. By arranging arches in a row, they invented the barrel vault, a structural element that was used for roofing many buildings. By intersecting the barrel vaults, they formed the groin vault, which was used in a variety of structures including public baths. In the 6^{th} century, arches evolved into pointed arches. With pointed arches, flying buttresses, smaller-sized stones, and thick mortar joints, the creation of more elastic and slender structures was made possible.

Developments in masonry continued throughout the Gothic Era and the art of spanning space with masonry was nearly perfected. However, with the arrival of the truss in the 16th century, along with the developments in structural analysis and the usage of steel and reinforced concrete in the 17^{th} and 19^{th} centuries, the need for masonry in spanning spaces decreased. With the invention of Portland cement by Joseph Aspin in 1824, masonry was partially revived and returned to its ancient role of filling gaps, separating areas, and cladding [9].

Figure 1.6. Roman concrete [12]

Figure 1.7. Puente romano de Mérida, a Roman bridge constructed from arches in 117 A.D. [13]

Behavior of masonry

Expectations from a masonry building are the same as any other building, these are three qualities that have not been changed for more than 2000 years: “firmitas, utilitas, and venustas” [14]. Firmitas is the strength and stiffness of the building to preserve its structural integrity. Utilitas is the adequacy of a building’s plan and space arrangement to fulfill its functions. Finally, the venustas stands for aesthetics of the building as a reference to the Roman Goddess of Beauty Venus. Out of these three qualities, the one with the top priority is the strength and stiffness to preserve structural integrity, as it would most likely be impossible for a building to conserve its utility and beauty without being intact and the biggest threat to a masonry structure is earthquakes.

Before getting tested against external forces, a structure must function under its weight properly. For this, external and internal forces must be in equilibrium. External forces can be static or dynamic. Static forces that can affect a structure are gravity, dead loads, live loads, snow loads, wind loads, and soil pressure while dynamic forces are seismic forces, impact loads, vibrations, and blast effects. Among all these external loads, forces induced due to earthquakes are generally the only ones that are given importance during the preservation of historical structures. Internal forces are the forces that are developed within the structural elements to equalize the external forces. These forces are axial forces (tension and compression), shear forces, bending moments, and torsional forces. In a structure, all forces must eventually be transmitted to the ground or other supportive structures to preserve structural integrity. This occurs by each structural element transmitting its self-weight along with the loads it is already transferring to the other structural element or elements that are responsible for carrying it, which take place through joints. Events occurring during this chain reaction can be described as the "behavior" of the structural element and the combination of behavior of each structural element creates the structural system or the structural behavior of the building. In this complex system, the behavior of each structural element is linked to each other. This means an "irregularity" in behavior of any structural element can cause a chain reaction that disrupts the entire structural system.

Irregularities in structural elements' behavior are described as "failure" which means that structural element is and will behave unusually. This is caused by a stress that conceives a force that exceeds limit of the structural element which means that it can not be equalized by the internal forces. Disturbance in this equality causes "micro-cracks" to form in the structural member. The stones and bricks that are used in the construction of masonry usually already contain micro-cracks and these cracks can grow under the forces that have been applied to them. The cracks that are conceived this way will not grow if the load is stable and generally does not pose a threat to structural integrity of the building. However, great lateral forces generated due to earthquakes can form "macro-cracks" by loading structural elements with forces that exceed their limits, which are unstable and will grow uncontrollably until the force decreases enough. Macro-cracks can cause major instabilities by forming new joints in the structural element, which usually results in failure and thus collapse of the structure.

To prevent a structural system from failing, its behavior against internal and external forces must known well. In the age of masonry, a time when mankind did not have access to the simulations that could analyze behavior of complex structures under their own weights and external forces, strategies against earthquakes in construction technology were mainly developed by trial and error and were passed down from generation to generation through the master-apprentice relationship. However, these severe limitations could not stop masonry from evolving even in ancient times. Even before classical and numerical analyses and laboratory tests, builders knew that masonry was a building system with very small tensile strength and very large compressive strength. This pushed engineers and architects to a corner where they either had to develop new structural elements that would form structural skeletons which only contain compression media, or they had to enhance tensile strength of their structural elements by developing better masonry units or mortars. A breakthrough in this sense came from the Romans, the form of "arch".

Figure 1.8. Masonry arch [15]

Figure 1.9. Masonry vault, Saint-Séverin Church, Paris, France [16]

Figure 1.10. Masonry dome, Santa Maria del Fiore, Florence, Italy [17]

The invention of the arch led the way to the birth of more complex forms such as vaults and domes, which are also the progenitors of modern wide-span structures. With these two forms, masonry was finally a reliable and complete way of building wide-span structures.

Innovations in these span-passing technologies were followed by the invention of buttresses and improvements in masonry units and mortars. This resulted in more slender and strong structures that possessed almost infinite compressive strength and made possible the erection of big and elegant historical masonry buildings that are adored even today. Even though these innovative masonry buildings were very good at bearing the compressive strengths that are conceived inside the building, they were, even after all these enhancements, still endangered greatly by the earthquakes. Because the enhancements made to the masonry were solely made to increase the compressive strength and the minor improvements to the masonry unit did not provide enough elasticity and tensile strength to the structure to withstand the tensile stresses that developed inside the structural elements due to earthquakes.

Figure 1.11. Habib-i Neccar Mosque, a masonry structure failed due to earthquakes (inside view) [18]

Figure 1.12. Habib-i Neccar Mosque, a masonry structure failed due to earthquakes (top view) [18]

Landmarks/breakpoints of Finite Element Method

Finite Element Method (FEM), as a technique, utilizes many aspects of mathematics and continuum mechanics. This makes the definition of a starting point for FEM impossible. The "mathematical modeling of continua by discrete elements" aspect of the FEM is linked with the works of, Hrennikoff and McHenry. In these works, McHenry and Hrennikoff utilized frame element assemblages to simulate plane stress systems. The procedures arising from this research were tried to be adapted to analyze wing of a delta airplane in 1952, by Ray W. Clough, for a project that he had been hired by Boeing Airplane Company. The techniques remained insufficient to solve complex configurations of shell elements used to represent the wings of plane, however, it was also this problem, which would lead to the invention of FEM [19-21].

Another significant development for the FEM was the matrix generalization of the structural theory. With this approach, applications of matrix and linear algebra concepts were adopted in the modeling and analysis of structures. This conceived the concept of stiffness matrices. In structural analyses that were carried out with this method, every structural element had its stiffness matrix. These stiffness matrices described the relation between forces and displacements for each structural element individually and the combination of the stiffness matrix of each element yields the global matrix of the structure. Obtaining the matrix of the structure makes it possible to analyze and predict its behavior systematically. The earliest known research related to this aspect of the FEM was done by Falkenheimer and Langefors (cited in Clough, 1980:362) [21]. However, the work that brought clarity to the whole process was the series of articles that were published in Aircraft Engineering by Argyris and Kelsey [22]. In these articles, classical structural analysis concepts were adapted to aircraft wings. This was also evidence that classical structural analysis concepts can be used for not only trusses and beams, but any type of structural element or assembly.

However, the true focus of finite element method is the process of discretization, not the process that analyzes the system after the identification and evaluation of discrete elements. To open the subject, in the finite element method, it is assumed that within each element of the discretized, the distribution of strain and stress is not uniform and can vary. This results elements behavior in analysis to be more similar with its real-world behavior. Moreover, unlike some other methods in FEM, continuum remains unchanged. Each finite element is considered a piece of the continuous structure, and stress-strain fields are used to display the behavior of structure in detail without disturbing the continuum. From the perspective of Ray W. Clough, the first steps of the computerized FEM were taken during his employment in 1953 when he was hired by Boeing Airplane Company. The result of the company's first experiment to analyze wing of a delta airplane was considered lacking. M. J. Turner, the head of the Structural Dynamics Unit in the company, suggested to discrete the membrane of the wing into triangular shell elements. To calculate their stiffness properties, constant normal and shear stress states are assumed within each element, and Castigliano's theorem was applied. Then, according to the method, to calculate the stiffness matrix of the whole wing (global matrix), stiffness matrix of each element was (including the frame elements within the wing, like stringers and spars) combined. Thus, in the summer of 1953, the concept of the FEM was truly defined. However, Ray W. Clough comments that the acceptability of the FEM's complete utility was delayed due to its first experiment's objective being limited to the stiffness and deflection analysis only. In addition, there was also the fact that the processing capacity of the computers of that time greatly hindered the advancements and the pace of the work done with computerized FEM. One of the biggest advancements in the computerized FEM was the successful application of the triangular elements to the FEM by Turner *et al.* [23]. This was such a *game changer* for FEM that many authorities considered it a *"quantum leap"* and a *landmark*.

Through the developments due to these and many contributions of other works, by the early 1950s, FEM was already a reliable approach to solving real engineering problems in civil engineering and aeronautical engineering departments. FEM was a standard to test the designs and study their behavior against the forces they encounter most likely, like testing the planes against the stresses they withstand in flight or assessing resistance of the buildings to withstand earthquakes, nuclear blasts, or other forces. Thus, the path to designing cost-efficient and durable products of many kinds was opened with the help of FEM [21, 24].

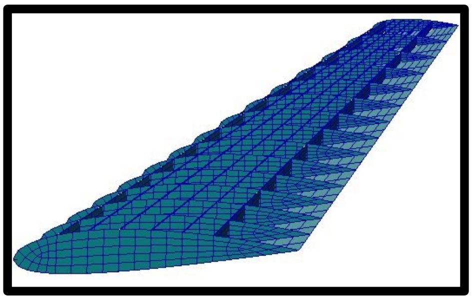

Figure 1.13. Finite element model of a plane wing [25]

Development of FEM software

The development of FEM software started as early as the 1960s. The starting point of this development was the Symbolic Matrix Interpretive System, SMIS. SMIS was a structural mechanics computing code created by E.L. Wilson and R.W. Clough, a code created to sew the connection between the matrix method and hand calculating in structure mechanics while unintentionally sparking the onset of FEM software like SAP2000 and many others. With the help of a large market along with accessible FEM tutorials and textbooks, computerized FEM became viable quickly for academia and industry globally [24].

Contributions of FEM in the aspect of earthquake engineering

Application of FEM to structural dynamics is considered revolutionary for structural engineering. With the works of Clough, FEM also became viable for problems other than static stress analysis, thus making it available for vibration problems. This essentially commenced the discipline of Earthquake Engineering, which is involved in not only designing structures that can resist the forces that occur inside them due to earthquakes but also making the structures that already exist stronger against earthquakes, like "historical masonry structures" while making the Clough the first researcher that utilized the FEM in solving earthquake engineering problems. The work of Clough named "On the importance of higher modes of vibration in earthquake response of a tall building" (Cited in Sullivan, 2010, p. 677) is generally considered the starting point of this field. Another breakpoint was his graduate-level instruction named "Dynamics of Structures and Earthquake Engineering" (Cited in Sullivan, 2010, p. 677), because, with this instruction, the earthquake phenomenon was treated as quantifiable forcing functions [26].

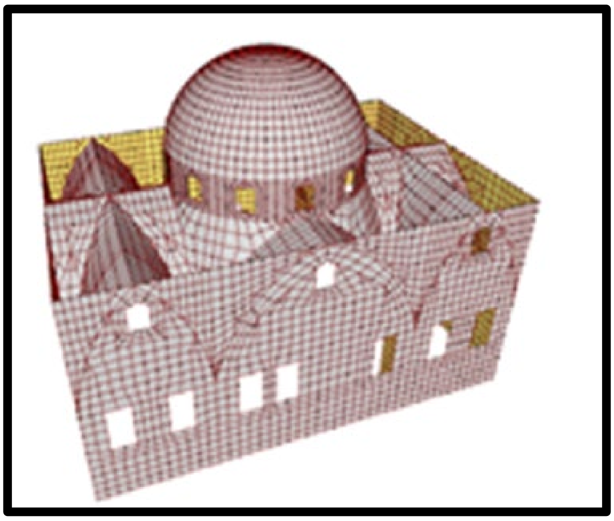

Figure 1.14. Finite element model of Habib-i Neccar Mosque

Defining the problem

Considering its popularity and the amount of research put into it, it was inevitable that FEM would be involved in the analysis of historical masonry structures. However, it immediately came with questions concerning its adequacy. Despite this, researchers produced many simulations and reports to answer these questions, confirming adequacy of FEM for analyzing historical masonry buildings. FEM, as in every other engineering problem, continued benefiting from innovations in technology in this field too.

Today, modern analyses that are conducted in historical masonry buildings and use FEM mostly rely on the software in discretization and meshing process. Algorithms of these software are usually designed to create a mesh of any given geometry and apply the same process to every part of the body. While this may be alluring at first look because of "automation", it is not always optimal. Finite Element Modeling presents many types of elements to the user. The most accurate usage of these elements in necessary conditions will enhance the clarity of the model and therefore results. Even if the output is appropriate, relying solely on a mesh generation like that can complicate the model unnecessarily and make interpretation of the results more difficult for the disciplines who work with the data of it in the conservation process of the historical masonry structures, such as architects with this subject [27, 28].

Purpose and importance of the research

This study is aimed to propose a simplified finite element modeling process that uses different types of elements provided by FEM in necessary conditions which results in a simpler yet accurate model, whose results would be easier to interpret. A model like this would present real-world behavior of the structure better and make the interpretation and the process of understanding real-world behavior of the structure easier for the conservators.

Limitations

In this research, due to its cultural significance and symmetric structural system, Şehzade Mehmet Mosque, located in Fatih, İstanbul, was chosen as a case study. And as in most cases for analyzing historical masonry structures, while constructing the finite element model of the Şehzade Mehmet Mosque, assumptions needed to be made due to lack of information on building materials of the structure and inside of the structural components.

2. MATERIAL ASPECT OF THE MASONRY

As stated before in the “brief history of the masonry” section, masonry was merely started with the urge of man to imitate the caves in which they lived. However, due to limitations of their environmental awareness, only a handful of materials were available, and because of this, in early times of mankind, masonry constructions were made primarily from local ingredients. This diversity in resources also resulted in a variety of construction techniques due to the strength of masonry depending on mechanical properties of its material thus triggering development from many branches at the same time.

These ingredients used for masonry as “masonry units” were first used as uncut and crude. Shaped masonry units like bricks and mud blocks were used much later. As we get closer to today, innovative units like hollow clay bricks, clay tiles, and manufactured concrete blocks were started to be used as a masonry unit.

Although the strength of the masonry varies with the mechanical properties of the masonry unit and the mortar, it also depends on their incorporated behavior. This makes not only the individual properties of the stone, brick, or mortar important but also the properties of the unit-mortar interface, increasing the number of parameters of masonry strength [29, 30].

Stone

Stone was one of the oldest materials available to mankind almost everywhere. Therefore, it was widely used throughout most of the early workmanship of any kind, including masonry. Stone also has crushing compressive strength, varying from 3 N/mm^2 to 60 N/mm^2 which makes it a perfectly suitable material for a system that works mostly with compression such as masonry, because even limestone, which has the least compressive strength among the masonry (3.6 N/mm^2), can be used in a single-story masonry structure, which hardly gives a stress of 0.15 N/mm^2. This value depends on the type of the stone, formation conditions, and mineral structure. For example, volcanic rocks generally have more compressive strength, hardness, and atmospheric resistance while sedimentary rocks have less. However, in general, rocks have weak tensile strength that seldom exceeds 10% of their compressive strength.

Figure 2.1. Machu Picchu: Inca dwellings, Peru [31]

Figure 2.2. Stone Henge, England [32]

For a stone to be adequate in masonry, first, it must have sufficient compressive strength, and resist atmospheric conditions and freezing while being able to be processed easily and adhering well to the mortar. Another issue that should be noted is that even though they have great compressive strength, some types of rocks disperse and soften when they get wet, making them unusable for masonry construction. This problem can be avoided either by using experience from the past to learn the quarry that contains the hardest stones or by observing the outside area near the quarry to look for unspoiled rocks, which is an indicator for the atmospheric resistance of the rock. Cracked and veined rocks like granite, sandstone, and rocks that contain clay and mica can be fractured in a very short time if they are frozen and dissolved rapidly. This property of rocks can be tested in the laboratory by replicating the freeze-dissolve process 25 times, cooling the rock down to -15^{0}C then heating it to +15 ^{0}C then measuring its compressive strength. However, the non-contemporary and practical method of testing this was yet again observing the outer area of the quarry and looking for rocks with moss on them, which is considered a sign of frost resistance. One last feature to look for in the usable stones for masonry is when the stone is cut its section must look scaly. Stones with scaly inner sections are usually hard but they do not adhere with the mortar very well, making them inadequate for masonry [29, 33, 34].

Figure 2.3. Granite [35]

Figure 2.4. Basalt [36]

Figure 2.5. Marble [37]

<u>Structure of the stone</u>

The structure of a stone can be layered or unlayered. Stones used for masonry in the superstructure must be shaped easily, therefore layered stones such as limestone and sandstone are preferred. Because they can not be shaped easily, unlayered stones like granite and basalt are usually used in foundation work.

Texture of the stone

Because they look more homogeneous and aesthetic, fine-grained stones are usually used for cladding. Stones like this are also stronger and more durable.

Figure 2.6. Soapstone [38]

Figure 2.7. Serpentine [39]

Specific weight of the stone

Specific weight increases the strength of stone, so the heavier stones are stronger. The specific weight of the stone is such a defining factor for construction that stones with a specific weight less than 2,4 are generally not considered suitable for building.

Strength of the stone

Stone as a material possesses large compressive strength along with weak tensile and shear strength, both being approximately %10 of the compressive strength. However, since the main strength type that masonry depends on is compression, the stone is a quite reliable material. Along with these, shrinkage, workability, hardness, and durability are also properties that affect the stone in the construction of masonry [40].

Table 2.1. Mechanical properties of the stones [34]

Type of stone	Unit weight t/m^3	Compressive strength MPa	Shear strength MPa	Tensile strength MPa	Modulus of Elasticity MPa
Quartz		29,4-68,6	13,7-32,3	3,9- 6,8	14 700- 68 600
Marbel	2,73	24,5- 63,7	8,8- 44,1	0,9- 1,4	24 500- 68 600
Limestone	2,57	17,6-63,7	5,8-19,6	1,9-5,8	9 800-53 900
Sandstone	2,24	4,9-29,4	1,9-9,8	1,9-3,9	12 700-49 000
Granite	2,65	9,8-29,4	2,9-9,8	2,9-3,9	14 700-53 900
Serpentine		6,8-29,4	1,9-9,8	5,8-10,7	22 500-44 100

Brick

Brick is a ceramic, inorganic, non-metallic material that contains silicates and metal oxides. They can be processed at high temperatures or left to dry under the sun like in ancient times. Like stones, brick is one of the main materials of masonry and has been used for centuries. Its reason for fitting to the masonry is similar to the stone, great compressive strength, easy to shape, raw material found in great quantities, and powerful binding with mortar. It also has the same disadvantages as stone. While the compressive strength of the typical brick varies from 8 N/mm^2 to 31 N/mm^2, its tensile strength varies between 5% and 10% of this value. Brick also has a near-linear stress-strain curve, due to its mate. These values can change, depending on the quality of clay in the brick and the heating process brittleness [29, 41].

Figure 2.8. Ancient adobe brick [42]

Figure 2.9. Mud brick [43]

Porosity of the brick

Depending on the quality of the brick that is wanted to be produced, firing heat of the brick can vary between 800-1200 °C and this process generally requires a significant amount of fuel. In the firing process of the brick, many fractures and holes form inside the brick. These fractures and holes are identified as “porosity”. Porosity is defined by the ratio of space volume to exterior volume of the brick (Porosity = Ratio of space volume / Exterior volume of the brick) and expressed as percentage (%). Porosity greatly effects the mechanical properties of the brick and should be less than %25 for adequate compressive strength. For the bricks that are exposed to the weather and not covered with plaster, porosity must not exceed %15.

Compressive strength of the brick

Compressive strength, like in any other material in masonry, is the most important attribute of brick. Compressive strength of the brick is affected by its porosity, firing heat, production method, soil type, and direction of the load. If it is a hollow brick, it is also affected by the holes' shape and positioning.

Frost resistance of the bricks

If a brick's holes are filled with water, there is a probability that (depending on the amount of water in the hole) when water freezes, it can fracture and destroy the brick eventually. If the brick is partially saturated, due to frozen water compressing the air inside the brick, the probability of fracture decreases. For a non-plaster covered brick to be frost-resistant, 40% of its voids must be filled with air and it must have a compressive strength of more than 49 MPa.

Water absorption of the bricks

Bricks with high water absorption rates can absorb the water from mortar and cause it to harden before reaching its full strength. To prevent this, bricks must be soaked wet enough to not absorb water from mortar.

Bricks in historical buildings

Bricks that are used in historical buildings are very different from contemporary fabricated bricks. The bricks that are used in historical buildings were produced in primitive ways, resemble blend bricks, and have much worse quality. They also are fired in lower heats and, therefore generally have less compressive strength. They have a thickness of around 5cm and they can be laid laterally in stone masonry walls and used as a girder to regulate the load distribution or can be used vertically in curvilinear surfaces like vaults, pendentives, and domes [23].

Figure 2.10. Ancient City of Sardes, Turkey [44]

Figure 2.11. Babylon Ruins, Iraq [45]

Mortar

The primary function of the mortar is to act as a joint between the units and hold them together. It also creates a continuum along the units and stabilizes the masonry element and compensates for the deformations due to thermal expansion and shrinkage. Before cement, the primary materials used for mortar were sand and lime. Basically, two requirements to produce a basic mortar are a cementitious material and aggregates. Lime in the lime-sand mortar emits carbon dioxide from the air, becomes calcium carbonate, and hardens. This hardening process occurs very slowly, however, so does the masonry process. This ensures the harmony between the hardening, strengthening, increase in masonry's weight, and the erection of the masonry. Eventually, with the use of cement, mortars' compressive strength increased drastically along with their hardening speed, however, the speed of mortar's hardening can be slowed down by adding lime to the mix.

Another crucial function of the mortar is to smoothen the *mortar-unit interface*. Because of their rough surfaces, the faces of two bricks that are on top of each other do not completely touch each other. This limitation in the contact area causes increased compression and decreased vertical weight carrying capacity. Applying mortar between two faces smoothens the contact area and enables force transfer from all areas in the brick-mortar interface. Mortar also has greater tensile strength than bricks, thus, applying mortar to a masonry wall also increases the wall's strength against the tension that occurs due to lateral forces.

Due to bricks used in the past being smaller, the ratio of the mortar's volume to the volume of the entire wall was between %30-%40. With the usage of larger bricks, this ratio was lowered to %20. The moisture shift between the brick and mortar and the deprivation of the mortar also affects its properties. Mortar's mechanical properties are influenced by so many different factors that the properties of mortar vary even in the masonry wall itself.

Relation between the mortar mix and mortar strength

Mortar must be able to be shaped easily without falling apart and easily workable. These traits of mortar cannot be quantified scientifically and are usually evaluated by the bricklayer's instincts and experience, however, increasing the quality of the aggregate and amount of water generally increases the workability of mortar. Workability also affects the construction process. Workability can be considered one of the most important assets in masonry because it directly influences cfartsmanship, which directly affects the mechanical properties of the masonry structure.

Another important feature of the mortar is that it must not lose its workability after being laid on top of a brick that absorbs water. To prevent this, mortars with more clay and lime in their mix, which can make mortar more absorbent than bricks, can be used, however, bricklaying with these kinds of mortars is usually harder.

If a mortar's mix contains big grains of sand, it usually has more strength than a mortar with smaller grains of sand. Increasing a mortar's resistance also increases its frost resistance. Increasing the particle size, strength of the aggregate, or cement/water ratio increases the mortar's strength.

Durability of the mortar

The durability of the mortar is measured by its ability to withstand exposure and the atmospheric conditions.

Compressive strength of the mortar

The compressive strength of the mortar depends on the amount of cementitious material and the workable amount of water in the mix.

Modulus of elasticity

The modulus of elasticity of the mortar is connected to its compressive strength. The bond between the mortar and the masonry units is also an important asset for the masonry and this bond is directly affected by the quality of the mortar, its properties, and the moisture of the unit at the time of laying. The mortar-unit bond is also considered the weak link of the masonry under loads [29, 34, 46, 47, 48].

3. STRUCTURAL ELEMENTS IN MASONRY: TYPES, DESIGN PRINCIPALS, AND BEHAVIOR

3.1. Design of Masonry Walls

The most important aspect when designing and building masonry walls is the understanding of the unique behavior of mortar and masonry unit. After the masonry unit and the mortar are composed and formed a wall, material properties of the wall will not be the same either as the unit or the mortar. Because even though properties of the mortar and masonry unit severly affect the behavior of the masonry wall, their influence is not 100%. For example, when the tensile strength of the mortar that is used in the wall is doubled, the tensile strength of the wall will not be increased with the same multiplier. The coefficient of how much tweaks in the mortar or units will affect the strength of the wall can be calculated and formulized through contemporary experiments today, however, when it comes to the ages of ancient masonry, the only way to figure out these effects were experience and trial and error.

The primary function of the masonry walls in the structure is to bear the vertical loads, this naturally makes the compressive strength of the wall the main concern in masonry wall design. However, situations like earthquakes, wind loads, or bearing lateral load of another element due to failure can result in a masonry wall facing lateral loads. This puts the shear and tensile strength into consideration and makes it a limiting factor in the design due to masonry’s huge disadvantages in the tensile and shear strength department.

In the design of masonry walls, formation of direct tension in the walls is nearly impossible due to masonry’s nature. Most of the tensile stresses in the walls occur due to flexure and create a problem. In flexure, the slenderness ratio of the wall and the eccentricity of the load that applies affect the magnitude of the shear and the tensile stress. Compressive failures are rather rare in masonry walls; however, they can happen in short and thick walls due to extreme compressive stresses. Slender walls on the other hand generally fail due to the eccentricity of the loads.

Even though tensile strength is crucial for masonry’s resistance against earthquakes, masonry wall’s primary function is to support the structure by carrying other structural elements' weight and transferring them to the soil along with the forces they carry. This makes compressive strength responsible for most of the masonry wall's strength and its most important feature.

In the design of masonry walls, it is important to evaluate the compressive strength that is needed from the wall. This depends on the eccentricity of the load that will be applied to the wall and the slenderness ratio of the wall, along with the used masonry unit's strength. This evaluation process is not simple on a theoretical level because of the complexity of interactions between the masonry elements and their inhomogeneity. Detailed evaluation on a virtual platform of masonry structures is possible with structural analysis software, however, making these evaluations in the age of masonry with limited technology was a much harder task. This fact further emphasizes the beauty and ingenuity behind the historical masonry structures [29, 49].

3.2. Mortar

In a masonry wall, the mortar constitutes 5% to 20% of the wall. However, the influence of mortar on a masonry wall's strength is much more than this ratio. The quality of mortar like workability and strength must be adequate to develop proper adhesion to the units. Mortar should be set without excessive shrinkage, otherwise, cracks can form which reduces the strength of the wall. Another role of the mortar in a masonry wall is to increase the flexibility of the wall up to a reliable degree. Considering all these factors, the importance of the mortar on masonry wall's strength, despite the small proportion it occupies, cannot be overstated. However, on the contrary, using a mortar stronger than necessary can also cause problems [34].

3.3. Masonry Brick Walls

The masonry units used in masonry walls are selected according to their specific characteristics as well as their type. These features are surface texture, unit weight, porosity, water absorption rate, compressive strength, tensile strength, heat and moisture movement for bricks. However, some properties are considered for aesthetic, comfort, and safety issues like thermal conductivity, fire resistance, and color [34].

3.3.1. Effect of joint thickness on compressive strength

The ratio of the thickness of the mortar joints to the height and thickness of the brick affects the compressive strength of the masonry wall. Proper configuration of mortar joints plays an important role in masonry strength because it is crucial for accurate load transfer between units and providing stability. Increasing the joint thickness generally decreases the compressive strength of the masonry wall. This is caused by the increase of the mortar’s ratio in the wall. Because mortar’s compressive strength is lesser than the brick, increasing the mortar’s influence on the wall decreases the wall's compressive strength. Thicker mortar joints also increase the risk of shear failure in the mortar joints, especially in situations where the mortar is weaker than the brick. This type of failure occurs when the mortar is insufficient to transfer the loads between adjacent bricks [34].

3.3.2. Influence of relation between brick and mortar’s strength on masonry wall’s strength

Compressive strength of a masonry brick wall can practically be defined as a percentage of its masonry unit’s strength. Compressive strength of a masonry brick wall generally varies between 25%-50% of its masonry unit’s compressive strength. As mentioned before, increasing the unit’s strength by a factor will not increase the wall’s strength by the same factor. This goes the same for the mortar too. Multiplying the mortar’s compressive strength several times only increases the brick wall’s compressive strength by 25%-30% percent. If the objective is to maximize the increase in the masonry wall’s strength, a proper ratio between the unit and the mortar’s strength must be ensured. Research on this issue shows that using a mortar that has more than %70-%80 of the brick’s compressive strength limits its contribution to the wall’s strength [34].

3.3.3. Compressive failure in masonry brick walls

Axial compressive failure in masonry brick walls formed by vertical splits resulting from horizontal tensile stresses. This is due to the different strain properties of brick and mortar. In masonry walls, usually, bricks are more rigid than mortar, so under axial compression, mortar tends to expand horizontally more than brick because it has a higher Poisson ratio. Even though the bond between brick and mortar can compensate for the consequences of this difference up to a degree. When the tensile stresses in the brick exceed its limit due to the compression in the mortar, they crack.

Excessive vertical loads that exceed bricks' compressive strength can also cause failure in masonry brick walls. In slender walls, excessive axial compression can make the wall undergo buckling and instability, eventually causing major lateral displacement and bending, which results in failure [34].

3.3.4. Shear-tensile strength of masonry brick walls

Shear-tensile strength of a masonry brick wall refers to the wall’s resistance shear-tensile stress combination. The shear-tensile strength of a wall is useful when the structure is subjected to loads that carry both shear and tensile stresses simultaneously, such as earthquakes.

3.3.5. Factors that affect shear-tensile strength of the masonry brick wall

The shear-tensile strength of the wall increases as the mortar’s shear-tensile strength and the strength of the bond between the brick and the mortar increase. If the brick used in the wall has vertical holes, its rate also affects shear-tensile strength.

Before the 1970s, there were some bricks produced with cavities on their upper surface. These cavities were named “mortar pockets” and they acted as wedges while the wall was being built. These pockets increased the shear strength of the wall however, no research still quantifies its contribution to the wall’s shear strength. Research made on the masonry brick walls also shows that the compressive strength that the wall is subjected to increases its shear strength while increasing the vertical holes in the bricks decreases it.

<u>Conditions that affect masonry elements after the settlement</u>

Masonry structures, as experienced by the fact that they have survived for centuries, structures are very durable. However, their long lifespan makes them vulnerable to long-term outer effects too. Chemical reactions that occur over time can damage masonry, decrease its strength, and “durability”, and cause additional stress and strain. For masonry or any other structure, durability can be defined as "remaining serviceable for an acceptable amount of time without excessive or unexpected maintenance". This is a crucial aspect of historical masonry structures, because historical masonry structures, as explained in the introduction, are invaluable assets for mankind, and the "primary objective" in their "preservation" is to maintain them without excessive or unexpected "restorations". Restoration means "the process of returning something to its earlier good condition" and this process can not be perfected on historical buildings because even if the material is repaired to its previous condition, the value it had attained can not be recovered. This requires minimizing the need for restoration or retrofitting in the first place to prevent transforming historical buildings into "Theseus' ship-like replicates" [49, 50].

Carbonation

Carbon dioxide (CO_2) in the atmosphere gradually enters a chemical reaction with the alkaline components in cement-based materials of masonry, which is calcium hydroxide ($Ca(OH)_2$), like concrete or mortar. When carbon dioxide is absorbed by the material it reacts with the moisture content in the material, which is water (H_2O), and forms carbonic acid (H_2CO_3). Carbonic acid then reacts with the alkaline components and produces calcium carbonate ($CaCO_3$). Calcium carbonate is solid and can form in the pores or on the surface of the material. This changes the material's properties and can affect its long-term performance and durability [50].

Poor soil conditions

Poor foundation conditions in masonry can lead to issues in stability, safety, and the performance of the entire structure. Differential settlements on the foundation where different parts of the structure settle at different rates can result in cracking in walls and other structural deformations. These cracks form due to excessive movement of the structural member and can alter structural integrity. Also, water can infiltrate from these cracks and can weaken the structure over time by chemical reactions or expanding in cold weather. Soil movement or settlement is another factor that can heavily affect foundation conditions and can put excessive stress on load-bearing elements, this is a major concern for any type of structure and can lead to a considerable amount of structural damage [49].

Frost action

Frost action is another phenomenon that can affect masonry's durability. Frost action affects masonry buildings, particularly in regions with freezing temperatures. The phenomenon of frost action refers to the expansion-contraction movement of the water in freeze-thaw situations. If this process is cyclical, it can cause damage to masonry. As stated in the inadequate foundation conditions, water infiltration in the structure is always a concern. Because water expands on freezing. This means that when water fills the pores of masonry material and when the temperature drops, water inside the pores freezes, expands, and cracks the material. Cracking the pore means that it enlarges, so when the process occurs again, more water will be in the pore, and when it freezes it will expand the crack even more. This creates a *snowball effect* and as time passes, the cracks that occur due to frost action become endangering to the structural integrity of the masonry [49].

3.4. Masonry Stone Walls

Stone walls are rarely used in masonry structures that are constructed today. However, most of the the historical masonry structures contain masonry stone walls. As in the scope of this thesis, these structures are vulnerable to earthquakes and to assess them properly, which is crucial for increasing earthquake resistance, their mechanical properties and behavior under loads must known well [34, 51, 52, 53].

3.4.1. Types of masonry stone walls

To predict the behavior of historical masonry walls their type/properties must known. For brick masonry walls, their units are generally uniform in size and have a regular geometry. In stone masonry walls, however, the irregular patterns of masonry can even be arbitrary, and the plane created by mortar and stone is rather discontinuous. Masonry units in stone walls are not uniform in shape and size. If we characterize a masonry brick wall as isotropic or orthotropic, we can also characterize a masonry stone wall as amorphous. A row of bricks can be inserted into stone masonry walls to compensate for amorphousness and homogenize the vertical weight distribution [29].

Rubble masonry walls

Rubble masonry walls are made of irregularly shaped stones that are generally uncut or minimally processed. Stones in this type of wall are placed together regardless of uniformity and the gaps between them are filled with small stones or mortar. Rubble masonry walls are generally used in situations where the possibilities of shaping the stone are limited. They have good resistance against lateral loads due to interlocking surfaces created by irregular arrangement of stones. However, they have less structural integrity compared to more well-constructed walls and the unconventional nature of wall can cause uneven settlement and unexpected movements after construction [29].

Figure 3.1. Rubble masonry, Sacsayhuaman, Cusco, Peru [54]

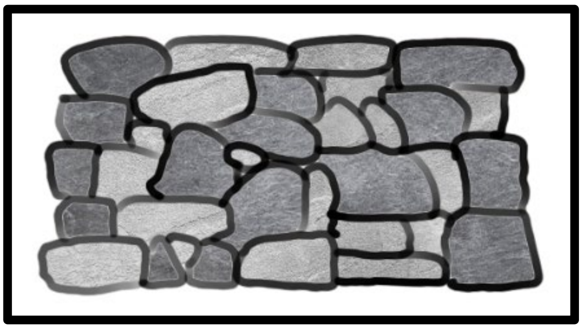

Figure 3.2. Rubble masonry exterior view

Ashlar masonry walls

Ashlar masonry walls contain cut stones that are arranged in a way to minimize the joint size. This type of wall is more visually appealing than rubble masonry and it can be coursed or randomly depending on the uniformity of the bricks' height in the same row. Ashlar masonry walls bear great structural integrity and stability due to their precise shaping, but they can be expensive due to craftsmanship requirements [29].

Figure 3.3. Ashlar masonry wall [55]

Figure 3.4. Ashlar masonry wall exterior view

Coursed rubble masonry walls

Coursed rubble masonry walls can be defined as a hybrid of ashlar and rubble masonry walls. In this type even though the stones in the same course are uniform in height their width is random. This strikes a balance between the natural appearance of rubble and the organized appearance of ashlar. This type of wall provides more stability compared to rubble masonry and can be built faster than ashlar masonry. However, due to varying shapes it can cause uneven settlements [30].

Figure 3.5. Coursed rubble masonry [56]

Figure 3.6. Coursed rubble masonry exterior view

Dry stone masonry walls

Dry stone walls, as the name implies, are constructed without the use of mortar. Instead, in the construction process, stones are arranged to be interlocked as much as possible. To achieve this, stones are precisely selected to increase interlocking. This type of wall relies on friction and the weight of the stones to achieve integrity and stability. It is generally used for landscaping and retaining walls. Dry stone masonry walls are mostly insusceptible to freeze-thaw effects due to lack of mortar. They also are naturally flexible and unaffected by minor ground shifts. However, the lack of mortar limits their structural strength, and settlement and displacement of stones can conceive the need for periodic maintenance [35].

An example of ancient dry masonry: The Great City Zimbabwe

Although there are different theories on the word *Zimbabwe*'s etymology, one theory suggests that the word is derived from the *Dzimba-dza-mabwe*, when the term is translated to English, it means "*large houses of stone*". There are also many dry-stone buildings in the area named The Great City Zimbabwe which are known as *Zimbabwe's*. As can be seen, they were so significant for the culture that they gave their name to the country of Zimbabwe. Builders of Zimbabwe were experts on dry stone masonry and utilized it fully in their constructions. When building dry stone masonry, if the building didn't have much importance, the course of laying was usually arbitrary. Builders also used dry stone masonry to transform hilltops into artificial floors that are flat and adequate to build on. This was usually done when the elevation difference was between 50-200 centimeters. It is a high probability that because a house was going to be built on top of this partially artificial surface the stones at the top were smoothened off to make the floor smoother. Sometimes it was even paved with a *dhaka* to further smoothen it. Dhaka was made by grounding the mud collected from termite mounds and mixed with water. It was set hard upon drying, had a brown-reddish color, and was used for flooring.

In the area the most common building material was granite. To put the granite rocks in the area in adequate shape to build the rock's surface was heated with fire for hours and rapidly cooled. This process enabled the rocks to be split into slabs. Builders had already mastered this process; however, it must have required a lot of wood to produce the needed heat for enough rocks to build a city. Thankfully the granite in Zimbabwe had a *plane of exfoliation* (a term used for the natural splitting paths of rocks) suited to produce neatly shaped units, which must have eased the process quite a bit. When it came to building passages and span the lengths of buildings units that are specifically shaped vertically or horizontally long and rather flat were used along with wood (See Figure 3.10).

The Great City Zimbabwe was built without mortar. The city consisted of many houses, halls, dining places, and offices. It is assumed that the construction and settlement in the area started around 800 A.D. A strong theory on the absence of mortar in the city's masonry is that the city was used only for trading, so there was no risk of war, it was also not exposed to any seismic activity or natural disasters due to its geolocation. So, people didn't see the need for enhancement to their structures by using mortar.

Figure 3.7. Dry stone hallways that lead to different buildings [57]

Figure 3.8. Drystone stairways [57]

Figure 3.9. Drystone wall [57]

Figure 3.10. Example usage of flat stones as beams [57]

Double-leaf masonry walls

A double-leaf masonry wall consists of two separate layers of masonry walls generally made of stone. The outer layer forms the external face of the building while the inner layer forms the internal. The cavity formed between the two layers can be left with a gap or filled. Leaving the cavity empty can contribute to thermal insulation and moisture drainage. If the cavity wants to be filled, mortar is a common option as a filling material. Its primary function is to bond two layers together and increase cohesion, thus the stability between the two walls. It also helps to distribute the loads evenly between the leaves. Using smaller stones or aggregates as a filling contributes to structural integrity and decreases vulnerability to settlement and differential movements.

Regardless of the material used, the filling must possess adequate strength and provide proper cohesion between the layers to bear the loads and maintain the integrity of the wall. If the filling is brittle and weak it will result in failure and structural instability. It also must be compatible with the masonry unit to ensure proper bonding, otherwise, deterioration may occur over time. Filling material's thermal expansion-contraction properties being similar to the units is also beneficial for the wall's performance because uneven expansion-contraction can cause excessive stress. Moisture resistance is also another must-have feature for filling because if the water reaches the filling it can cause mold and decay.

In summary, the properties of the filling material have a significant influence on the double-leaf masonry wall's structural performance. It affects the bond between the layers, load distribution, moisture management, and thermal insulation. Therefore, the choice of the filling material is essential for a durable and stable double-leaf wall, especially in regions where historical masonry structures are exposed to seismic activities. Also like in any type of stone masonry wall interlocking the stones is beneficial to the wall's strength. In the double-leaf masonry wall's case promoting interlocking between two layers increases the wall's strength and prevents displacements.

An issue that needs to be mentioned in the double-leaf masonry walls is generally, the outer layer of the wall is built intentionally with higher compressive strength stones while the inner layer is built sloppier and with weaker stones. Regardless of the material used, modulus of elasticity and compressive strength of the fill is weaker than both layers. Under the vertical loads, while the walls bear most of the load fill compresses more under the load tends to expand more. Expansion of the fill enables more load transfer to the walls, thus increasing the load carrying capacity of the fill with the support of the adjacent walls. However, if lateral and shear forces exceed strength of the fill, it would fracture, thus damaging the bond between the layers and cutting off the lateral support to the outer wall provided by the inner wall. This increases the fragility of the outer wall, and it starts to buckle outwards which can result in failure. This can yet again be prevented by interlocking the stones between the two walls [34].

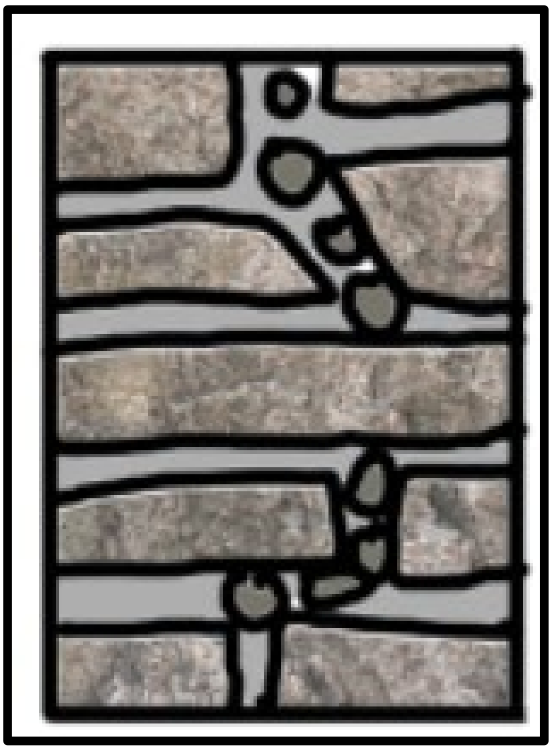

Figure 3.11. Double leaf masonry wall, full connection between leaves

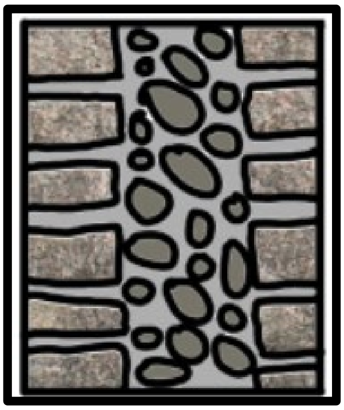

Figure 3.12. Double leaf masonry wall, no connection between leaves

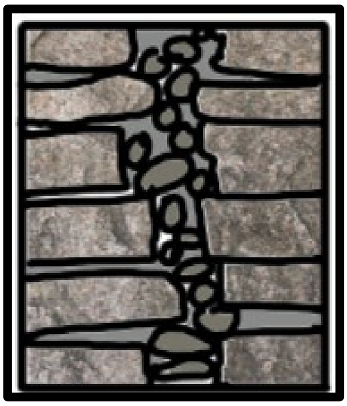

Figure 3.13. Double leaf masonry, connection between leaves with interlocking

3.4.2. Unit weight of the masonry stone walls

The unit weight of stone masonry walls depends on the unit weight of the stone, the unit weight of the mortar, and the ratio of masonry unit-mortar volume. The size of aggregates that are used in the wall affects the amount of mortar used which affects the wall's unit weight. If the aggregates are big, less mortar needs to be used and the cavity inside the wall increases. Decreasing the size of aggregates increases the mortar used while decreasing the cavity.

3.4.3. Classification of masonry stone walls

According to Dusi *et al.* (Cited in reference, pg 99) masonry walls can be classified as "high, medium, and low" depending on their strength. The masonry units of high-strength walls are of the same size and similar shape. There is almost no cavity between the stones and the gaps are filled properly with mortar. It is also constructed with very strong mortar and stones which also have excellent resilience against atmospheric conditions. If it is a double-leaf wall, the stones are interlocked, and they contact on a very large surface. In low strength walls masonry pattern is disordered. The stones contact at a very small point, have cavities between them, and the cavities are not properly filled with mortar. If it is a double-leaf wall it also has many gaps in the fill and inner and outer walls are not interlocked with stones. When it comes to medium strength walls, they contain some characteristics of both high and low strength walls and provide strength on an intermediate level [34].

3.5. Masonry Columns

Columns are basic and crucial structural elements for masonry for providing additional vertical support to the structure and transmitting forces from upper elements like domes, arches, and vaults to foundation. Columns are traditionally constructed using stone due to their durability and compressive strength capacity; however, brick and adobe also can be used in regions where they are more accessible than stone.

Columns are constructed primarily to resist vertical loads through axial compression. Columns' material strength and geometric dimensions greatly influence their capacity to resist vertical loads. Although columns in masonry, and in other construction techniques, are generally considered as structural elements transmitting only axial loads, columns may be subjected to vertical loads due to large bending moments and shear forces, especially in earthquakes. When a column is subjected to bending moment, it conceives both compression and tension inside the column. It is crucial to consider bending moment as both compression and tension in case of earthquakes and wind, where the direction of the force is not known. Stability is another concern for masonry columns which is influenced by the slenderness ratio, effective length, and support conditions. If the conditions are not achieved, columns can buckle and fail under compressive loads due to instability [58-61].

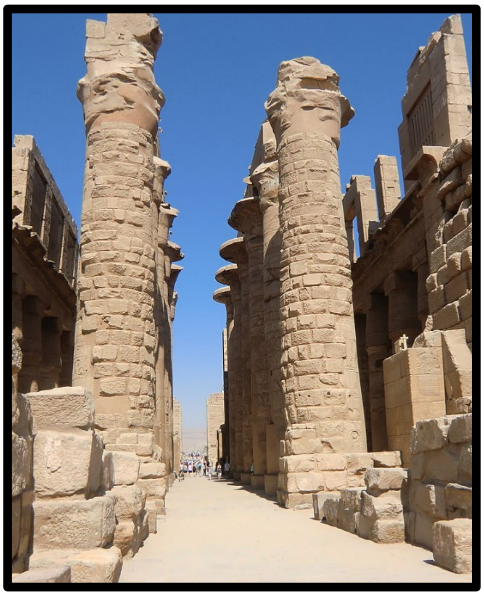

Figure 3.14. Pillars of the Great Hypostyle Hall, Egypt [62]

Figure 3.15. Temple of Segesta, Italy [63]

3.5.1. Behavior of masonry columns under compression

Masonry as a unique material can be considered anisotropic, heterogeneous, and quasi-brittle. Masonry is already a heterogeneous material due to it being an assembly of mortar-unit and things like the composition of mortar and unit, damage, degradation cracks, *etc.* further contribute to its heterogeneity. All these conditions make the mechanical characteristics of masonry more complex and unpredictable.

Generally, failure of a compressing masonry column is followed by vertical cracks due to horizontal tensile stress caused by the mechanical property difference between mortar and the masonry unit. Mortar in historical masonry structures generally has a lower young modulus than the unit and strains greater than the unit laterally when compressed, this lateral strain in the mortar joints causes tensile stress on the masonry units which results in vertical cracks (Explained in more detail in 3.3.3).

3.5.2. Structural performance of masonry columns

As explained before, columns are often subjected to lateral loads. Columns are even more vulnerable to tensile stresses than other masonry structural elements and are not tolerated to them. If one of the main columns forms a joint or fracture, the structure's integrity can fall apart resulting in the collapse of the whole structure. Therefore, in such elements, very large cross-sectional dimensions are required so that no tensile stress can occur in a section more than one-third of the dimension of the cross-section perpendicular to the bending axis. It is understood that columns with massive cross-sections in historical masonry structures are a result of trial and error on masonry structures that collapsed due to earthquakes.

3.6. Masonry Arches

Arches, since the beginning of masonry construction, are fundamental structural elements in most masonry buildings to span long distances and they are known for their large load-bearing capacity, durability, and contribution to the building's aesthetic. The true magnificence of the arches comes from their structural behavior. Due to the load distribution in the arch, the equilibrium of the forces, and its curved shape arches can efficiently transfer loads in compression. Arch's curved geometry allows it to transmit forces to the joints without concentrating at any point. The geometric form of the arches also ensures that the bending moments are very small, or no bending moment occurs [29, 58, 64-67].

Structural and stability of requirements an arch is mostly fulfilled with materials that have high compression strength, which makes the masonry a *perfect fit* as a construction technique. Arches also balance tensile stress with compressive stress, decreasing the risk of failure under tensile loads. All these features make the arch the most suitable structural element for masonry in spanning large openings.

Three main keys to designing a proper masonry arch are selecting appropriate materials, geometry, and construction techniques. For material, the stone or brick that is selected for the arch must possess high compressive strength and low porosity, and for the geometry, the arch's rise, span, and curvature must be decided carefully to obtain optimal structural performance and longevity.

Figure 3.16. Arch of Titus, Italy [68]

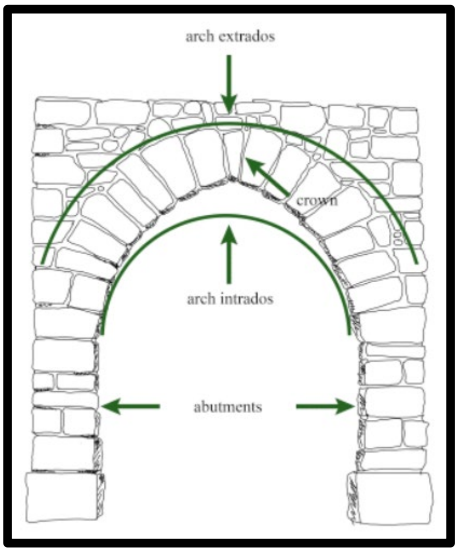

Figure 3.17. Parts of a masonry arch

3.6.1. Corbels

Corbelling is a masonry construction technique that is used for forming overhangs, primal arches, and primal domes. To construct a corbel, masonry units are laid in a row and on top of each other. In the rows, bricks go cantilever further and further in each row. If a corbel dome or arch needs to be formed, rows are added to the corbel until units of both sides are connected at the top, or the rows can be left as a cantilever to form an overhang. Corbelling as a technique or corbels as structural elements can be seen as ancestors of masonry arches, vaults, and domes.

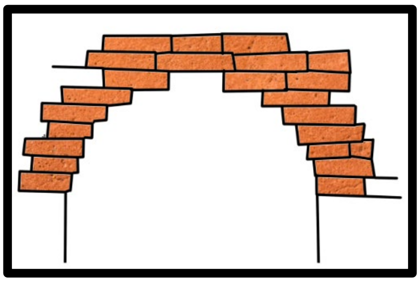

Figure 3.18. Section of a dome constructed by corbelling [74]

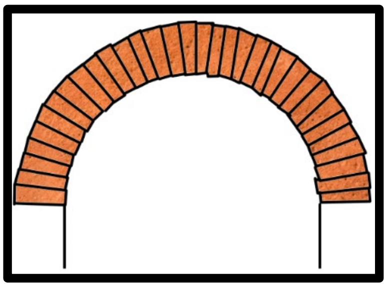

Figure 3.19. Section of a true dome [75]

3.6.2. Thrust line in arches

In arches, needs related to compression are mostly satisfied by arch's geometric form. The line of thrust is a crucial concept for compression needs and to understand the structural behavior of the arches. The line of thrust is an imaginary line that goes along the path where the resultant force of the loads and reactions on an arch is transmitted. Ideally, this line must be along the centerline of the arch ring. The significance of the line of thrust is linked with its direct influence on the stability of the arch. The load-bearing capacity of an arch is limited by its ability to maintain the equilibrium of the forces that conceive in it, which is the condition of resultant forces acting on the arch remaining in the borders of the arch's outline. When a load is applied to the arch, it causes deformations and transmission of the forces along the arch's structure. The line of thrust is the representation of the path these forces act along. It is crucial to keep the line of thrust inside the confines of the arch, especially between its two ends. If the line of thrust overflows outside the arch, it could cause instability and failure.

The overflow of the line of thrust outside the arch also conceives tensile stresses inside the arch. This occurs frequently when the loads acting on the arch are unsymmetric or concentrated because the arches carry primarily their own weight.

"Voussoir central or clé de voute" (keystone), the wedge-shaped brick or stone placed in the center of the arch, is another important element in establishing the line of thrust. The keystone is often the key to maintaining the stability in the arch because it locks other units in place.

Understanding and anticipating the line of thrust is essential for the design and construction of the arches. Predicting and analyzing the forces correctly to ensure that the line of thrust remains within the boundaries of the arch outline is vital for the arch's performance. Factors like the geometry of the arch, properties and the quality of the materials used, and magnitude and eccentricity of the applied loads are all significant on the line of thrust.

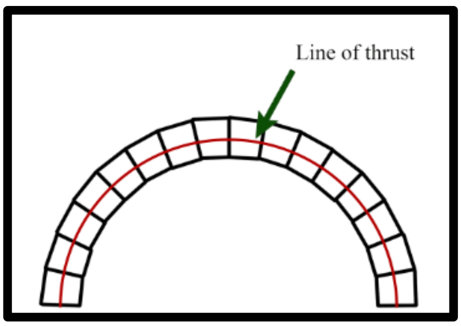

Figure 3.20. Line of thrust in a masonry arch [76]

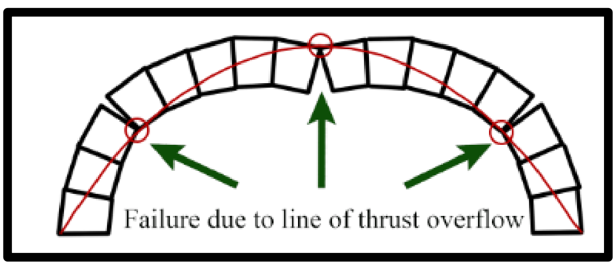

Figure 3.21. Failure in a masonry arch due to line of thrust overflow [77]

3.6.3. Vertical-horizontal load interaction in masonry arches

Theoretically, an arch with a parabolic curvature only bears axial loads. However, practically this is impossible because an arch is subjected to many different loads and interacts with other structural elements. Especially in a complex masonry structure, arches must bear lateral loads conceived by other structural elements' push and carry both lateral and vertical loads simultaneously. If the vertical load the arch is subjected to is larger than the lateral load, the vertical load reduces the tensile stress inside the cross-section. Because of this behavior, arches have considerably large cross-sections to maximize the advantages coming from the own weight of masonry unit.

3.6.4. Failure in masonry arches

The units that are used for masonry arches' construction (like brick, stone, and mortar), are weak against tensile stresses. The occurrence of any tensile stress that exceeds the material's limit at any point of the arch forms a crack. However, the forming of one or a few cracks doesn't necessarily mean the failure of the arch. The major factor that causes arches to fail is the formation of new joints along the span. To prevent this tension members are used in many historical structure's arches.

3.6.5. Types of arches

To satisfy both aesthetic and structural needs in a masonry building understanding the geometry of arches is crucial. The specific configuration chosen for an arch depends on factors such as intended purpose, design preferences, and structural requirements of the building. Each arch type comes with its own unique visual feature and structural implications.

<u>Semicircular arch</u>

A semicircular arch is a basic half-circle shaped arch. Its center is located at the midpoint of the span and the span of the arch equals the circle's radius. Semicircular arches provide a simple but aesthetic form. Its plain yet effective geometry ensures even distribution of the loads among the arches.

Segmental arch

Segmental arches are very similar to semicircular arches. The center of the circle is still located at the span's midpoint however, their radii are shorter than semicircular arches. These arches are often used when a large span needs to be covered with a curved profile.

Elliptical arch

As the name suggests, elliptical arches are shaped like an ellipse. These types of arches are particularly preferred for their aesthetics and flexibility in their configuration where the visuals are the priority.

Horseshoe arch

Horseshoe arches have a shape that is reminiscent of a horseshoe. It is typically more than a half-circle and can feature steeper curves at the bottom. Horseshoe arches were often used in Moorish and Islamic buildings. They are usually combined with ornaments and decorations to enhance their unique visuals.

Basket handle arch

Basket handle arch is a three-centered arch type that is characterized by its three different radii. This results in an elongated ellipse-like shape with a slight rise at the crown. These types of arches can be considered as segmental-semicircular arch hybrids.

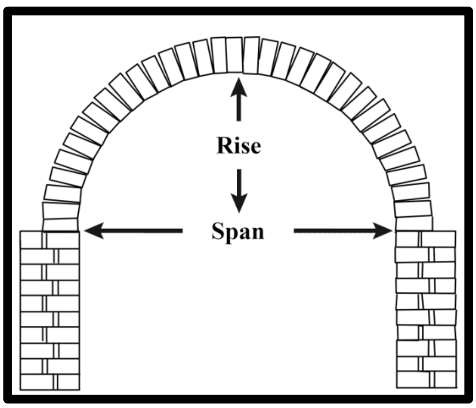

Figure 3.22. Semicircular arch [78]

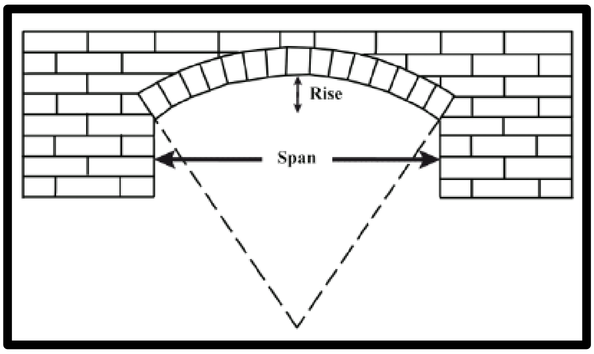

Figure 3.23. Segmental arch [79]

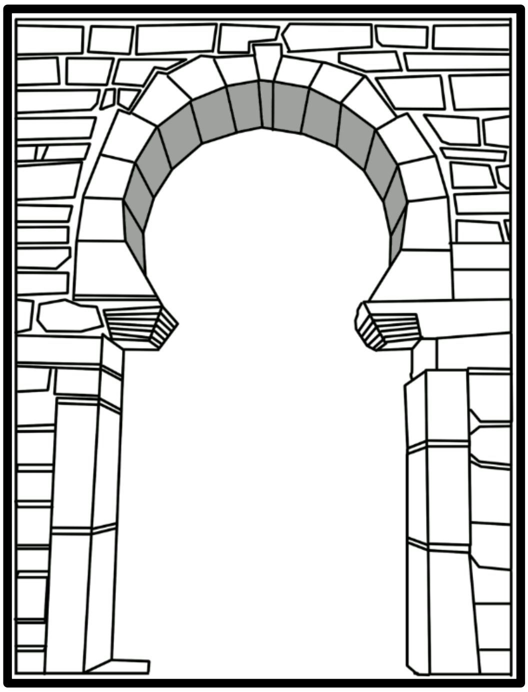

Figure 3.24. Horseshoe arch [80]

3.6.6. Significance of arch

Arch, as a structural element, can be considered one of the most significant developments in the history of masonry building. Arches not only enabled and unlocked many construction methods by itself, but also laid the foundation for vault and dome's conceiveing. In masonry construction arches possess many advantages over horizontal elements like beams or lintels. Compared to these elements, arches can span much larger distances and can be constructed easier with relatively smaller masonry units. Arch also can carry much more loads in compression than other lateral elements. These capabilities of arches come from their own form. As explained in previous sections, when the arch is under compression voussoirs (stones that the arch is constructed from) get closer and become more stable. Arch's geometry allows it to convert vertical loads to a horizontal thrust, which conceives compression in the element. Vertical loads in arches also relieve the tensile stress in the element yet again due to its geometry. This synergy of arch with compression and not being entirely vulnerable to the tensile forces makes it a seamless implementation into any masonry structure.

Arches had a wide use range throughout history. They used to cover spans of any kind and to build large and impressive structures like aqueducts and bridges. Arches also helped mankind to achieve significant vertical heights in structures, especially when they were combined with other masonry elements like columns and buttresses.

Arches were not only significant for mechanical purposes. They added a new dimension of elegance and grace to the masonry structures. Over time, arches became a symbol of triumph and magnificence. Big and monumental arches were erected to honor military victories or important events. They also became emblems for specific architectural styles. Like horseshoe arches for Islamic architecture and pointed arches for Gothic architecture [69, 70].

3.7. Masonry vaults

As mentioned in every other structural element, masonry structures are very complex, and they can be in situations where they have to bear loads that they are not primarily designed to oppose. For arches, this problem arises in the form of forces that affect the arch perpendicular to its plane. When an arch is subjected to a force that is perpendicular to its plane, it causes instability in the arch. Vault, as a masonry element can be considered as a solution to this issue. Another easy way to visualize and define a vault's structural behavior is to think of it as arches that are merged to provide lateral stability.

The forces that are conceived inside the vaults' cross-section are generally compression. The factors that enable the vaults to carry their own weight, stabilize lateral forces inside their plane, and withstand external forces are their continuous and curved form. This means their macro and micro geometry have a major impact on their structural behavior and performance.

The structural behavior of vaults heavily influences the whole structure's behavior and is crucial in earthquakes. However, analysis of their behavior can be challenging due to their response being also heavily influenced by the elements they are connected to [29, 58, 71-73].

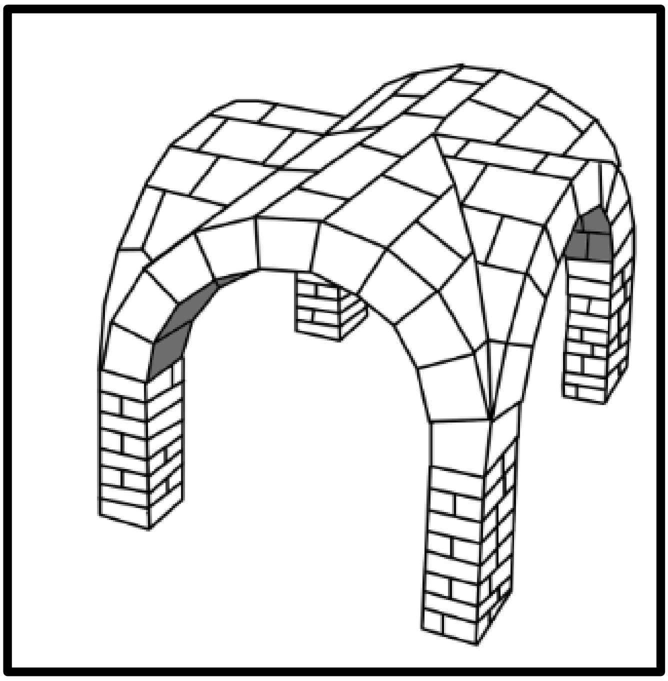

Figure 3.25. Groined masonry vault

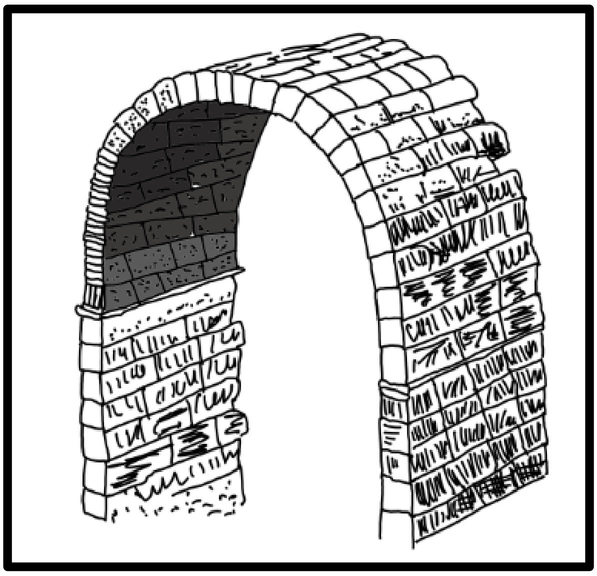

Figure 3.26. Semicircular masonry vault

3.8. Masonry domes

The problem of spanning has been a major issue for architects throughout the history of building. As interior space gained more importance for mankind larger and larger interior spaces needed to be created thus increasing the lengths that must be spanned.

Timber was a good way to span greater lengths with ease. However, it was vulnerable to pest infestation and fire which was a natural phenomenon and used a lot in warfare. Masonry was the other option, but it was greatly restrained in spanning due to its units' low tensile strengths, which was a trait that affected masonry structures' very forms.

Due to these limitations, considerable lengths can only be spanned with arches, domes, and vaults, structural elements that partially bypass the tensile weakness of their material by operating mainly in compression.

It is easily visible to the human eye that nature discovered the structural potential of the shells way before mankind and already perfected its usage by spanning large areas with very thin shells. Shells of eggs and sea creatures can be used as examples in this manner. Although vaults are also considered as shell elements domes are the shell elements in masonry that put masonry's maximum potential to use given that they are also an evolved version of arches, and they almost completely function in compression. They can also cover larger spans and are culminations of complex geometric, technical, and static solutions. The word dome comes from *domus dei*. Which means house of God. The word *dom* in German and *duomo* in Italian also means cathedral, which can be associated with the term house of God.

Domes are the structural elements that can bear in double-curvature planes which provides more stability than single-curvature planes. Domes generally have very small thicknesses compared to their other dimensions. Their geometry also allows them to transfer forces mainly with axial loads. Unless specific conditions of load and constraint rotational domes (spherical, hemispherical, and elliptical domes) are *funicular* or *catenary* (idealization of the curve that a chain or cable takes under its own weight when supported only at both ends) under the distributed loads. This behavior of domes further shows their structural efficiency. A theory suggests that the domes are erected in this specific funicular shape because the used materials were weak against the tensile stresses, and they could not operate in any other shape that conceives considerable tensile stresses. It is also believed that the optimum shape for domes was found through trial and error [74-79].

3.8.1. Hooping tension in domes

Domes are structural elements that can operate smoothly in compression due to their geometry. However, when it comes to horizontal forces, domes have their own problems. Throughout the dome's meridians, the dome's own weight and additional loads like wind or snow can be transferred as compressive forces. However, if the lateral movement of the dome is not restrained at the bottom tensile stresses called *hoop tension* can occur at the lower parts of the dome, an issue that becomes even worse due to the low tensile strength of masonry reducing to almost zero by creeping. Hoop tension is the name of tensile forces that occur horizontally in the parallels of the dome. In contrast to the forces in meridians, stresses in the dome's parallels are not solely compression or tension. At the top, they start as compressive stresses, and as they reach the lower parts they turn into tensile stresses (hooping tension). The point where this transformation occurs is called *springing point* and it plays a crucial role in the dome's design process.

Hoop tension affects the dome similar to when a hemisphere orange peel is pushed from the top. Due to the peel's tendency to spread and deform outward, at a point, vertical tears occur at the peel. It can also be said that hooping tension tries to transform the dome into a sequence of rotated arches almost like reversing its evolution.

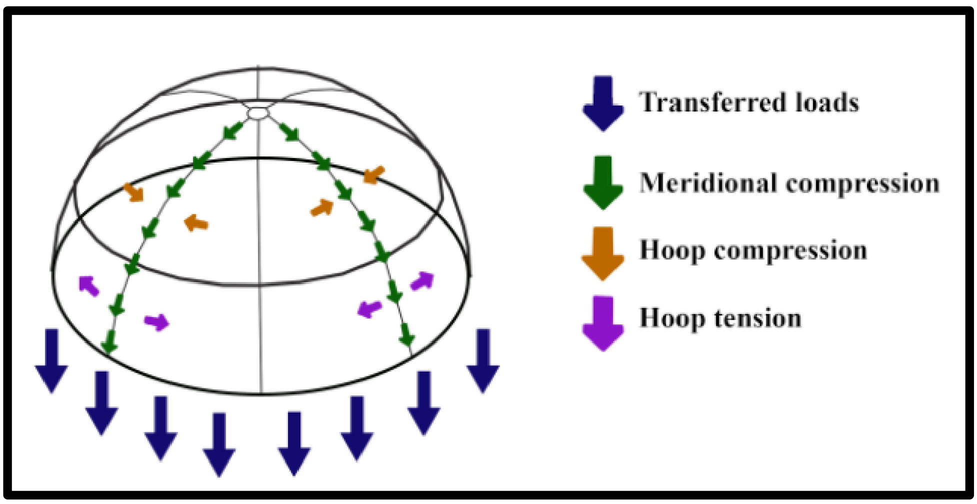

Figure 3.27. Forces acting on a dome

3.8.2. Solutions to hooping tension in domes

Hooping tension is a phenomenon that can threaten the dome’s structural integrity. One of the most common ways to increase resistance against hooping tension in masonry domes is by using *hooping rings*. Hooping rings (also known as tension rings) are structural elements that are used to enhance the masonry domes' strength against the tensile forces and increase stability by encircling the dome with a material that has adequate tensile strength. In masonry structures with large domes, hooping rings neutralize the tensile forces at the lower part of the dome with their heavy and massive bodies.

For tension rings metal is considered very suitable because of its high ductility, workability, and strength/cost efficiency. However, it needs to be protected from corrosion. Corroding can be prevented by using stainless steel, but it increases the cost which can cause other concerns. Reinforced cement concrete is another material option that costs less than stainless steel without the risk of corrosion. However, its detailing due to the brittle nature of concrete can require complex detailing.

Although it has a weak tensile strength due to joints, stones like igneous rocks that have better tensile strength than other stones used for masonry can be used for tension rings by interlocking with tongue and grove joint system. A ring constructed like this cannot compete with the other types of tension rings and can fail against sudden increases in tension due to the stone’s fragility. However, it can still be a cheap and accessible option for resisting relatively small tensile stresses. Despite all this, a masonry hooping ring can be constructed to neutralize the tensile stresses of a dome, but it needs to be heavy and massive to have adequate tensile strength which creates additional loads for other structural elements to bear.

The hooping tension issue can also be solved by making the lower dome shallower. This eliminates the hooping tension however it gives an angle to the thrust coming from the dome thus creating a horizontal component of the vector. This horizontal component must be resisted by buttresses or other structural elements like smaller domes. An example of this can be observed in Hagia Sophia. Shallower domes like this are generally used in Byzantine and Muslim architecture.

Another option to solve the hooping tension issue is to not resist it and let the dome relieve the tension itself by cracking. The cracking process ends after the dome achieves stability and transforms into a form like a circular array of arches. This method is uncommon and even though there are structures that are still standing with this method, visible cracks on the structure can cause stability and usability concerns for people.

3.8.3. Types of masonry domes

Masonry domes have been an important part of the many masonry structures for many centuries. With their long-time enduring massive bodies, they represent their time's state of art architecture and structural engineering. Domes, like any other structural element in masonry, can achieve unique characteristics influenced by the history, culture, and engineering of their region. From the onion domes of the east to the shallow dome of Hagia Sophia, all masonry domes reflect their cultures' construction techniques and artistic ingenuity. Different forms of domes distribute the loads they are subjected to in different ways. Therefore, understanding dome types and their characteristics plays an important role in the analysis of their behavior.

Rotational domes

Rotational (or spherical, hemispherical, elliptical) domes are domes that are derived from the projection of the circular rotation of an arch. As explained in previous sections, rotational domes create outward thrusts due to their shape that need to be managed by the use of other structural elements. Two of the most known examples of rotational domes are the Pantheon in Rome (perfect hemisphere) and the Hagia Sophia in Istanbul (shallow hemisphere).

Ribbed domes

Ribbed domes are, as their name suggests, domes that are distinguished by visible ribs on their exterior surface. These ribs contribute to the dome's both aesthetic and structural aspects, enhancing its stability and decorative appeal. With the help of the ribs, loads on the dome are distributed more efficiently and the risk of cracking and deformations are reduced. Radial ribs go from the beginning to the end of the dome, providing resistance for both outward thrust and compressive forces. With ribs, as discussed in the hoop tension section, additional hooping rings can be attached to the dome to further increase stability against the lateral spread tendency of the dome under its own weight.

Onion domes

As their name suggests, onion domes have a distinctive onion resembling shape that narrows to a point at the top of the dome. They are widely used in Eastern Europe and Russian architecture. From the 16th century onward, they become a signature element of Russian Orthodox churches. Their shape symbolizes the Holy Spirit's flame and heaven. They are usually constructed with wooden frames and the shape is created with metal cladding. Another characteristic of onion domes is their colorful exterior painting. Great examples of onion domes can be observed in Saint Basil's Cathedral in Moscow and the Church of Savior on Spilled Blood in St. Petersburg.

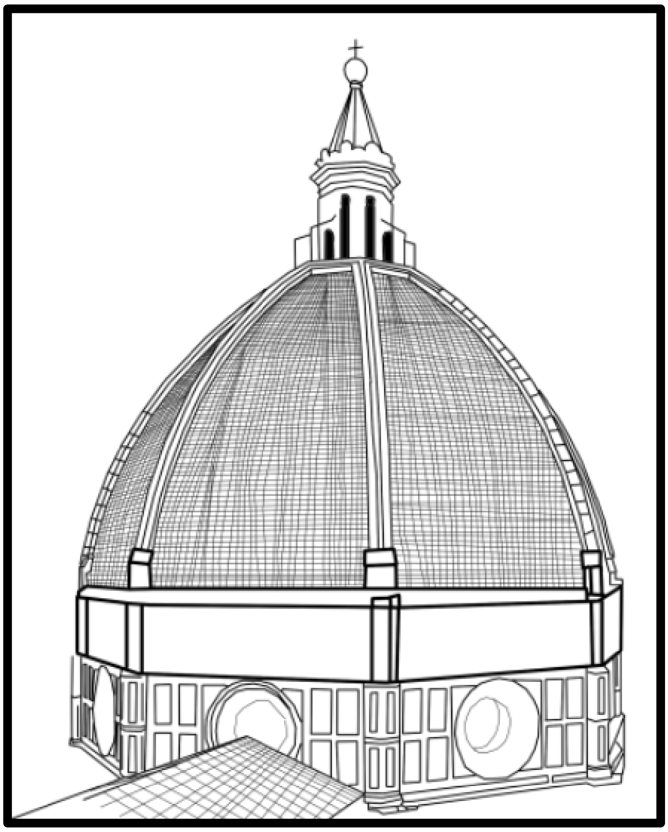

Figure 3.28. Ribbed masonry dome

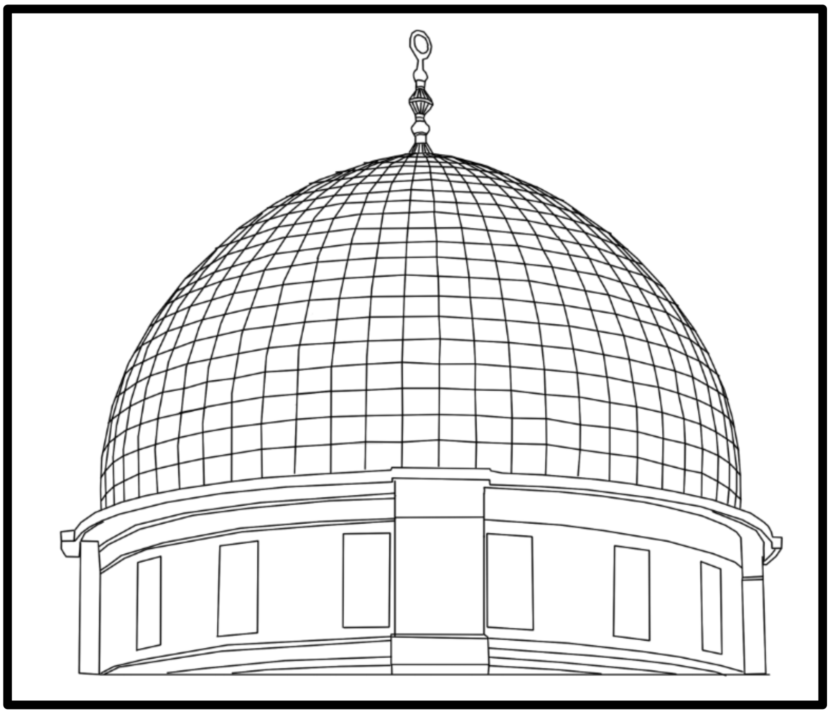

Figure 3.29. Hemispherical dome

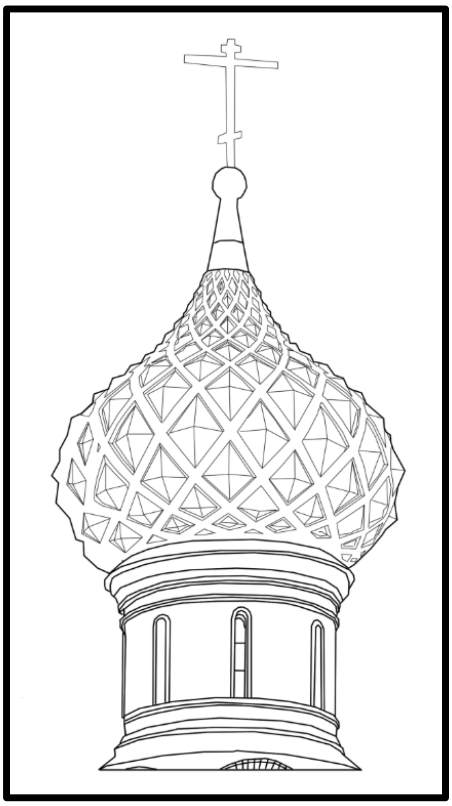

Figure 3.30. Onion dome

3.8.4. Aggregate size in domes

Aggregates that were used in the construction of masonry domes varied greatly in size and shape. While constructing massive domes like the Pantheon the sheer weight of the dome becomes so massive that even the properties of the aggregates used in the mortar gain importance. When it comes to massive domes, lower sections of the dome bear most of the compressive forces. Therefore, using heavier aggregates with less porosity, which have a higher compressive and tensile strength at the cement that binds the units at the lower sections becomes more beneficial. In contrast, the upper sections of the dome bear much less compressive stress compared to the lower sections of the dome. For this, aggregates with lower weight and porosity become adequate for the cement that binds units at the upper parts of the dome. Because this type of aggregate can bear the loads they are subjected to at the upper part of the dome and decrease the weight lower parts must bear.

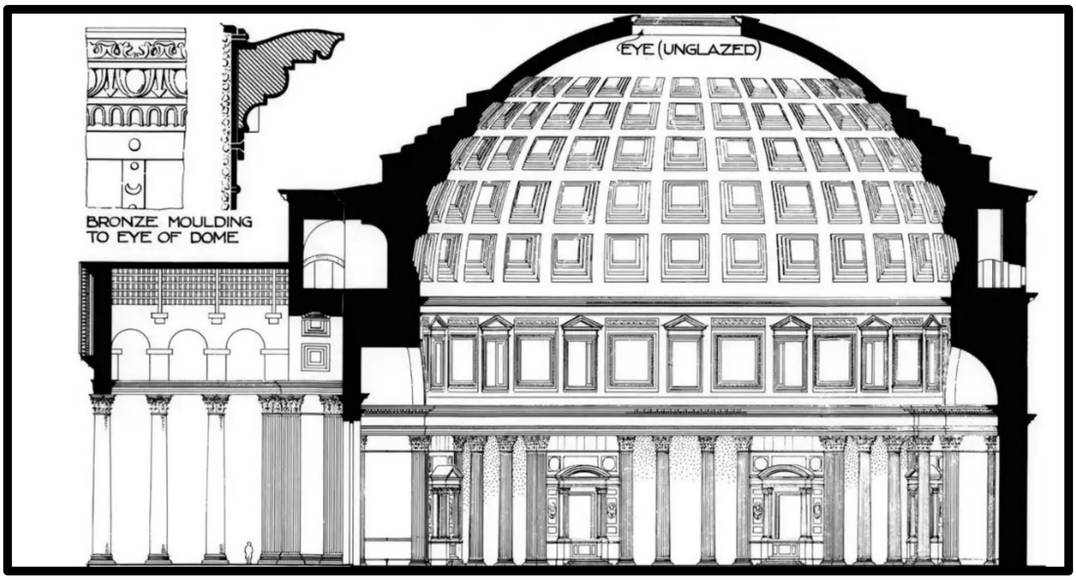

Figure 3.31. Section of The Pantheon, Italy [80]

3.9. Dome Transition Elements

Dome transition elements or transition elements are structural elements that enable the insertion/construction of a dome to a square or polygonal shape created by other structural elements of the building by transforming it into a more adequate polygon (like an octagon) or circle. These elements also smoothen the progression of the structures from the bottom to the top and enhance the interior visual of the building.

3.9.1. Squinches

A squinch is a transition element that fills the corners of a square or polygon to form an adequate geometry for the dome construction. Squinches are constructed with the corbelling technique and are often used in Byzantine architecture and Gothic architecture. With heavy influence from the Byzantines, Islamic architecture also utilizes squinches in their masonry buildings [81].

Figure 3.32. Odzun Basilica’s dome, supported by 4 squinches, Armenia [82]

3.9.2. Pendentives

Pendentive is another type of transition element that transitions a square or a polygonal space to a circular one for the construction of the dome. Pendentives are curved triangular elements that support the dome and transmit its load to the columns while supported by arches on each side. Pendentives are crucial for transferring the dome’s loads to the soil because their structural behavior prevents the occurrence of concentrated loads on arches. The curve and the shape of the pendentives must be in perfect harmony with adjacent arches and the dome itself. They may depend on temporary support until their construction is finished. Due to all these challenges, the insertion of the pendentives into the structure requires advanced workmanship and engineering.

Pendentives are widely used in Byzantine, Gothic, Islamic, Romanesque, and Eastern Orthodox architecture. Examples of pendentive utilization can be seen in Hagia Sophia in Istanbul, Notre-Dame Cathedral in Paris, and the Church of the Holy Sepulcher in Jerusalem [83].

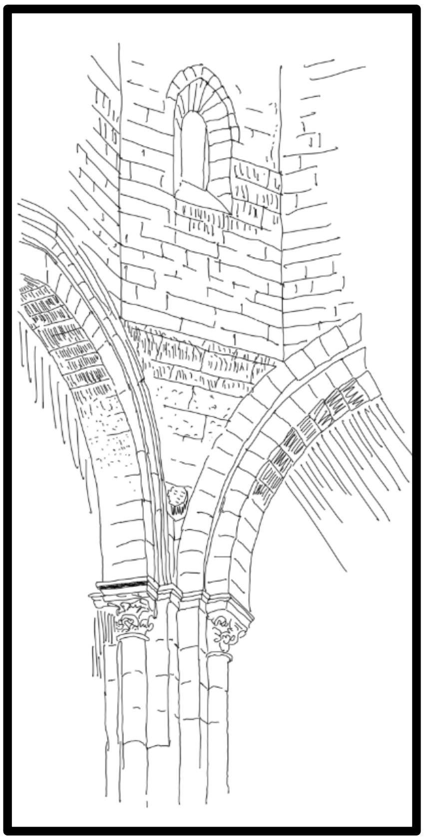

Figure 3.33. Illustration of a pendentive

3.10. Buttresses

Buttresses are structural elements of masonry that support masonry walls perpendicular to their plane that look like protrusions. In masonry, buttresses are commonly used to provide support to the walls to bear the loads transferred from the vaults, domes, and arches. The main purpose of the buttresses is to resist the lateral loads, either conceived from the other structural elements, lateral component of the thrust, or the loads conceived due to earthquake and wind.

When a dome or vault collapses in a masonry structure, the integrity of the whole structure is not endangered. However, if the buttress systems fail in a masonry structure, consequences can be catastrophic for the whole structure. As explained, buttresses are designed to bear the lateral loads and provide stability in masonry. Failure of buttresses makes the structure much more vulnerable to lateral forces, which is already a major weakness of masonry. Therefore, correct assessment of the buttresses' safety and proper understanding of their structural behavior plays a crucial role in preserving historical masonry buildings.

Types of buttresses can be divided into two, classical and flying buttresses. Classical buttresses can be connected to the wall they support the whole way through or up to a certain height. Flying buttresses on the other hand contact the wall in a small area. They transmit the thrust they receive from the wall to a stronger vertical buttress located at their other end. When it comes to geometrical shapes, buttresses can come in many. The most known shapes for buttresses are rectangular, trapezoidal, stepped, combined, and curvilinear. Other than these shapes when a custom design is needed, buttresses can be constructed in special forms [84].

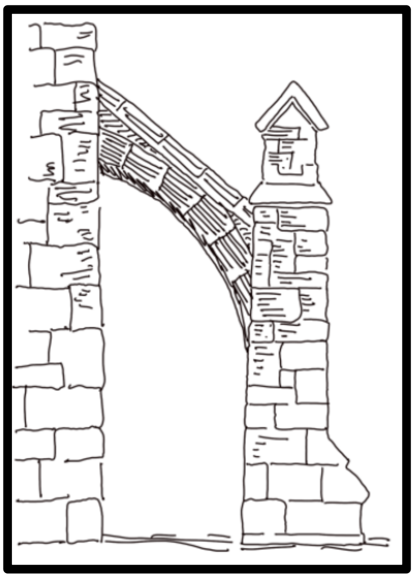

Figure 3.34. Illustration of a flying masonry buttress

3.10.1. Design rules of masonry buttresses

Before the sufficient development of statics, the only possible approach for designing adequate masonry structural elements, including buttresses, was empirical rules. However, it can be suggested that despite the lack of science, masonry buildings of ancient times were still the results of successful experiments, given the fact that they are still standing after centuries. These established rules for buttress design codified many aspects of buttress design like their thickness or depth. However, rules were specific for each buttress type. That means the rules for designing the light buttresses of Gothic architecture couldn't be applied to the heavy buttresses that were used in Renaissance or Baroque architecture [85].

3.10.2. Effects of buttress dimensions on its structural performance

Although there are empirical rules for buttress design, buttresses that are used in masonry structures have a huge variety of dimensions and shapes. In this section, the effect of the buttress' form and dimensions on its structural performance will be explained. Results that are used in this section are taken from Avcıl *et al.* [84].

Effect of the form

Experiments on the topic show that buttresses with triangular and curvilinear concave shapes possess the highest out-of-plane resistance. Out-of-plane resistance is a term used for a structural element's capability to bear the loads perpendicular to its primary plane without failing. Experiments also show that out of other shapes, semi-cylindrical buttresses have the least out-of-plane resistance. According to results, semi-cylindrical buttresses can bear a maximum base shear force (highest horizontal force applied to the structural element's base in an earthquake) of 1549 R_{max} / kN while for the triangular and curvilinear concave buttresses, this value is more than 5400 R_{max} / kN. This shows how even a design choice like shape can increase a structural element's strength more than three times.

Effect of depth

The strength of a buttress is strongly influenced by its depth. For example, a buttress with a depth of 4.4m is more than twice as strong as a buttress with a depth of 2.2m. This growth of strength is related to the increase in the moment of inertia (a shape's resistance to bending or flexural deformation) of the cross-section of the wall and buttress combination.

Effect of thickness

Like depth, increases in thickness increase the resistance of the buttress against the lateral forces yet again due to the moment of inertia of the cross-section. However, it is not as effective as increasing the depth, therefore increasing the thickness of the buttress of additional strength is a less effective option than increasing the depth.

Effect of height

As the ratio of wall height/buttress height decreases the strength of the buttress increases. However, experiments show that if the buttress' height is between 80%-60% of the wall's height it provides almost as much support as if it was as high as the wall. Another significant breakpoint is that even if their height is around half of the wall's height buttresses still provide considerable support. This shows that the growth in the buttress strength as its height increases is parabolic. Therefore, making it higher after a certain point is ineffective.

When the buttresses of the historical masonry buildings are investigated their design correlates with the experimental results. This means that builders of ancient times already had a good grasp of buttress design even under limited circumstances which raises admiration and respect.

4. UNDERSTANDING THE FINITE ELEMENT METHOD

In the modern world, there are many approaches to solving mechanical engineering problems. The problem can be as simple as the deflection of a cantilever beam, or as complex as a structural analysis of a suspension bridge tower. If it's anything like the first, it can be solved by hand. However, as geometry, loads, and materials get more and more complex solving the equations by hand becomes impossible and the help of computational power becomes necessary. Finite Element Method (from now on referred as FEM) is a numerical technique that uses computational power to solve complex problems in engineering that may be tasking or even impossible to solve analytically. FEM is a technique that is used widely in all disciplines of engineering. FEM can be used for static, dynamic, buckling, or modal analysis in solid mechanics and many other purposes in fluid mechanics, heat transfer, and electromagnetics.

In this chapter of the thesis, concepts that are used in the finite element method will be explained briefly. The purpose of this is to give a grasp of the FEM to disciplines that are involved in the restoration and conservation of historical masonry structures so that they can understand the structural behavior of the building better and make clearer decisions regarding their profession on the building. This strengthens the connections between the conservators and brings clarity to the understanding of inter-discipline data coming from the numerical models of the building [86, 87].

4.1. Understanding the concepts used in FEM

Lots of structural mechanics concepts are involved in the modeling and analysis process of Finite Element Models. Getting a grasp of these concepts is very beneficial for professionals involved in the restoration and conservation of historical buildings. These concepts make it easier to understand a historical building's structural behavior, analysis results, and communication between different disciplines in the process. This means better interpretation of the results thus making the decisions on the building healthier.

4.1.1. Equilibrium of forces

If the forces acting on an object don't cause any linear movement or rotation the object is in equilibrium. This also means that the sum of all forces acting on the object is zero. An object in equilibrium remains motionless if it is initially motionless. If it is moving at a constant speed, it continues the motion without accelerating. To further ensure the equilibrium of the object, the sum of the components of the forces in different directions can be checked (See Eq 4.1).

$$\begin{aligned} \sum F &= 0 \\ \sum F_x &= 0 \\ \sum F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned} \tag{4.1}$$

Equilibrium is an important concept for structural design because by implying the equilibrium to the structural elements forces and moments acting on them can be detected. This enables designers to assess structural elements' stability and safety depending on the loads that they are subjected to or will be subjected to [62, 103, 104].

4.1.2. Reactions and internal forces in general

The forces that affect a structure are generally analyzed in two types. The first type is the forces that affect the structure due to natural phenomena or usage requirements. The other type is the reaction forces that occur on the joints of the structural elements. Knowing reactions and internal forces in a structure is important to assess a structure's safety and is a major goal in structural analysis [58, 86, 87].

<u>Reaction forces</u>

Reaction forces act on the joints of structural elements, and they differ depending on the joint's (support's) *boundary conditions (or constraints)*. The most common boundary conditions for the supports are pinned (hinged), free (unrestrained), fixed (rigid), and roller support. Pinned supports react to perpendicular forces that are coming from vertical loads but allow the body's rotation. Free supports don't constrain the element in any direction, allow completely free movement of the element, and don't produce any reaction. Fixed supports can be considered as the opposite of the free supports. They resist both horizontal and vertical forces, and moments, and prevent the movement of the element completely. Roller supports resist vertical loads and allow rotation like pinned supports, however, they also let the element move horizontally [62, [103, 104].

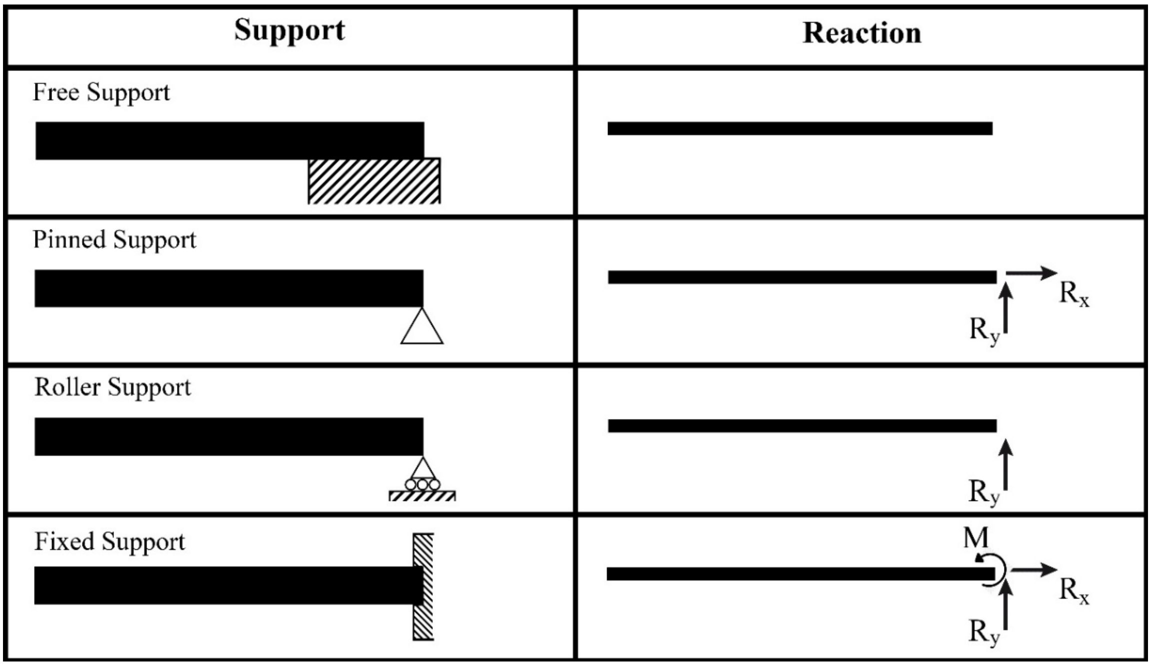

Figure 4.1. Support and reaction types

Internal forces

Internal forces and moments that occur in the structural elements are a result of external forces affecting the structure and their distribution process throughout the structure. Internal forces and moments stabilize the structure while enabling it to behave as a unit. These forces can be divided in two. First is the internal forces that occur in the joints of very complex structures that are a combination of many structural elements and the second one is the forces that occur inside the bodies of the structural elements that are forming the structure [58, 86, 87].

Compression

When an object is forced to shorten along its axis compression occurs inside the body. Compression forces atoms or molecules of the material to get closer to each other.

Tension

When an object is forced to lengthen or stretch along its axis tension occurs inside the body. Tension forces atoms or molecules of the material to get further apart from each other.

Shear

When a force acts parallel to the object's given cross-section it causes atoms or molecules of the object to slide past one another and deform the object with an angle.

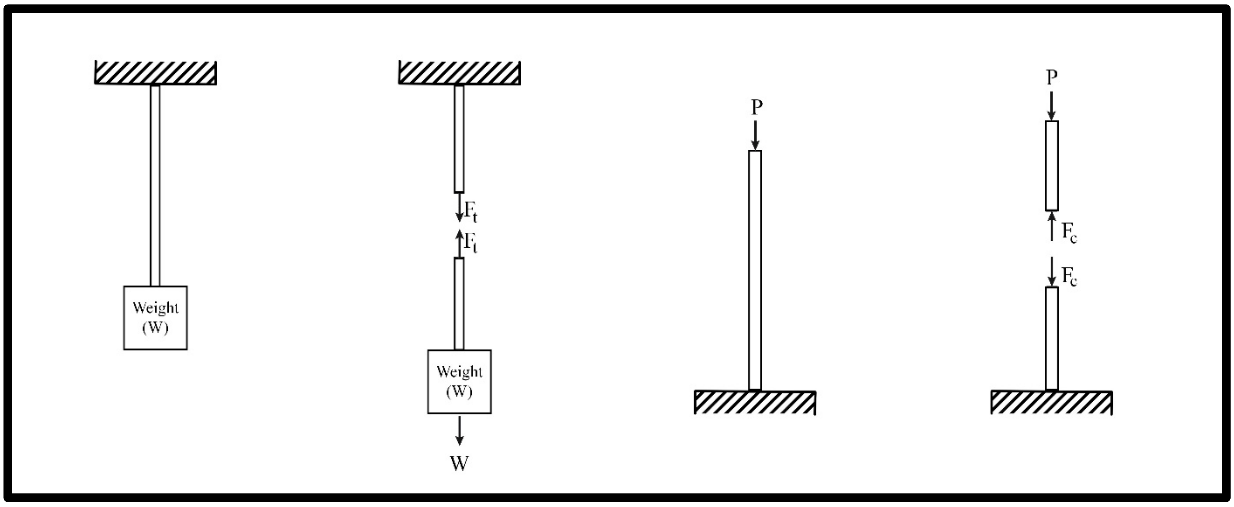

Figure 4.2. Illustrations of tension and compression

4.1.3. Understanding the stress and the strain

If a solid metal bar is loaded by two equal but opposite forces (uniaxial loading, all applied loads act along the same axis) and the forces are pulling the bar thus stretching (straining) it, the bar will develop internal forces (stress) within its body to resist the applied forces and stretching. As explained before, internal forces will develop in such a way that equilibrium will be maintained. Internal forces acting in the cross-section are displayed as 6 forces (6 orange arrows) in the figure. However, it can be described as any number of different forces because the internal forces are distributed over the whole cross-section, and it is impossible to examine one specific internal force thus the choice of display is irrelevant in this manner [86, 87].

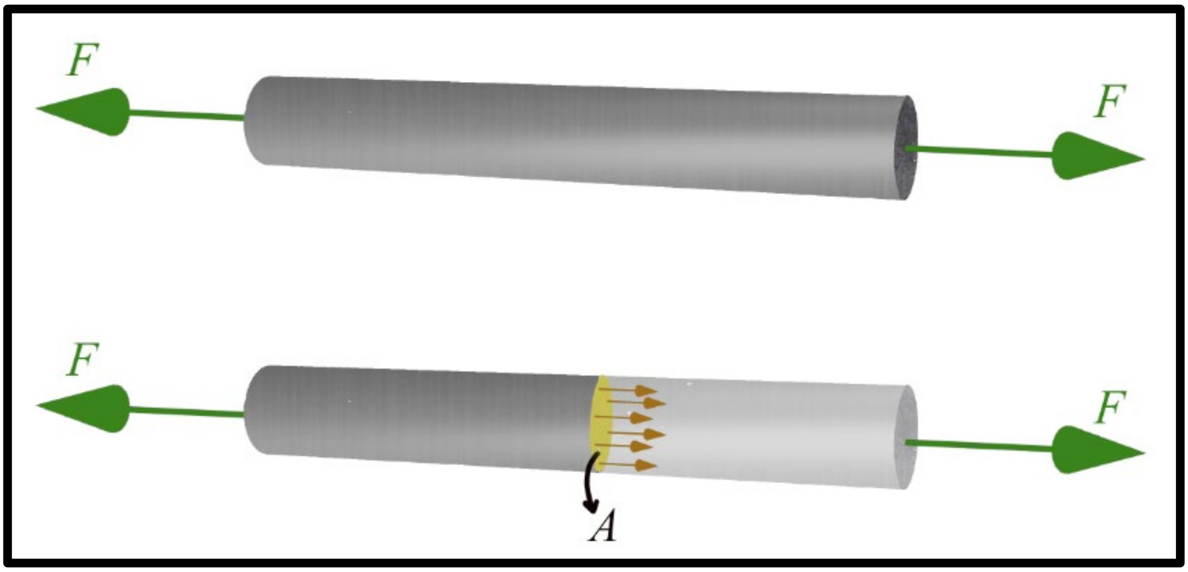

Figure 4.3. Forces inside a solid bar due to tension

Stress

Stress is a quantity that describes the distribution of internal forces within a body. It also makes it easier to understand the internal state of the body due to the responses it gives to external loads. Since the stress is a measure of the internal force per unit area it has units of Newtons per meter squared in SI units (See Eq 4.2). Newtons per meter squared are also called Pascals (Pa) [86, 87].

$$\frac{force}{area} = \frac{N}{m^2} \tag{4.2}$$

In the case of the given example, internal forces are acting perpendicular to the chosen cross-section. This type of stress is defined as *normal stress*. To calculate the normal stress acting on the bar, the force F can be divided by the cross-sectional area A (See Eq 4.3). Normal stress is denoted by the Greek letter sigma (σ). The most important aspect of calculating the stress inside an object is to gain information about how close the object is to its failure threshold. For example, a bar with a 0.5 m^2 cross-section and a tensile strength of 400 Pa (400 N/m^2) will fail if the tensile force the bar is subjected to is greater than 800N because tensile stress exceeds the given strength [86, 87].

Normal stresses can be tensile or compressive. Typically, the sign convention is positive for tensile stresses and negative for compressive stresses.

$$\sigma = \frac{F}{A} \tag{4.3}$$

<u>Strain</u>

Strain is a quantity that is used to describe the deformations occur within the body. If we fix the bar at the given example at one end (to simplify the situation) and keep applying the force at the other end, the force will cause deformation on the bar. Yet again the strain is perpendicular to the section, so it is a *normal strain*. To calculate the normal strain the change in length of the bar (ΔL) can be divided by the original length (L) (See Eq 4.4).

$$\varepsilon = \frac{\Delta L}{L} \tag{4.4}$$

Strain (ε) is a non-dimensional quantity and because of that it is often expressed as a percentage like stress, strains can be compressive (-) or tensile (+).

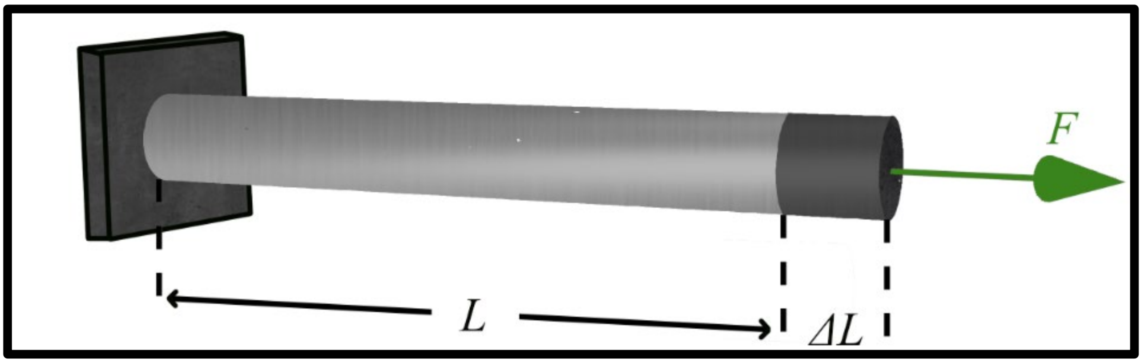

Figure 4.4. Strain on a solid bar

When studying material behavior under the loads, strain is a parameter that is as important as stress because it is used along with the stress to characterize a material's deformation by forming its stress-strain curve and its *Young's Modulus* (modulus of elasticity) [86, 87].

4.1.4. Stress-strain diagrams and Young's Modulus

The relationship between the two previous field variables can be understood through a stress-strain diagram. Stress-strain diagrams are graphics that show the relation between stress and strain for a material under axial loading. By looking at a material's stress-strain diagram knowledge about its material properties like its elasticity, ductility, Young's Modulus, yield strength, and ultimate strength can be gained. To obtain a material's stress-strain diagram, a compressive or tensile test can be performed on a sample (tensile or compressive tests are mechanical tests where a sample of the material is subjected to progressively increasing compressive or tensile forces until it fails) [86, 87].

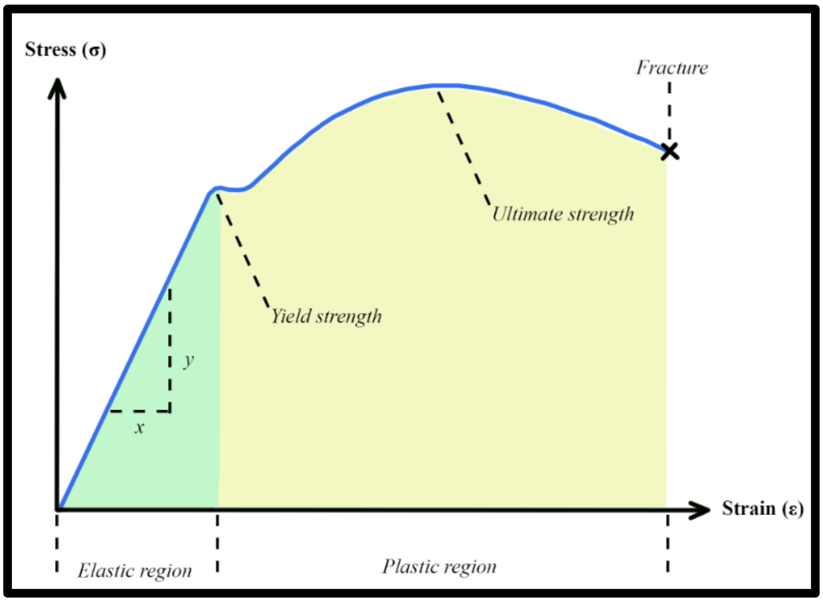

Figure 4.5. Stress-strain diagram of a material

Elastic region

Elastic region is the part of the stress-strain diagram where the relationship between two field variables is linear. Deformations occurring inside this region are reversed when the force is released. The linear relationship between stress and strain inside this region is defined by *Hooke's law*. This also means in this region strain is directly proportional to the stress in the body. Another material property that can be obtained from the elastic region is Young's Modulus (modulus of elasticity) (E). Young's Modulus can be obtained from the slope of the line that's in the elastic region which is an indicator of material's stiffness inside the elastic region (See Eq 4.5) [86, 87].

$$\frac{y}{x} = \frac{\sigma}{\varepsilon} = E$$
$$\sigma = E\varepsilon \qquad (4.5)$$

Plastic region

After stresses and strains exceed a certain point, the material's behavior against increasing stresses and strains becomes *non-linear*. The threshold where behavior turns into non-linear from linear is defined as the *yield strength* of the material. Yield strength is a crucial parameter in design because it defines the safe working limits of materials and is the limit stress a material can bear if it is intended to work without plastic deformations. Once this threshold is crossed the deformations in the material are no longer reversible. Some materials reach the maximum stress capacity they can resist during the plastic deformation phase which is defined as the material's *ultimate strength*. If the material keeps getting subjected to loads while in plastic deformation at a point it reaches its limit, fails under the applied loads, and *fractures* which causes separation or breaking of the material [86, 87].

Poisson's ratio

Poisson's ratio is a material property that gives information about materials' deformation under loading. To explain Poisson's Ratio, a rectangular prism made from a lump of material can be examined. If a compressive or tensile force is applied to the object, it will deform in both the direction the force applied and directions perpendicular to it. Because the object is under tension it will extend in longitudinal direction (same direction as the force) and contract in lateral directions.

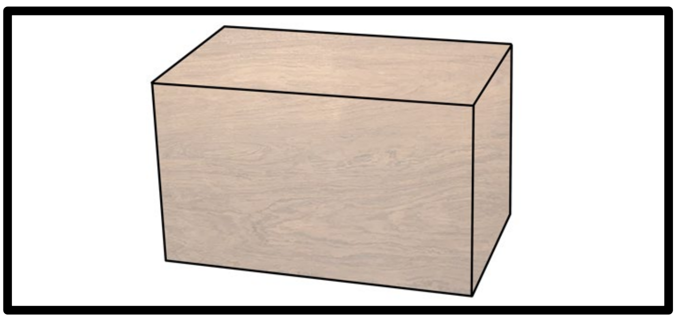

Figure 4.6. An unstrained material

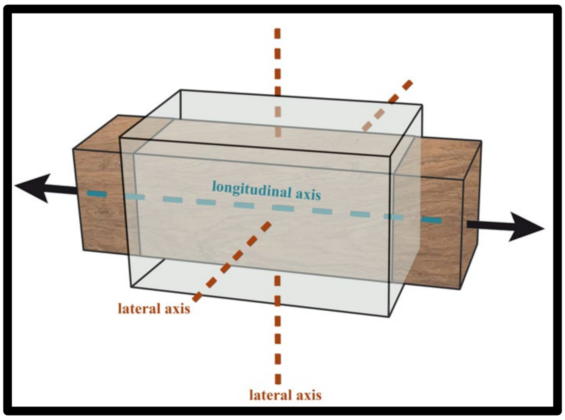

Figure 4.7. Axes of a strained material

As explained before, regardless of the force's direction, the object will deform in all directions. When a material's Poisson's Ratio and deformation in one direction are known, deformations in other directions can be learned by using Poisson's Ratio.

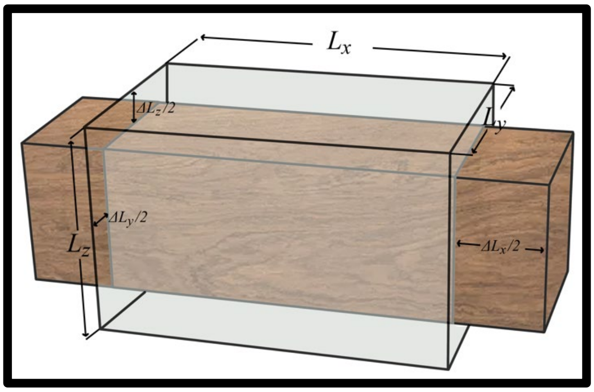

Figure 4.8. Directions of deformations in a strained material

In the figure, L_x, L_y, and L_z are the original dimensions of the prism before the deformation. ΔL_x, ΔL_y, and ΔL_z are the changes in the dimensions due to deformation. Changes in dimensions are shown as divided by two because the change in dimensions occurs on both sides equally. If the strains are calculated it can be seen that strains in the lateral directions are equal and proportional to the longitudinal direction. The ratio between the longitudinal and lateral strain in this proportion is a material constant and is named as Poisson's Ratio (υ).

$$\varepsilon_x = \frac{\Delta Lx}{Lx} \quad \varepsilon_y = \frac{-\Delta L_y}{L_y} \quad \varepsilon_z = \frac{-\Delta L_z}{L_z}$$

$$\varepsilon_y = \varepsilon_z \quad \varepsilon_y \propto \varepsilon_x \quad \varepsilon_z \propto \varepsilon_x$$

$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{\varepsilon z}{\varepsilon x} = \nu = \frac{-\varepsilon\ lateral}{\varepsilon\ longitudinal} \tag{4.6}$$

The reasoning behind the minus sign in the equation is because by convention tensile strains are positive and compressive strains are negative. In Poisson's Ratio's case compression causes an increase in lateral dimensions and decrease in longitudinal dimensions. So, with the minus sign Poisson's Ratio becomes positive where lateral strains are opposite in sign to the longitudinal strains, which is the common case. It must not be forgotten that the information provided by Poisson's Ratio is only valid in elastic regions of isotropic materials (isotropic materials are the materials that have the same properties in all directions).

Poisson's Ratio is a material property that must be known to properly design finite element models. In masonry structure's design and analysis, it becomes even more important. Because, as explained in the masonry walls section, masonry structural elements are a composition of brick, stone, and mortar. These materials' Poisson's Ratio can differ and major differences of Poisson's Ratio between the unit and the mortar can cause cracks and failures under compression in masonry [86, 87].

4.2. Finite Element Modeling Process Through a Basic Example

To understand the Finite Element Modeling process, we can use a fictional scenario that leads to a basic Finite Element (FE) problem. If we take a simple example such as a man hanging on a bar, we can take this scenario as an example and can carry out a static stress analysis to understand the basics of FEM.

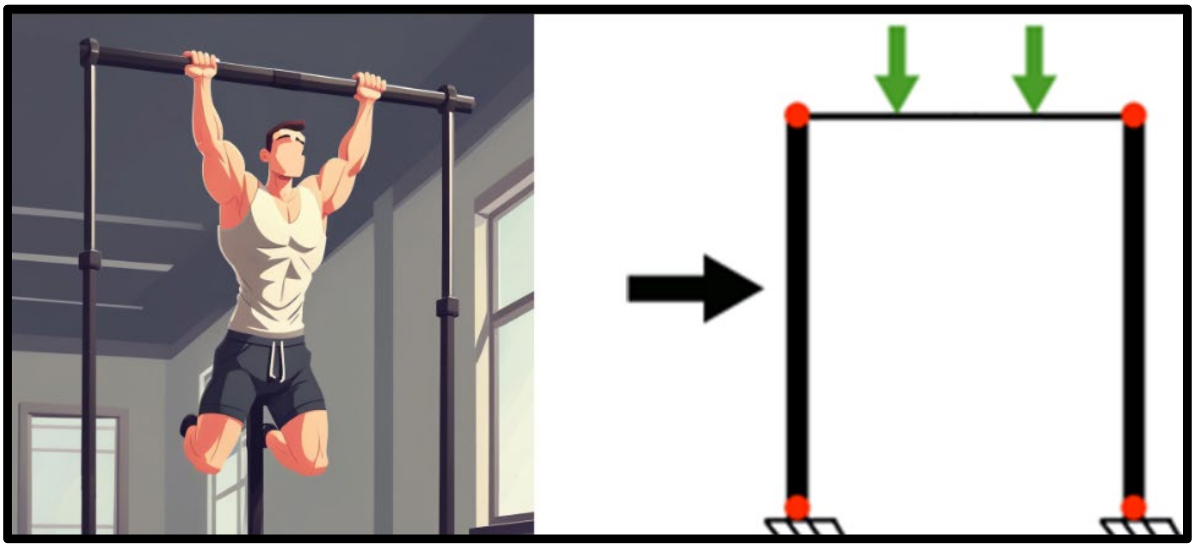

Figure 4.9. Basic example of a FE problem

As seen, the pull up bar that man is hanging on is a simple column-beam system and his weight is applied to the bar through his two grips on the horizontal bar which can be think of as 2 point loads. Generally, a static stress analysis is carried out to calculate stresses, strains, and displacements, also known as field variables, in the body. The 2 concentrated forces on the bar develop stresses in the structure in such a way that equilibrium is maintained at every volume of the body. So, we can apply the concept of equilibrium to calculate any field variable. Applying the equilibrium to find the field variables is easy for a simple beam or an example like this, which is a structure made of 3 frame elements. However, as the body that needs to be analyzed gets more complex or the detail of the model increases, the problem can get very multifaceted very fast up to the point that applying the equilibrium to the whole body either gets too complex or too time-consuming to solve analytically.

The finite element method solves this problem of complexity by splitting the body into several small elements which are connected by nodes. This process is defined as *discretization* and the collection of elements that form the total body is called *mesh*. Discretization can be seen as the essence of the FEM because, after discretization, the equilibrium requirement only needs to be satisfied for a finite amount of discrete elements instead of continuously through the whole body.

For example, when we model the system as one dimensional the finite element model contains 4 nodes and 3 frames. It is possible to solve this problem analytically if we have a basic one-dimensional model like this and computational power may not be needed. However, finite element models are usually a composition of hundreds of frame, shell and solid elements. Even if we start to make this model more complex by increasing the dimensions by a relatively low detail level numbers of nodes and elements thus number of equations must be solved increases rapidly thus a need for comp. In the first picture, system is modeled as one dimensional which contains 6 nodes and 5 frames, which can be considered simple. When the body is modeled as 2 dimensional by using shell elements it contains 22 nodes and 11 shell elements and because we are using shell elements plane stresses come into play. When the dimensions increased to three, it contained 1766 points and 972 and became far too complex to solve analytically. Even though the model conceives many equations to be solved, it is still a simplification in analysis aspect compared to a real-life 3d body.

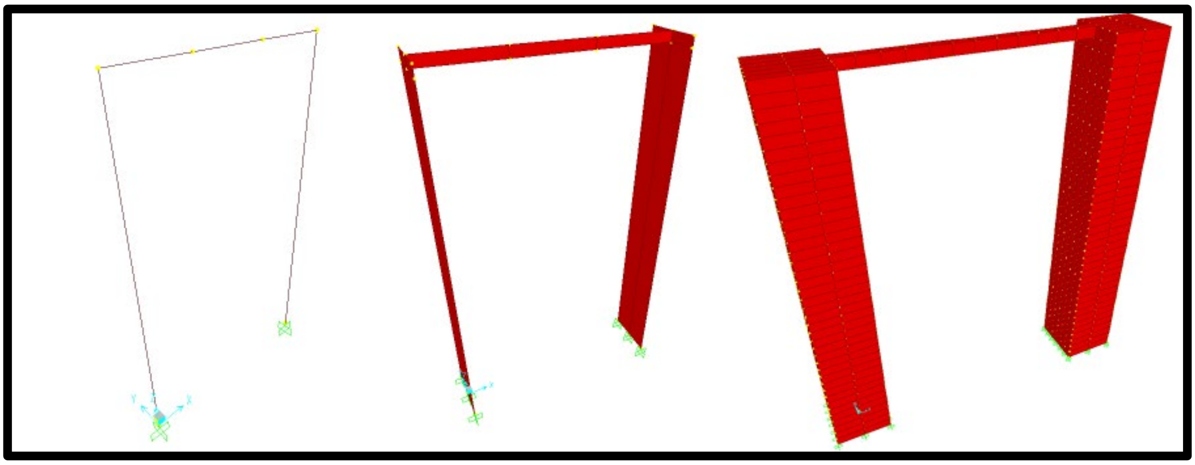

Figure 4.10. Different versions of a pull-up bar's finite element model (constructed out of frame, shell, and solid elements respectively)

As seen in the figures, a body's mesh can be made from different types of finite elements like frames shells, and solids. The choice of element type usually depends on the body's geometry, loading conditions, properties, and the desired precision and detail level in the analysis. Only one of these types can be used to model an entire body as well as different combinations of them.

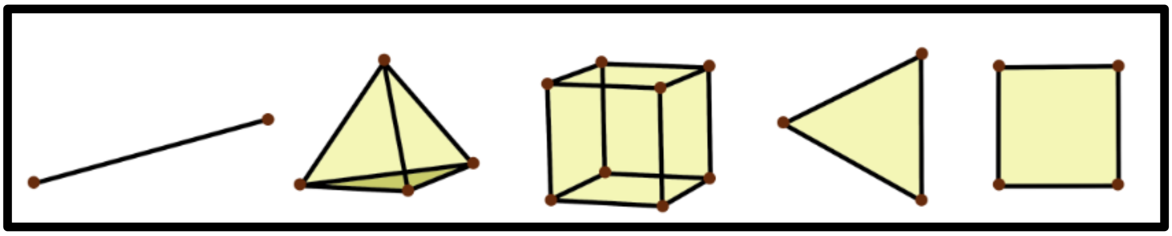

Figure 4.11. Different types of finite elements

In a stress analysis problem, the vital variable that needs to be known is displacement at each node. Because when how a body displaces is known to the applied loads, other variables like stress and strain become much easier to calculate. Let's imagine that we define a vector {u} for a single frame element that contains all its possible displacements for each node including rotations. In a frame element each node can translate along the X and the Y axis and can rotate about the Z axis. So, the vector {u} forms like this (See Eq 4.7).

$$\{u\} = \begin{Bmatrix} u1 \\ v1 \\ \theta 1 \\ u2 \\ v2 \\ \theta 2 \end{Bmatrix} \tag{4.7}$$

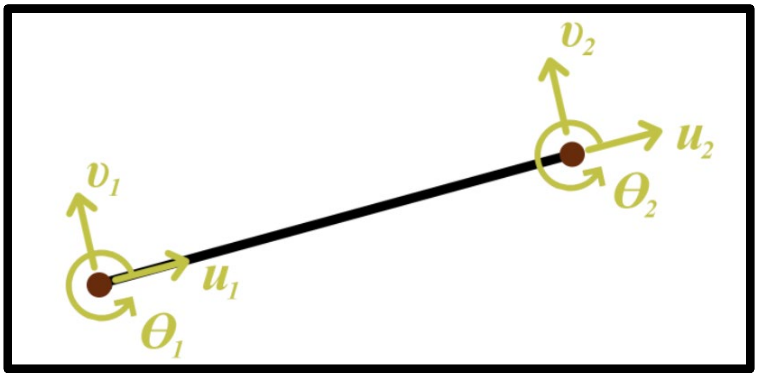

Figure 4.12. Degrees of freedom of a frame element

Each of these displacements is called a *degree of freedom* which is used to quantify the number of different independent ways a node can translate or rotate in a finite element model. Frame elements have 3 degrees of freedom per node which sums to a total of 6.

As same as a frame element, each node in a shell element can translate along the X and the Y axis and can rotate about the Z axis, thus has 3 degrees of freedom. However, due to containing more nodes it has more degrees of freedom in total. For example, a quadrilateral shell element contains 12 degrees of freedom. When it comes to solid elements, like frame and shells, each of their nodes have 3 degrees of freedom too. However, different from the other types, their nodes can translate along in X, Y, and Z directions but can't rotate in any direction (rotation of a solid element can still be calculated by displacement of the nodes). A rectangular or square prism-shaped solid element contains 24 degrees of freedom in total.

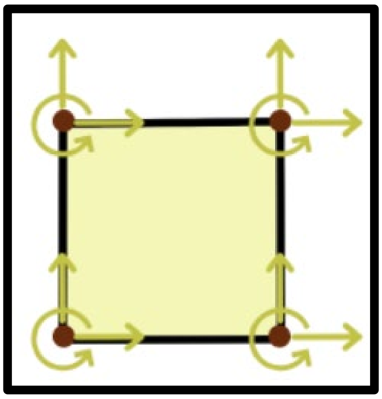

Figure 4.13. Degrees of freedom of a shell element

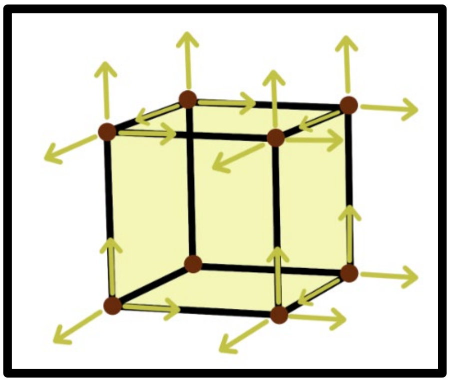

Figure 4.14. Degrees of freedom of a solid element

To calculate all displacements in a node for every degree of freedom the *stiffness* must known. Stiffness (k), in a spring, determines how far it will displace under a given force. Elements in a finite model mesh can be thought of in a similar way as the elements resists the deformations. In a spring, multiplying the stiffness by the displacement in the spring gives the applied force (See Eq 4.8).

$$F = -kx \tag{4.8}$$

By using the same equation, we can find the displacements in a finite element if we know the given force and the stiffness of the element (See Eq 4.9).

$$\frac{F}{-k} = x = \Delta L \quad \frac{\Delta L}{L} = \varepsilon$$

$$\varepsilon = \frac{\sigma}{E} \quad \Delta L = \frac{PL}{AE} \tag{4.9}$$

When this equation is modified for finite elements, it changes because now there are multiple nodal forces and nodal displacements. Also, the stiffness for each degree of freedom differs which turns it into a *stiffness matrix*. Stiffness matrix defines how much each node in the element will displace for a set of forces and moments applied to the nodes and is the key to finding the displacements (See Eq 4.10).

$$\{f\} = \{k\}\{u\} \tag{4.10}$$

Stiffness matrices of each element are merged to create a *global stiffness matrix* for the whole finite element model which defines the displacement of the entire structure under given loads. Global stiffness matrix is a square stiffness matrix and the number of columns and rows it contains is equal to the numbers of degrees of freedom in the whole model. As is clear, using more nodes, either by increasing the dimensions of finite elements or the detail of the model makes model more complex. Because the global stiffness matrix becomes harder to solve, thus more computational power is needed. The tendency towards a more complex finite element model due to the excessive finite elements usage or using the more complex element types in unnecessary parts of the model also reduces the readability and clarity of the results by creating data pollution. A simpler finite element model can be achieved by *using the adequate finite element types where they represent the structural element's real-world behavior the best.*

4.3. Comparing 2 Finite Element Models of a Masonry Column Modeled with Frame and Solid

In this section, a comparative analysis between the two finite element models of a masonry column is carried out. One of the models is solely constructed from frame elements while the other is from solids. Stresses and displacements are compared in both models to investigate if there is a difference in precision and reliability of the models.

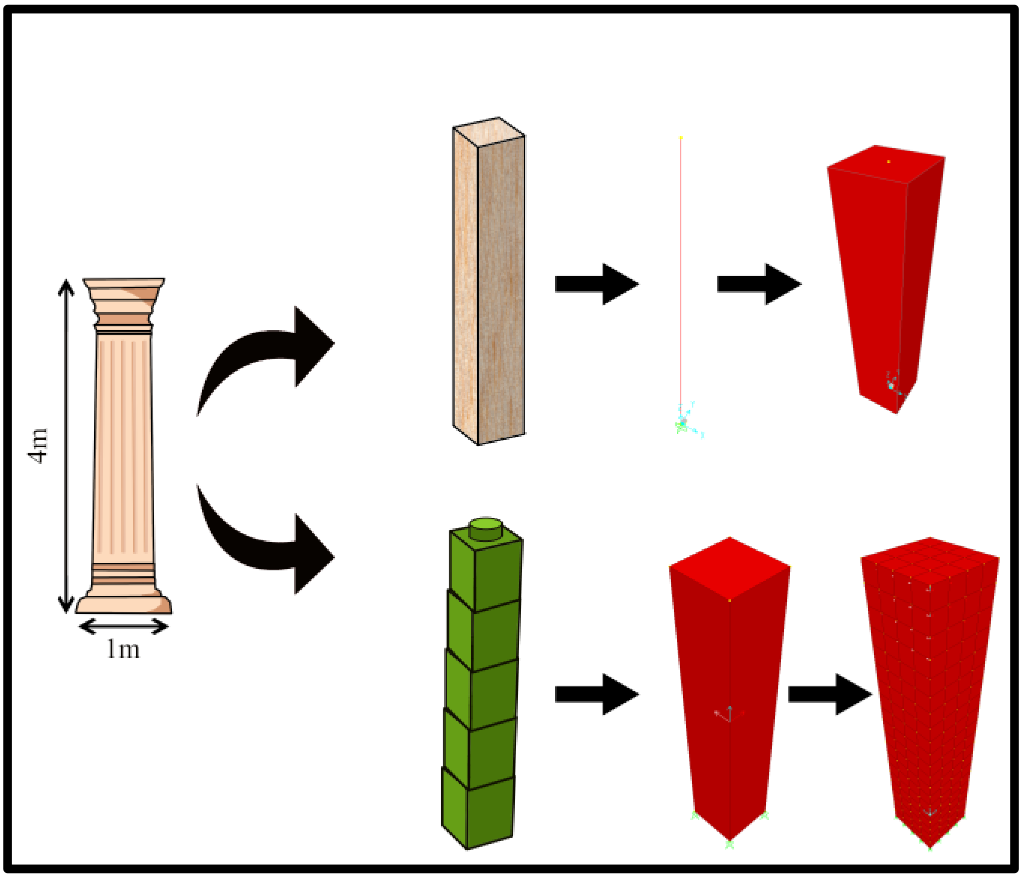

Figure 4.15. Comparison of two finite element models of a masonry column modeled with frame and shell elements respectively

In the first model, only one frame element is used to model the whole column, containing 2 points and one frame in total. In FEM frame elements are widely used to model slender structural elements like trusses and beams. Frame elements are a good choice for element type if the axial loads and bending are important variables for the element's performance. The second model on the other hand, utilizes solid elements to model the column. Solid elements in FEM can be used to create the mesh of almost any geometry, including the complex ones. The density, or the number of solids in the mesh, can be increased to increase the detail and to capture more complex stresses and strains inside the structural element in exchange for increasing the complexity of the power and the computational power needed. Solid version of the column contains two versions. One is with the less detail that is formed from one solid and 8 nodes. The other version contains 320 solids, and 525 nodes thus has more detail.

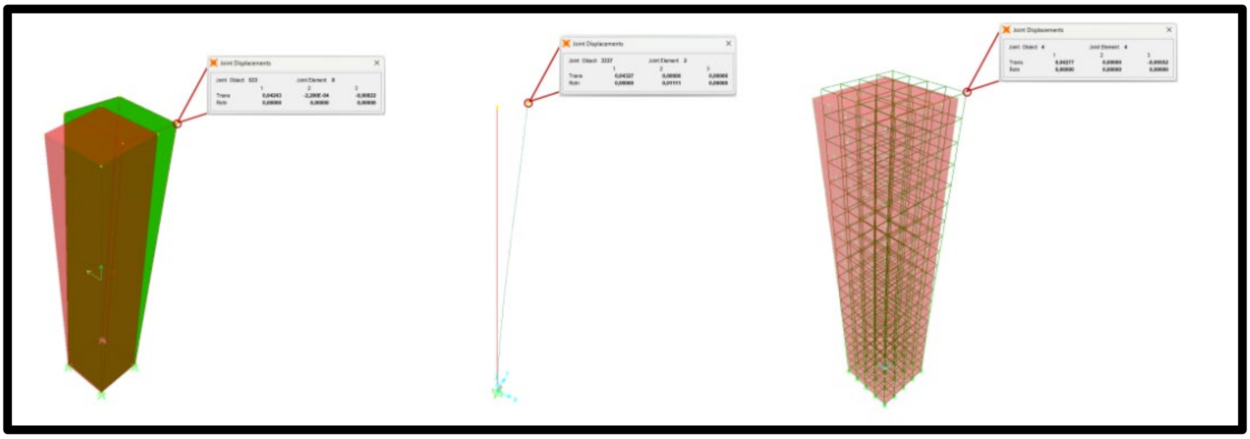

Figure 4.16. Comparison of deformations in three different finite element models of a masonry column

In each model stone column is subjected to a force equivalent to 1 time of its weight in the X direction. As seen regardless of the detail, type, or number of the elements used, the horizontal deformation in every model is approximately 43 millimeters. This means one frame element that consists of 2 nodes and one finite element performs same with the more detailed models that have more elements and nodes thus equations to be solved. It can be said that frame element reflects the real-life behavior of the column properly against the lateral loads without bringing much complexity to the model and can be considered an adequate choice of material type for modeling the masonry columns.

4.4. Importance of Conducting Finite Element Analyses on Historical Masonry Structures

Conducting Finite Element Analyses on historical masonry structures is crucial for their preservation against external threats. These analyses help conservators to understand structures behavior, weaknesses, and performance; thus they configure their retrofitting actions in line with this data. With the data coming from Finite Element Model, locations of reinforcements can be detected, and their effectiveness can be simulated through yet again the using the Finite Element Model, which further improves the decision making in the preservation process. Another perk of the FEM is that being a cost-effective analysis method that requires almost no physical action on the building, which also minimizes unnecessary physical intervention in the structure.

4.5. Different Approaches to The Masonry Modeling in FEM

In Finite Element Modeling different approaches to modeling can be taken depending on the detail wanted from the results and the scale of the structure or object that is getting modeled. For masonry modeling in FEM, approaches like macro modeling, micro modeling, and multi-scale modeling are used widely to capture different results depending on the situations.

4.5.1. Macro modeling

In macro modeling the finite elements that create the mesh of the masonry structural elements are configured as homogeneous material as it is a unique mix of masonry unit and the mortar. In this approach generally simplified material configurations and boundary conditions are used throughout the whole model, this also makes it easier to understand the structure's behavior. Macro modeling is efficient to analyze large-scale masonry structures and its components where phenomena on a detailed level is irrelevant for the analysis.

4.5.2. Micro modeling

In micro modeling masonry structural elements are considered heterogeneous, thus all the mortar, unit, and the mortar-unit interface is modeled separately. This approach is usually used for detailed analysis of cracks and separations in the structural element. Micro modeling requires detailed knowledge about the structural elements' dimensions, material, geometry, and overall expertise in mortar-unit interface behavior. Micro modeling is also widely used when the structural element's performance under different loading conditions, durability, and repair/strengthening options needs to be investigated.

4.5.3. Multi-scale modeling

Multi-scale modeling can be thought of as a model constructed from both micro and macro scaled parts. The purpose of multi-scaling approach is to combine the information of material behavior coming from the micro modeled parts with the information about the structure's behavior coming from the macro modeled parts and get a good grasp of the whole structure's behavior and making more accurate predictions on structural performance. This approach is generally used to understand the behavior of masonry structures with complex geometries, material properties, and element interactions.

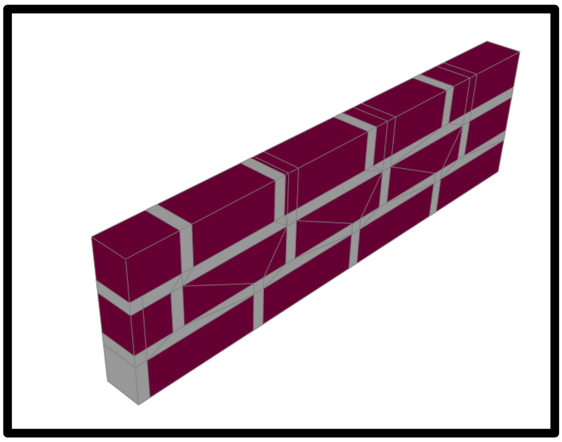

Figure 4.17. Micro finite element model of a masonry wall

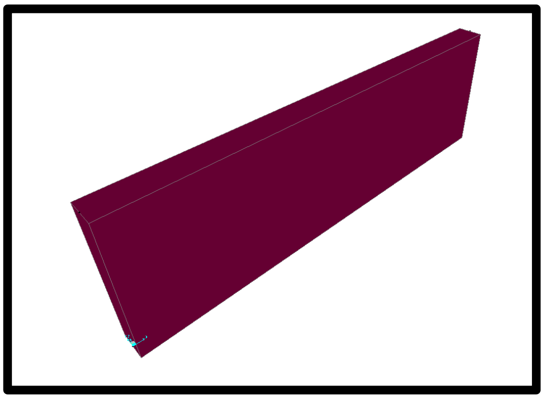

Figure 4.18. Macro finite element model of a masonry wall

5. CASE STUDY: A SIMPLIFIED FINITE ELEMENT MODELING PROCESS WITH FRAME AND SHELL ELEMENTS ON SEHZADE MEHMET MOSQUE

5.1. Brief history of Ottomans' relationship with domes

The Ottomans' involvement with domes goes as far as the roots of their empire. Their masonry domes were mainly influenced by the Seljuks and the Byzantines. The Seljuks were ruling Anatolia before the Ottomans, and they were already utilizing masonry domes in their buildings and monuments with the heavy influence of Islamic culture and the Byzantine Empire. When the Ottomans conquered Anatolia, they inevitably consumed the Seljuk culture, as well as the fragments of Byzantine culture with it thus making them utilize the masonry dome in their buildings too.

In the early times of Ottoman architecture masonry domes were generally small and simple compared to their descendants and despite their small size, they needed to be heavily supported by thick walls and buttresses. 16^{th} century and onwards the golden age of Ottoman architecture began. In the 16^{th} and 17^{th} centuries, Ottomans perfected their skill of masonry dome. They supplemented their domes that get more and more massive with structural elements like pendentives, muqarnas (more ornated Ottoman version of squinch), and semi-domes. The most iconic mosques of Ottoman architecture like Şehzade Mehmet Mosque, Süleymaniye Mosque, and Selimiye Mosque were built in this era. This mosque trio can be seen as climaxes of Ottoman architecture when it comes to domes and mosques.

It can be said that Ottomans, throughout their history, showed a huge affection for craftsmanship and art in the form of architecture and structural engineering. The massive domes they built not only showcased their architectural prowess but also showed their spiritual ambition in the form of metaphors through the structural elements in their religious buildings.

Figure 5.1. Top view of Şehzade Mehmet Mosque [88]

Figure 5.2. Top view of Süleymaniye Mosque [89]

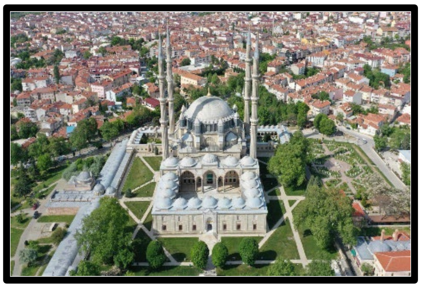

Figure 5.3. Top view of Edirne Selimiye Mosque [90]

5.2. Sinan the Great Builder

Mimar Sinan is considered by many people to be the greatest architect of the Ottomans. With his buildings, Sinan shaped and heavily influenced the golden age of Ottoman architecture. He is also the architect of all three iconic mosques mentioned in the previous section.

Mimar Sinan revolutionized masonry architecture with his ingenious solutions and innovative designs, especially in the construction of the domes. He unlocked the path for building bigger domes in the craft of masonry by supporting them with semi-domes, buttresses, and pendentives. Sinan possessed a tremendous amount of technical knowledge in structural mechanics and properties of the materials, which he gained when he was a military engineer. This background helped him considerably to push the physical limits of masonry when he was building his masterpieces. But despite being a structural mechanism expert, Mimar Sinan was not any worse when it came to artistry. His works were a flawless blend of engineering and artistic talent. All these qualities of Sinan along with his contributions to Ottoman architecture earned him the epithet of "*master builder*". Even after his death Sinan's techniques and solutions to the hardships of masonry became common practice in Ottoman architecture, immortalizing his legacy [91, 92].

Figure 5.4. Portrait of Mimar Sinan [91]

Figure 5.5. Statue of Mimar Sinan [93]

5.3. Adding Şehzade Mehmet Mosque to the equation

Şehzade Mehmet Mosque is one of the gems of prime Ottoman architecture built by Mimar Sinan during the time of Sultan Suleyman the Magnificent. Şehzade Mehmet Mosque was constructed between 1543 and 1548. From a cultural perspective, it reflects all the classical Ottoman architectural traits, including symmetry, proportion, ornaments, and a massive dome.

The Sultan Süleyman ordered the construction of the mosque to honor his son Şehzade Mehmet's early death during a military attack against Safavids in 1543. The task of erecting such an important structure was given to the Mimar Sinan. Sinan used all his knowledge and innovative techniques while incorporating the elements of Ottoman architecture with a touch of Byzantine architecture.

The mosque's plan was of an orthodox Ottoman Mosque, with a massive central dome that shows the trademark of Ottoman architecture supported by a complex system consisting of semi-domes, arches, muqarnases, pendentives, and many other smaller domes. As a tradition of Ottoman mosques,

Şehzade Mehmet Mosque contained an Avlu (Courtyard) that surrounded the Mosque. Avlu was used for social gatherings and events as well as a gathering point before and after the prayers. It also acted as a mid-layer before entering the mosque, preparing the prayers for the transition to a sacred space from outer space. The prayer hall (Cami) was embraced by the massive dome. Cami's plan is a square one that consists of 4 massive columns that support the dome while providing structural stability for the whole mosque. The prayer niche (mihrab) faces the qibla and the pulpit (minbar) is located at the qibla wall which is in the direction of Mecca. There are also 2 tall and slender towers that are called minarets at each corner at the north-west side of the mosque, which are used to spread the voice of prayer call (adhan) wider using the height advantage of the tower.

Overall, Şehzade Mehmet Mosque can be seen as a classic Ottoman Mosque archetype that integrates original elements of structural engineering that show the Mimar Sinan's mastery in structural engineering and architecture, containing all three of *firmitas*, *utilitas*, *venustas* [29, 88, 94].

Given the content of the thesis it must also be mentioned that Şehzade Mehmet Cami's structural behavior is previously studied. These works also contain its numerical model constructed through finite elements [29, 97].

Another issue that needs to be addressed is the damages and repairs Şezade Mehmet Mosque has suffered. Because it is inevitable that an almost 500-year-old masonry structure will be damaged and repaired over time. Even though there were several major earthquakes that occurred in İstanbul in 1557, 1690, 1719, 1766, 1894, 1912 and 1999, the only earthquake that documented to cause minor damage to the structure was in 1766. According to the documents, most of the damage to the building during its lifetime was caused by fires [98].

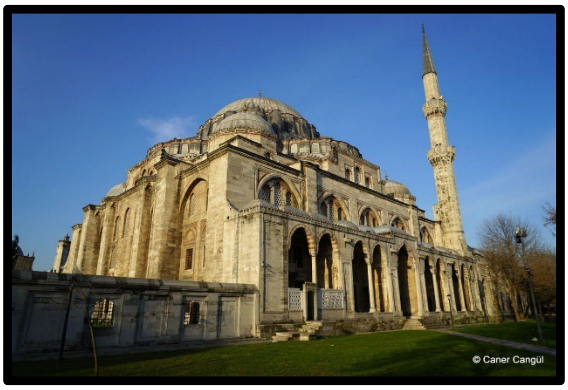

Figure 5.6. Şehzade Mehmet Mosque [95]

Figure 5.7. Avlu of Şehzade Mehmet Mosque [96]

Figure 5.8. Interior view of Şehzade Mehmet Mosque [95]

Figure 5.9. Half domes supporting the main dome [96]

Figure 5.10. Main dome [96]

5.4. Systematically breaking down: The Şehzade Mehmet Mosque's Structural System

In the 16th century, the mainstream way to erect a building was masonry. In masonry, domes and vaults were mandatory to span long lengths. The stress occurred between the masonry units due to spanning was resisted by the mortar between them. Therefore, in massive masonry buildings like Şehzade Mehmet Mosque, where massive lengths were spanned by masonry vaults and domes, the strength of mortar between the units was vital for the structure's integrity. At that time Ottomans had already perfected the mortar aspect of masonry construction and were able to produce mortars that contained egg whites and could create a flawless bond between the units and resist the stresses occurring due to spanning in their massive masonry structures, including Şehzade Mehmet Mosque.

The structural system of Şehzade Mehmet Mosque is unique in not only its mortar but in every other way too. Its smaller arches were strengthened with iron rods and the massive exterior walls were constructed from natural stones. To build its massive dome, a special scaffold and framework system was put in use. Şehzade Mehmet Mosque possesses uniqueness even in its acoustic system. To improve the acoustic quality ceramic pots were injected into the various structural elements of the mosque during the construction.

To further understand and break down the structural system of the building systematically, we can condense it to an assembly of masonry domes, arches, piers, and walls. A perfect three-dimensional symmetry can also be observed in the structure, which also contributes to a less complicated load transmission process.

The systematic breakdown of the structure can be started by separating it into 3 layers. The first and the bottom layer consist of thick exterior walls, 4 massive piers, and 8 small arches that form the base for the second layer elements along with squinches that allow the domes of the second layer to be erect on a square framework.

The second and the middle layer form the structural system that acts like a gate that allows it to transmit the forces coming from the massive dome to the first layer which then transmits it to the foundation and the soil. This layer consists of 4 big arches that are located in the middle of the mosque and on the top of the 4 massive columns of the first layer, 4 pendentives between the arches to permit the placing of the main dome, 4 half domes that connect each arch, 8 lesser half domes neighboring the 4 greater half domes at each side and 4 small domes that are located at each corner of the structure to further increase the stability. On a closer look, one can see that the second layer is a complex web of masonry structural elements that has its own sublayers with the sole purpose of dampening the horizontal force coming from the tension rings of the main dome throughout its layers and distributing it to the exterior walls in a diminished state. The necessity of this dampening process arises from the fact that masonry walls and piers, which consist of most of the first layer are not good at bearing horizontal loads.

The third layer of the structure consists of a massive masonry dome (main dome) and 2 tension rings (upper and lower) along with 8 buttresses to further provide support against the horizontal thrust. The 19-meter-wide dome that forms the majority of the third layer provides a very spacious interior for the prayer hall while creating a celestial effect on the ceiling, which is a common trait in religious buildings. The dome can also be described as an "apple of the eye" for the mosque because it is such an important part of the mosque that these 3 layered complex assemblies of masonry structural elements almost work entirely for transmitting the dome's weight to the soil.

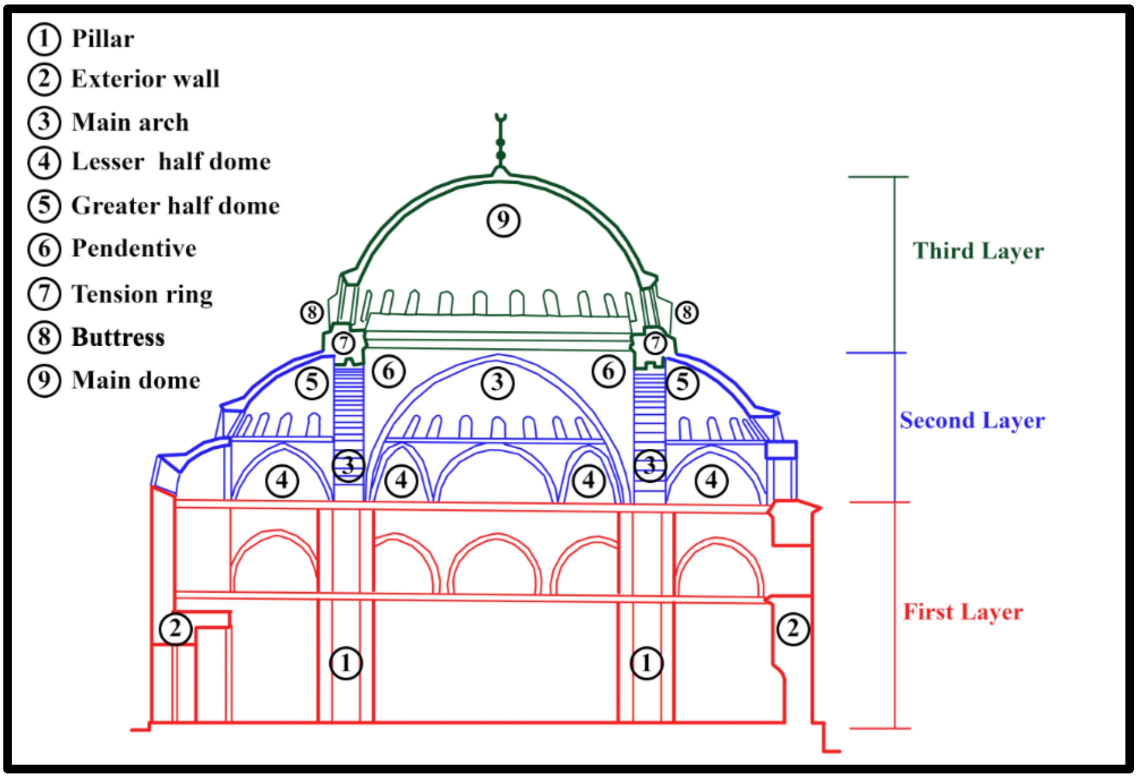

Figure 5.11. Layers of the building's structural system

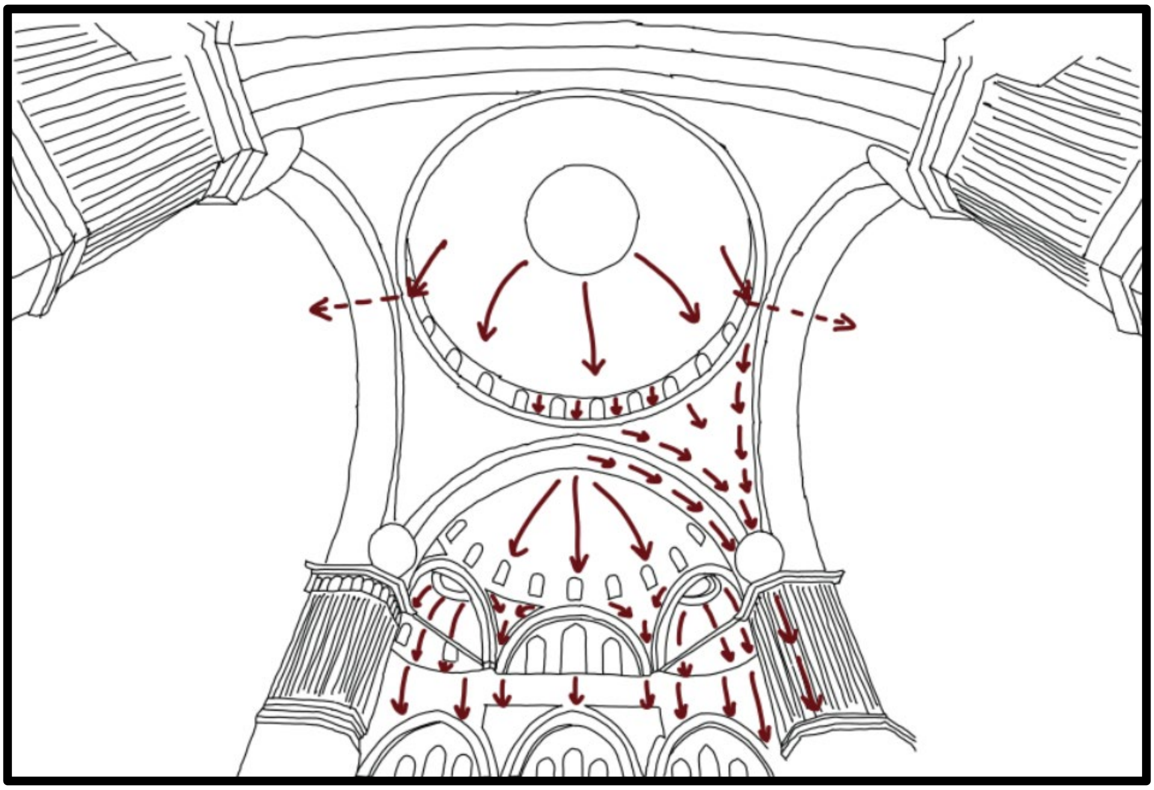

Figure 5.12. Weight transmission mechanisms of the structure

5.5. Using the layers to build a finite element model

The system and the layers used in the previous section to analyze and understand the building's structural system can also be used for building its finite element model. Preparing a finite element model by using these layers creates clarity that leads to a further structural understanding of the building not only for the developer of the model but also for any other conservator in the process. When the finite element model of a systematically broken-down masonry structure is in progress, starting with the bottom layer (first layer) and making your way up to the top layer (third layer) can give one further insight into the load transmission in the building because building a model from bottom to up mimics the real-life sequence of its construction process. When this sequence is followed, the first structural elements that will be modeled will be exterior walls, columns, and the first layer of arches that contains 8 arches. Then the second and third layer will be modeled, and the sequence will be finished with the assignment of the main dome on the model. As stated in the systematically breaking down section one can understand from this sequence that the whole structural system is built to dampen and transmit the loads coming from the main dome to the foundation. This also shows that in the real-life building process, the main dome cannot be inserted without the erection of every other structural element. As can be seen, the load transfer system of the structure can already be understood very well by using these results obtained from the modeling process.

A very important factor that should not be ignored in this process is the contribution of the usage of the frame and shell elements to the understanding of the structural system. When frame and shell elements are used in the finite element model of Şehzade Mehmet Mosque, each structural element is manually constructed and can be distinguished from each other. As stated in the fourth section, when the finite element model composed of frame and shell elements is run under the specified loads, the deformed shape of the model simulates the realistic behavior of the masonry structural elements, thus enable the inspection of the behavior of the building properly.

To build a finite element model of a masonry structure by using frame and shell elements, the modeler must first observe the building very well to understand its structural system. In this case, the impression of decorative elements in masonry buildings should be considered. In historic masonry buildings, the structural system is often hidden by decoration, ornamentation and cladding and it is not easy to obtain the dimensions of the raw structural members from technical drawings. To bypass this, the observer must develop a *Superman-like X-ray vision* to see the raw structural system through these decorative layers which can only be done by architectural intuition.

Completely relying on software to construct the whole mesh of the finite element model from a point cloud or a similar way strips the modeler of all the benefits stated above. In a mesh created through software, the whole model is constructed of solids that have very close dimensions which completely disables the distinction of the elements from each other. A model like this also, even though it later can be optimized by the modeler by hand, can not exclude decorative elements from the structural ones which can cause minor inaccuracies and create a situation like explained in section 4.3. where the behavior of the structural element begins to diverge from its actual state. It must also be stated that constructing a model like this can also disrupt or complexify the transfer of information between the disciplines that contribute to the restoration or conservation process which conflicts with the purpose of this thesis.

5.6. Finite Element Model of Şehzade Mehmet Mosque

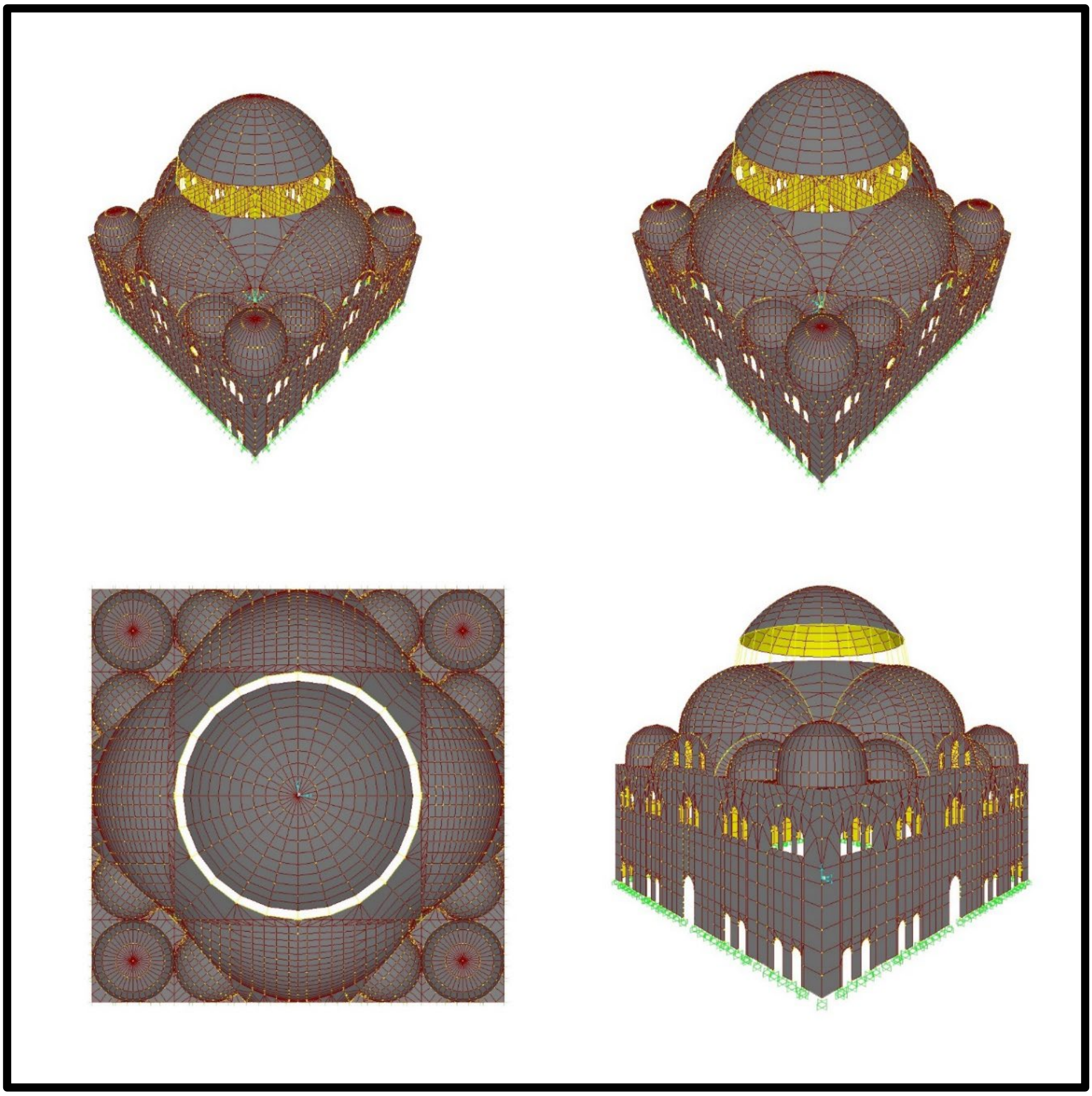

Figure 5.13. Different perspectives of Şehzade Mehmet Mosque's Finite Element Model

The finite element model of Şehzade Mehmet Mosque consists of 7130 joints, 873 frame elements, and 7842 shell elements. Pre-produced drawings were used in the modeling process to detect the building's dimensions. The images above show different views of the numerical model.

5.6.1. Modal shapes and periods

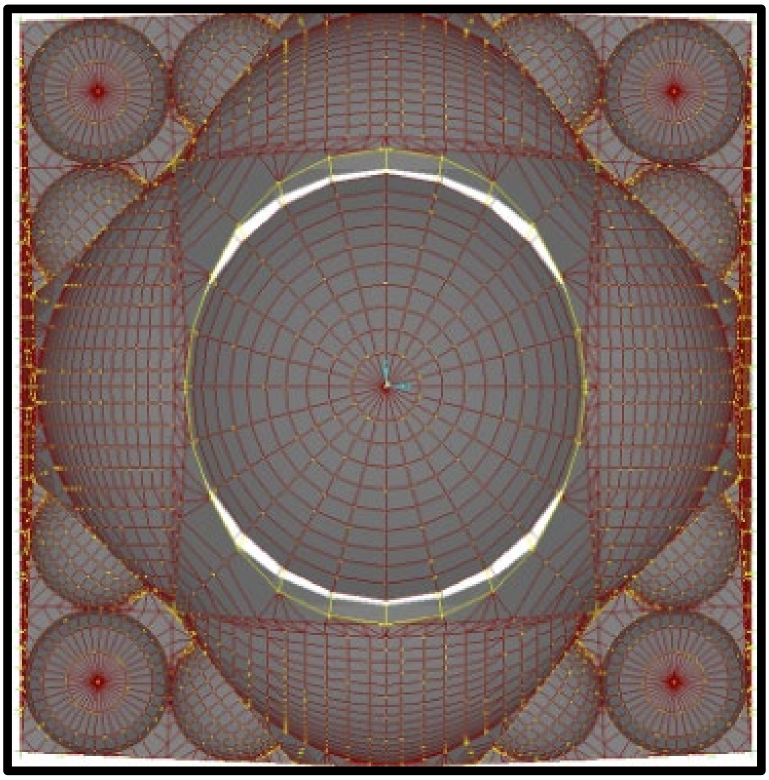

Figure 5.14. Mode: 1 – Period: 0,80 seconds

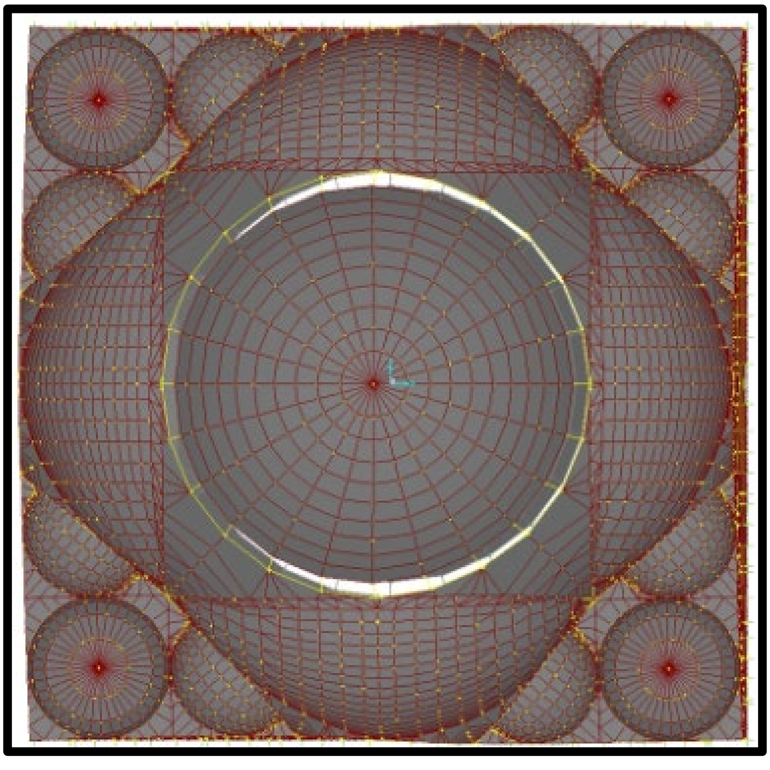

Figure 5.15. Mode: 2 - Period: 0,73 seconds

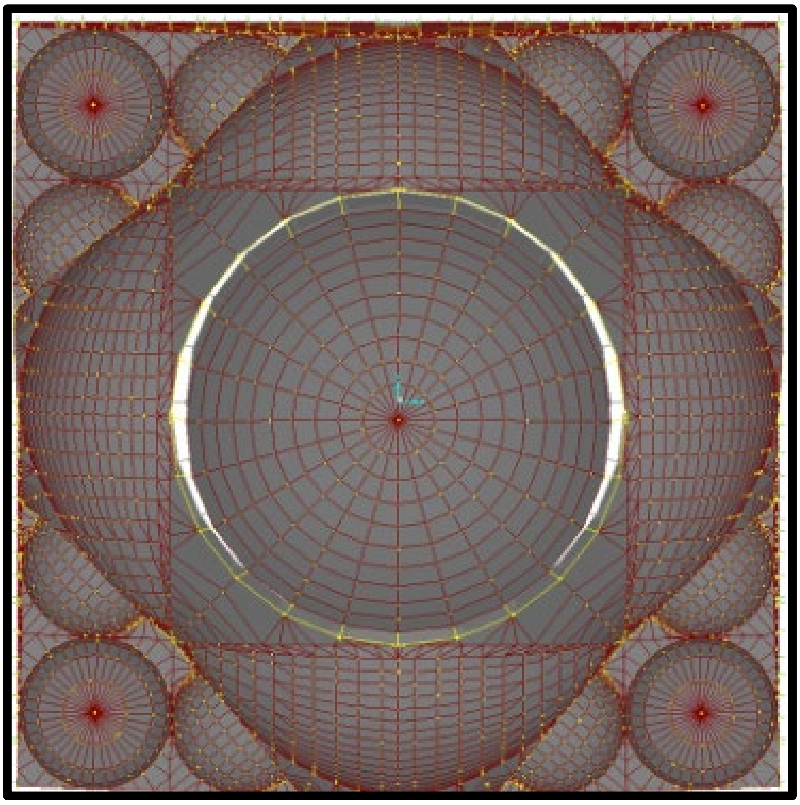

Figure 5.16. Mode: 3 - Period: 0,73 seconds

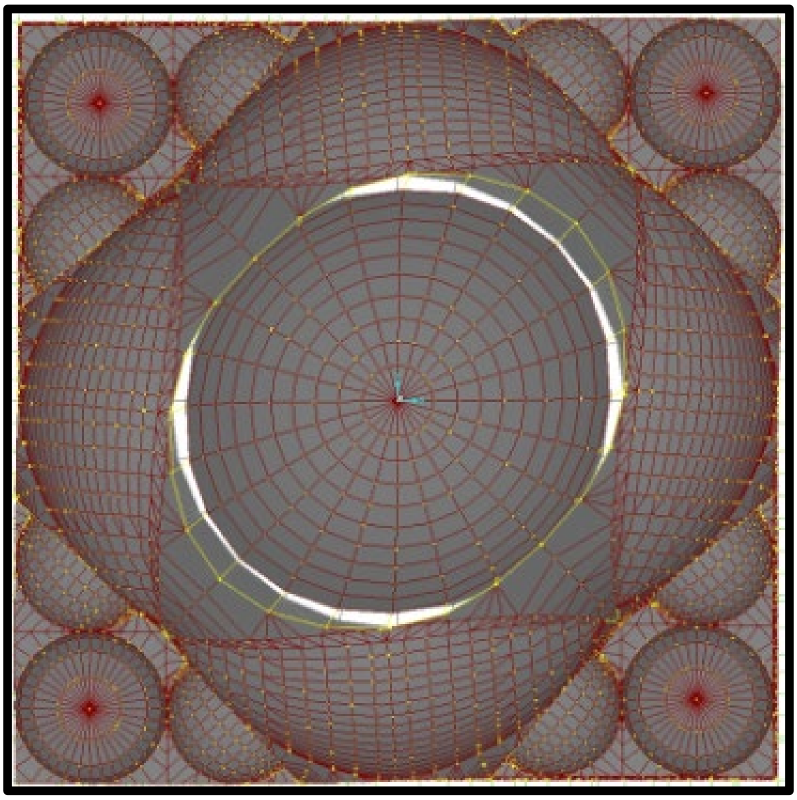

Figure 5.17. Mode: 4 - Period: 0,55 seconds

5.6.2. Structural element groups for detailed evaluation

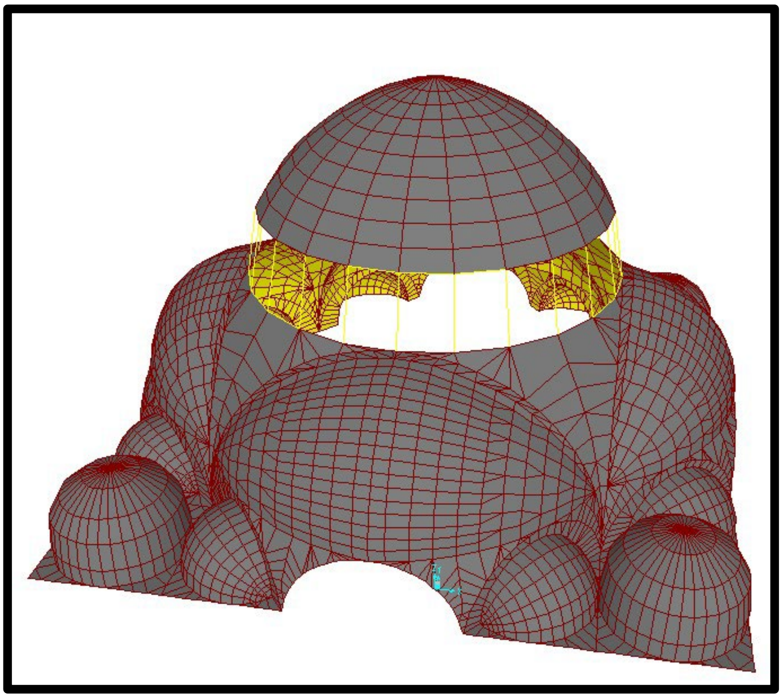

Figure 5.18. Domes, pendentives, and drum in the finite element model of Şehzade Mehmet Mosque

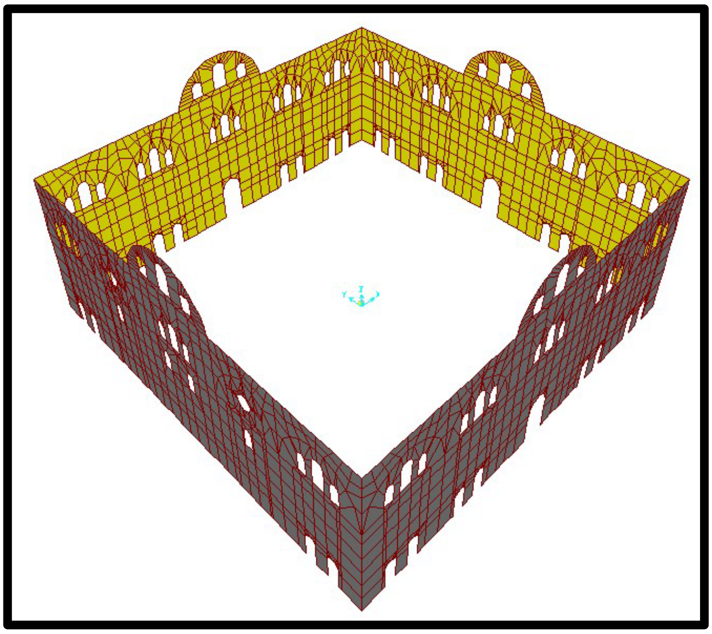

Figure 5.19. Exterior walls in the finite element model of Şehzade Mehmet Mosque

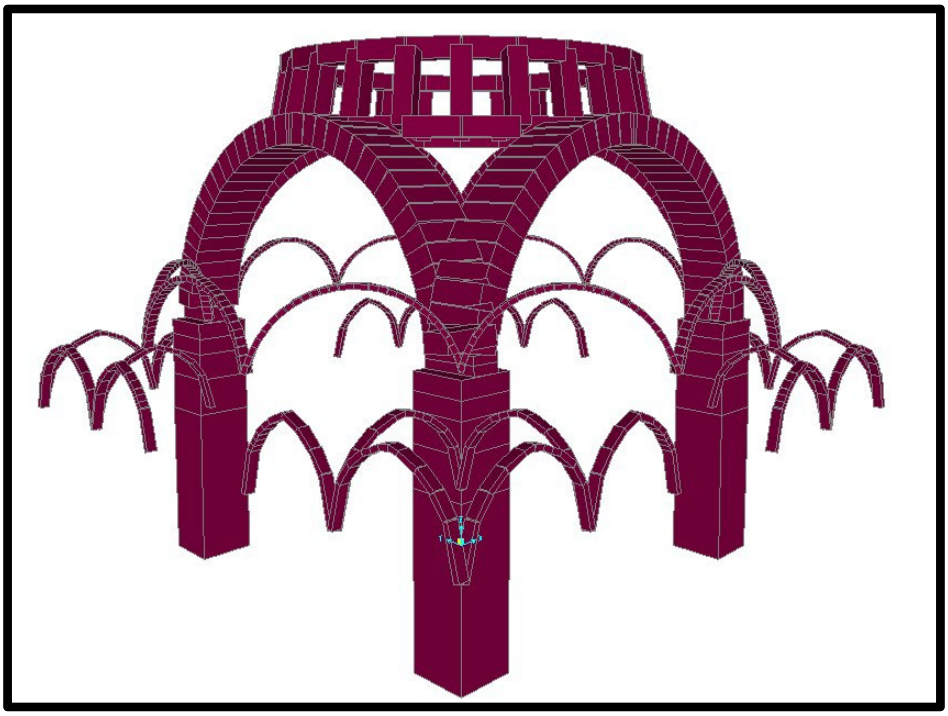

Figure 5.20. Frame system (columns, tension rings, and arches) in the finite elementmodelof Şehzade Mehmet Mosque

5.6.3. Layers of structural system in the model

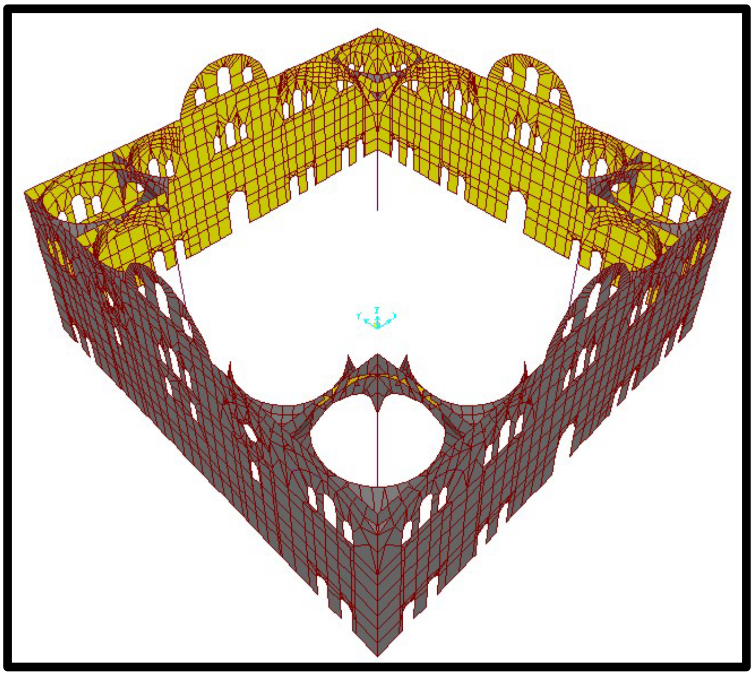

Figure 5.21. First Layer of the Şehzade Mehmet Mosque's structural system in the finite element model

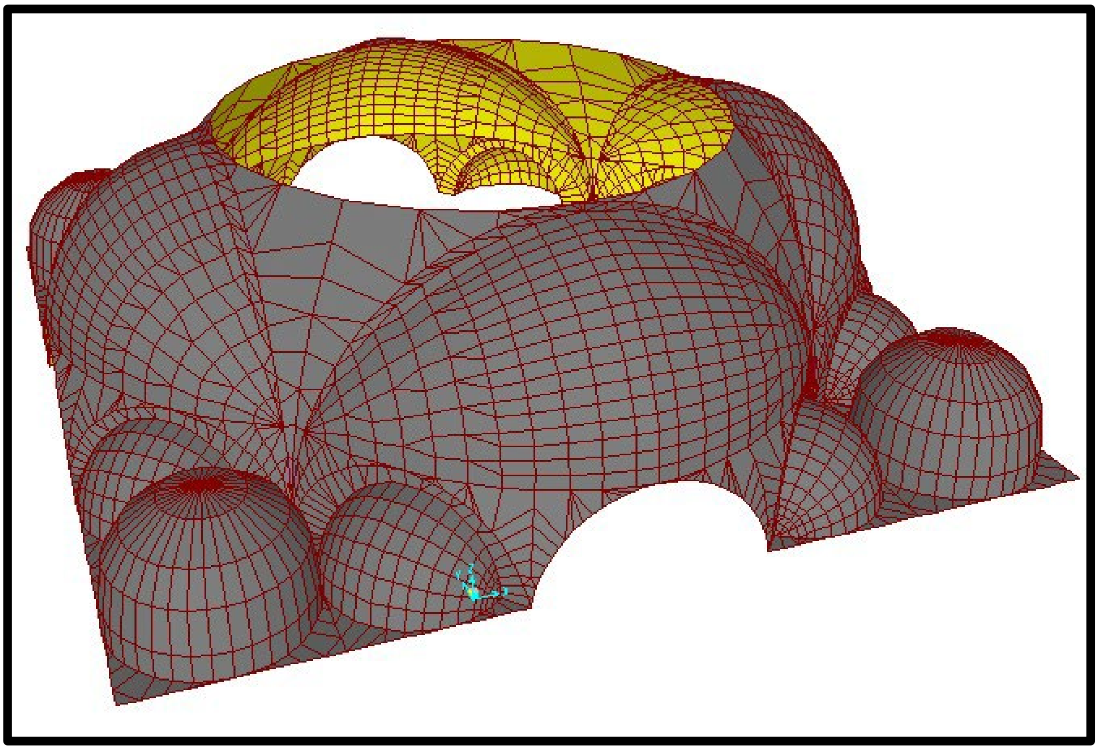

Figure 5.22. Second layer of the Şehzade Mehmet Mosque's structural system in the finite element model

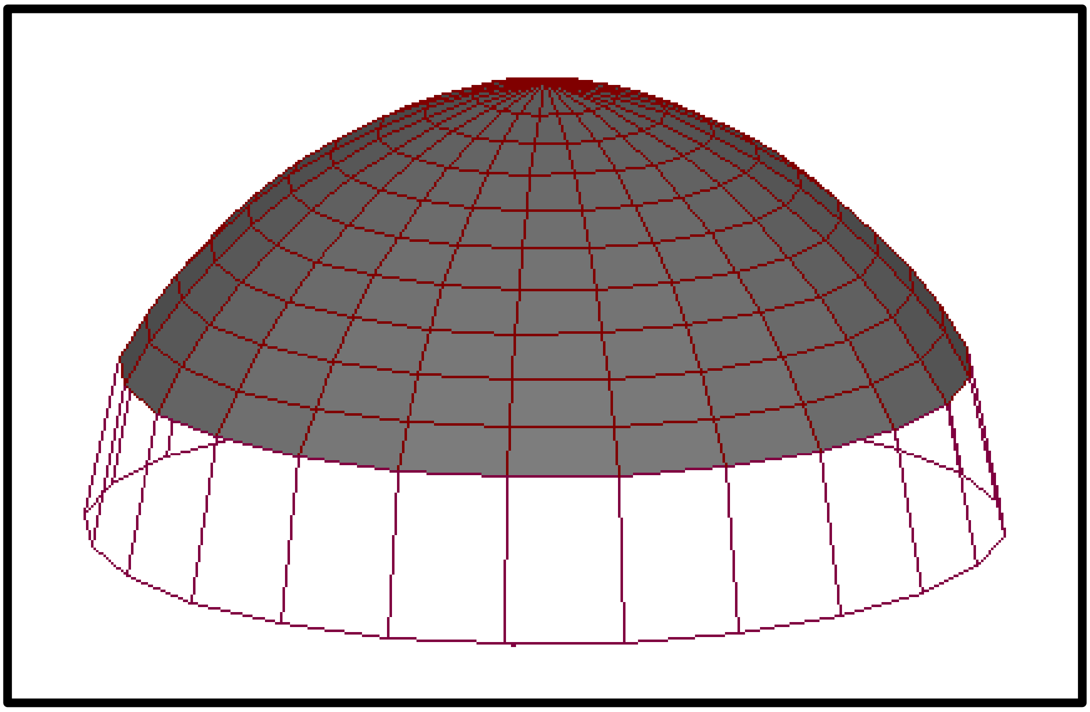

Figure 5.23. Third layer of the Şehzade Mehmet Mosque's structural system in the finite element model

5.6.4. Displacements due to G+EQx and G+ EQy loads

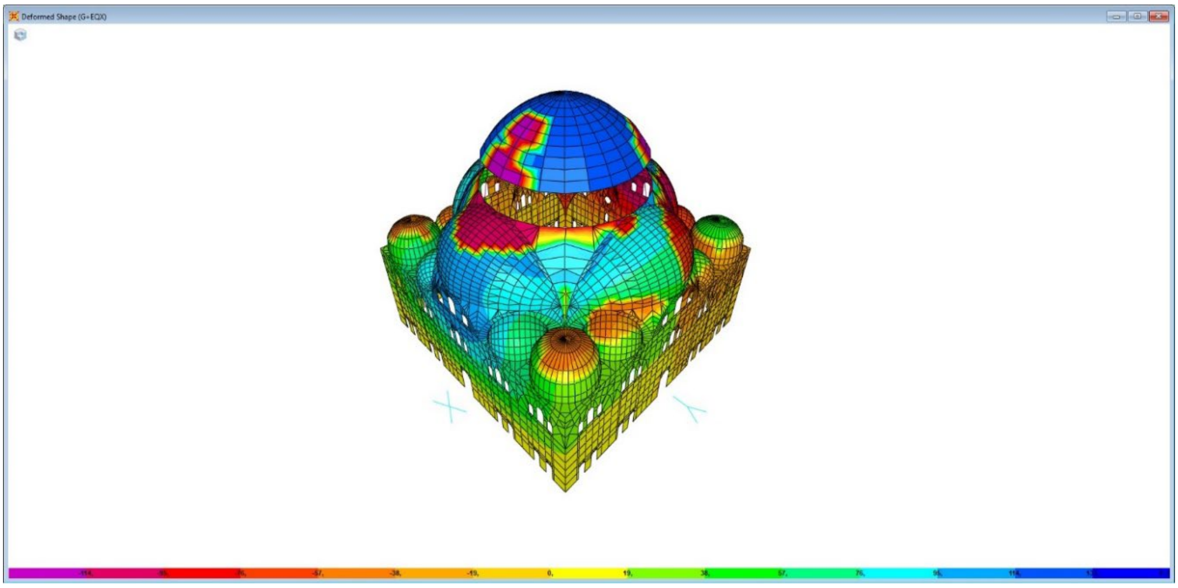

Figure 5.24. Displacements in X direction due to G+EQx Load

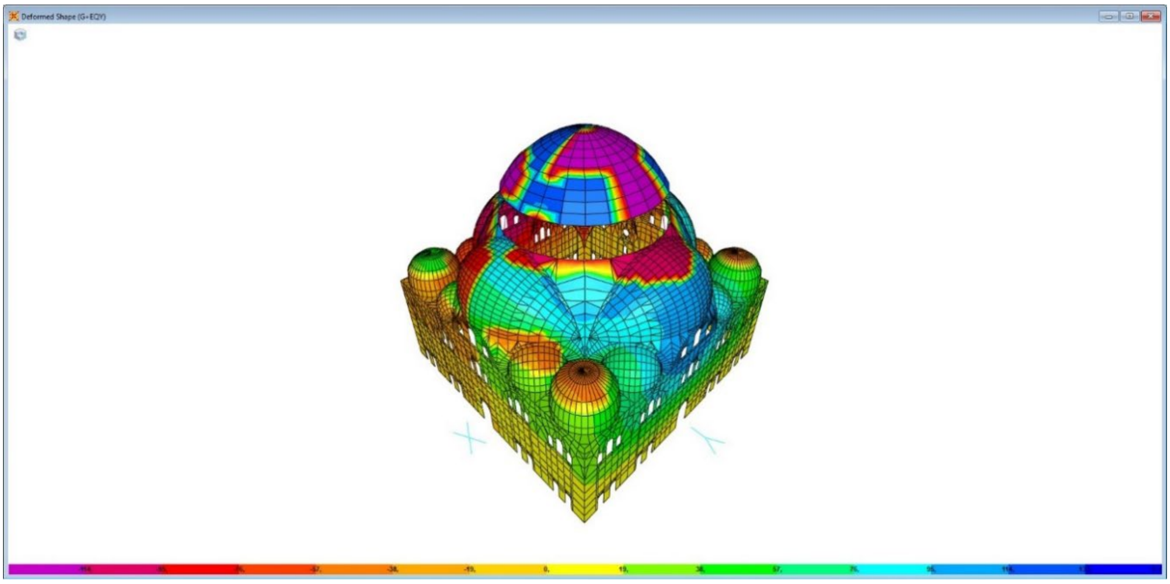

Figure 5.25. Displacements in Y direction due to G+EQx Load

According to the Figure 5.24 and Figure 5.25, the largest displacement in the X direction due to G+EQx load is Δ_x=133 mm while the biggest displacement in the Y direction due to G+EQy load is Δ_x=133 mm. On the scales above, displacements larger than 133 mm in the X direction and displacements larger than 133 mm in the Y direction are shown in dark blue.

5.6.5. Limit compressive and tensile stresses occurring in the third layer due to G+EQx load

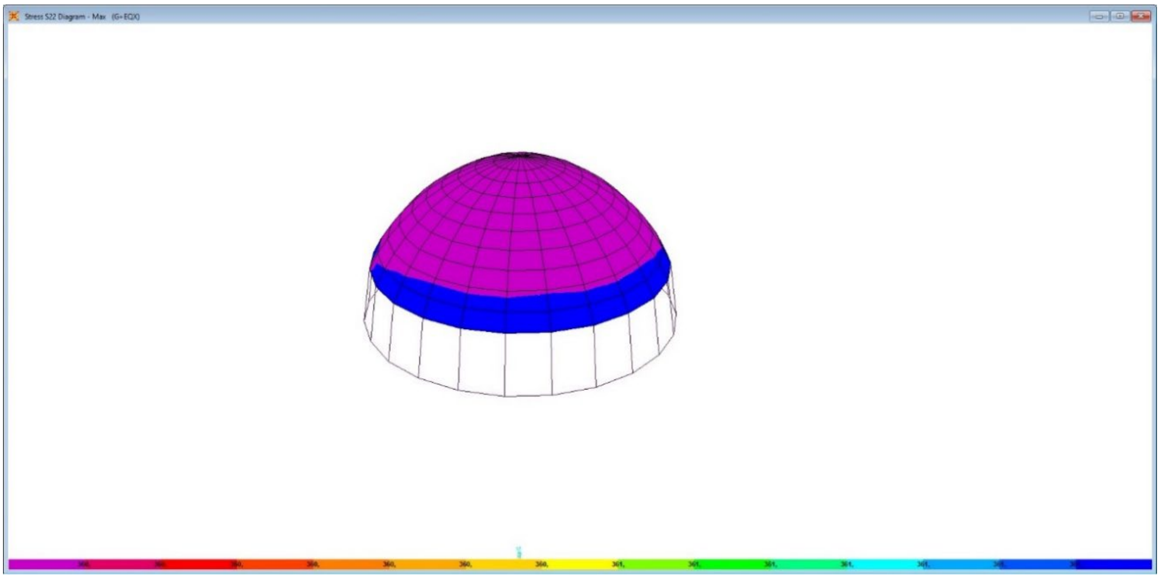

Figure 5.26. S22 tensile stresses in the third layer due to G+EQx load

Figure 5.26 shows the S22 tensile stresses in the third layer due to G+EQx load. The areas that are colored dark blue indicate tensile stresses larger than 0.36 MPa, which is the limit tensile stress for brick masonry ($f_m = 0.36$ MPa).

In SAP2000 software, S11, S22, S33, and S12 stand for specific stress components of a shell or solid element. S11 represents the stresses along the first axis, which is the length of the element. S22 represents the stresses along the second axis, the width of the element. S33 represents the plane that is perpendicular to the first and second axis and goes along the through-thickness of the element. S12 represents the shear stresses acting in the plane that is defined by the first and secondary axes.

As shown in Figure 5.27, the value of 2,4 MPa, the limit compressive stress for the brick masonry ($f_m = 2.4$ MPa), is not exceeded at any point of the third layer. The biggest compression in the area is around 0.08 MPa. The areas that are colored pink in the figure indicate the areas with compressive stress larger than 0.04 MPa.

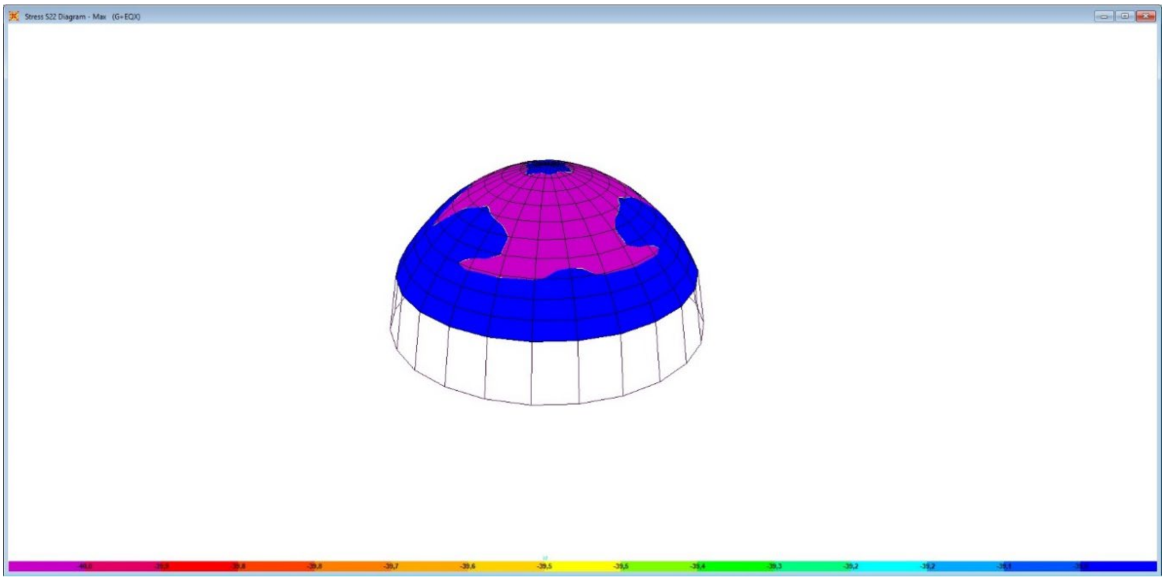

Figure 5.27. S22 compressive stresses in the third layer due to G+EQx load

5.6.6. Limit compressive and tensile stresses occurring in the third layer due to G+EQy load

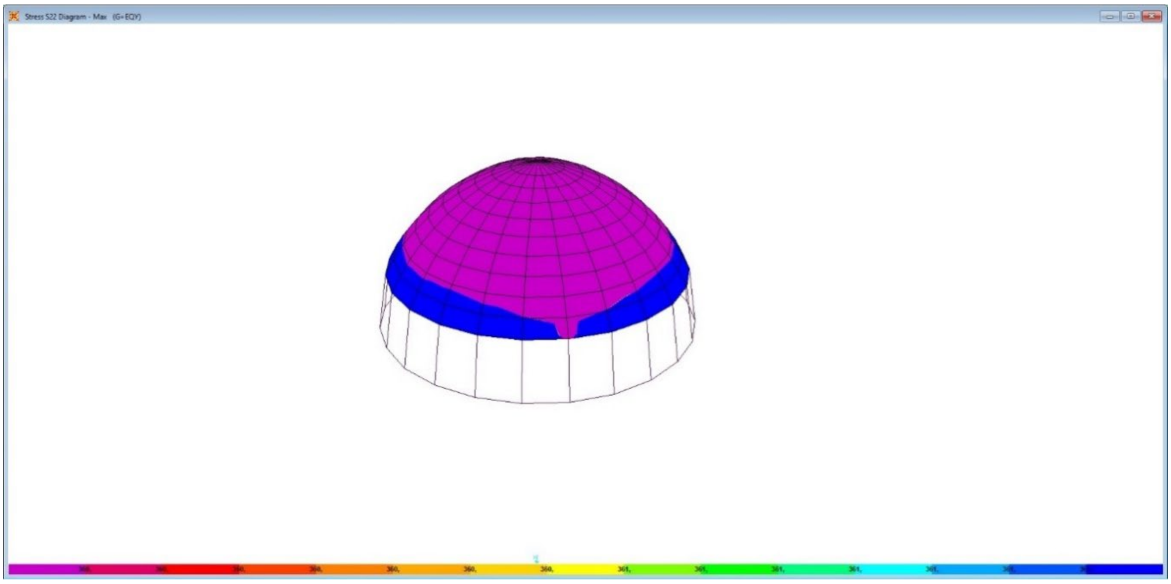

Figure 5.28. S22 tensile stresses in the third layer due to G+EQy load

Figure 5.28 shows the S22 tensile stresses in the third layer due to G+EQy load. The areas that are colored dark blue indicate tensile stresses larger than 0.36 MPa, which is the limit tensile stress for brick masonry ($f_m = 0.36$ MPa).

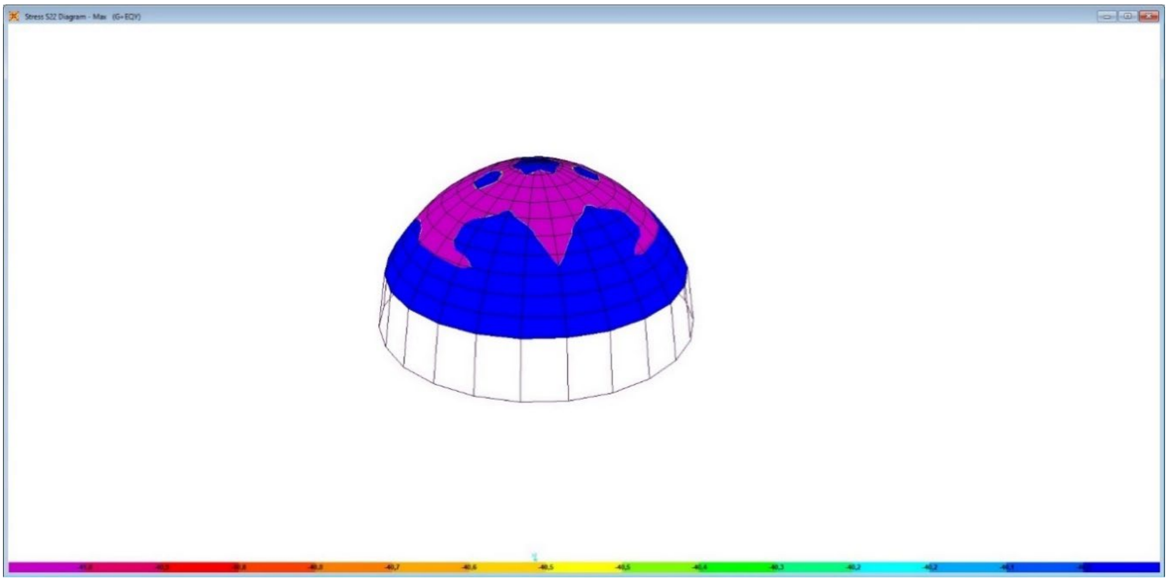

Figure 5.29. S22 compressive stresses in the third layer due to G+EQy load

As shown in Figure 5.29, the value of 2,4 MPa, the limit compressive stress for the brick masonry (f_m = 2.4 MPa), is not exceeded at any point of the third layer. The biggest compressive stress in the area is around 0.08 MPa. The areas that are colored pink in the figure indicate the areas with compressive stress larger than 0.04 MPa.

5.6.7. Limit shear stresses occurring in the third layer due to G+EQx and G+EQy load

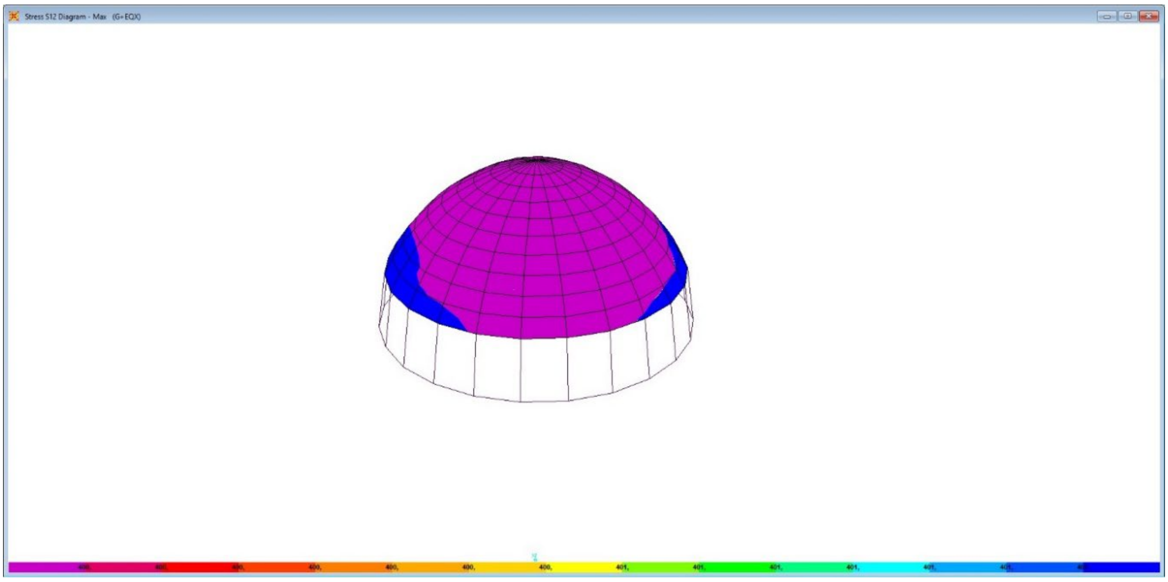

Figure 5.30. Shear forces (S12) occurred in the third layer due to G+EQx load

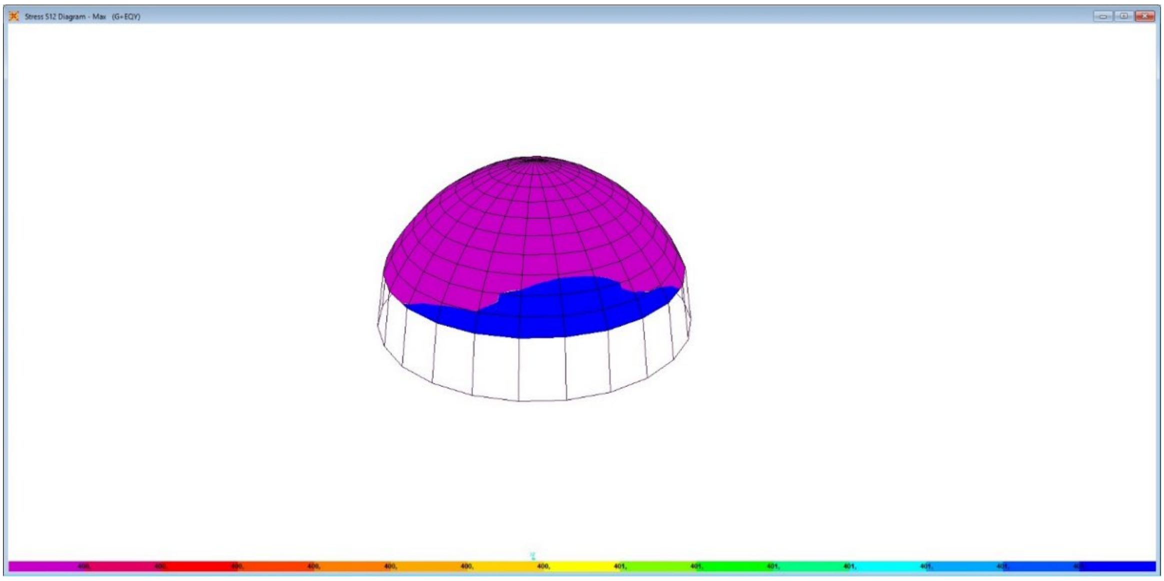

Figure 5.31. Shear forces (S12) occurred in the third layer due to G+EQy load

Figures 5.31 shows the shear forces (S12) occurred in the third layer due to G+EQx and G+EQy loads respectively. According to the results, the safety limit for the brick masonry domes (τm = 1.05 MPa) has not been exceeded in any point of the layer. To show any noticeable shear force on the layer, the shear forces larger than 0.4 MPa are colored dark blue in both figures.

5.6.8. Limit compressive and tensile stresses occurring in the second layer due to G+EQx load

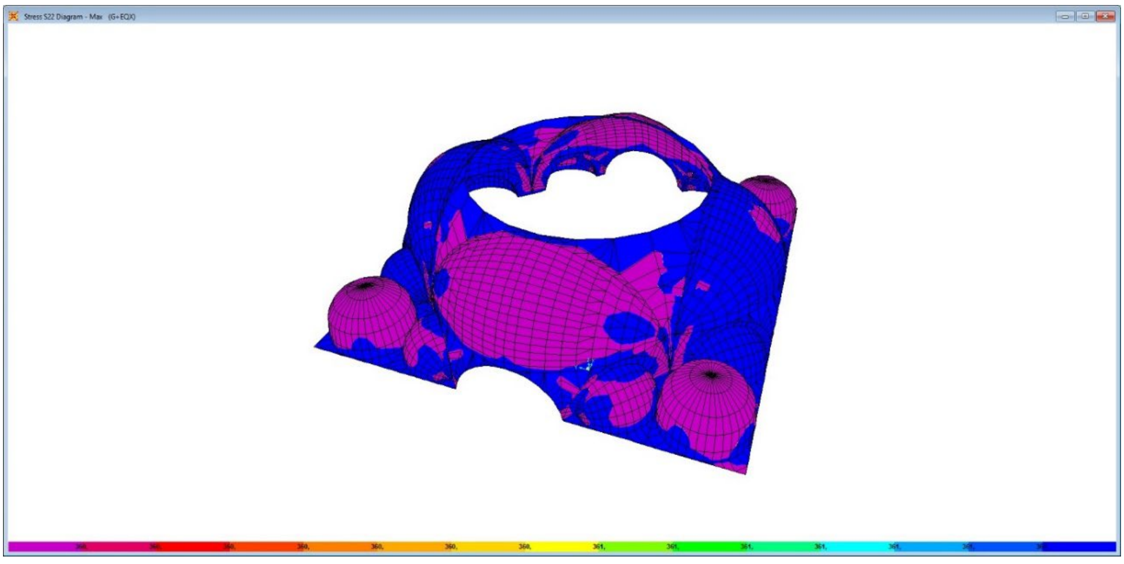

Figure 5.32. S22 tensile stresses in the second layer due to G+EQx load

Figure 5.32 shows the S22 tensile stresses in the second layer due to G+EQx load. The areas that are colored dark blue indicate tensile stresses larger than 0.36 MPa, which is the limit tensile stress for brick masonry (f_m = 0.36 MPa). Analysis shows that the majority of the second layer fails under the loads conceived in the structure due to given earthquake.

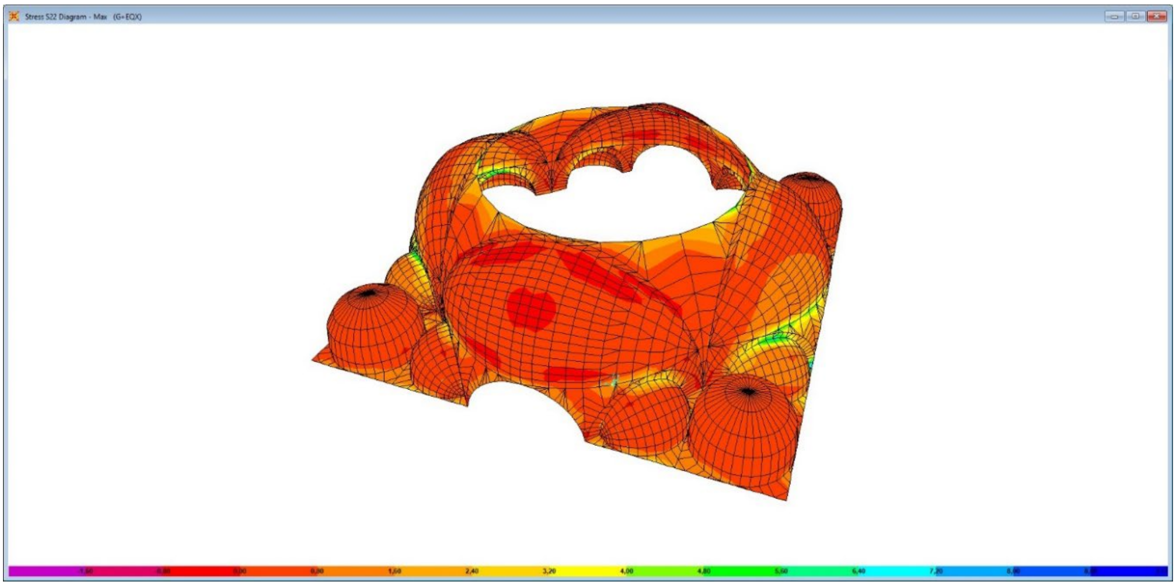

Figure 5.33. S22 compressive stresses in the second layer due to G+EQx load

As shown in Figure 5.33, the value of 2,4 MPa, the limit compressive stress for the brick masonry (f_m = 2.4 MPa), is not exceeded at any point of the second layer due to G+EQx load. The biggest compressive stress in the layer is around 0.05 MPa. The overall stress distribution of the second layer can be seen in the figure.

5.6.9. Limit shear stresses occurring in the second layer due to G+EQx and G+EQy load

Figure 5.34 and Figure 3.35 show the shear forces (S12) occurred in the second layer due to G+EQx and G+EQy loads respectively. According to the results, the safety limit for the brick masonry domes (τm = 1.05 MPa) has been exceeded in several points of the layer, which are colored dark blue. Like under tensile stresses, the second layer of the structure fails under shear stresses too.

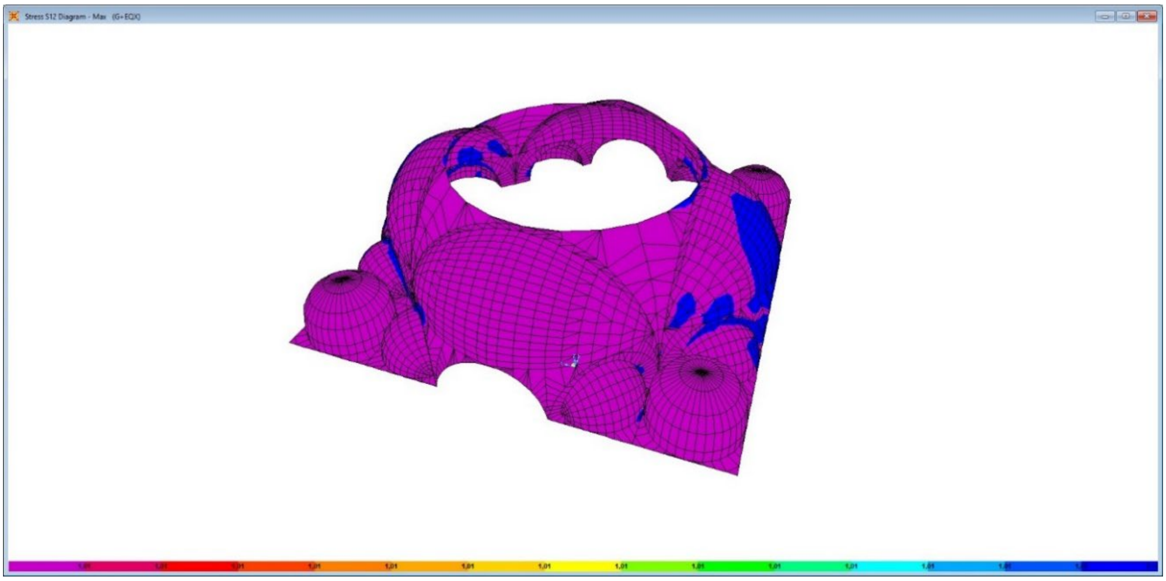

Figure 5.34. Shear forces (S12) occurred in the second layer due to G+EQx load

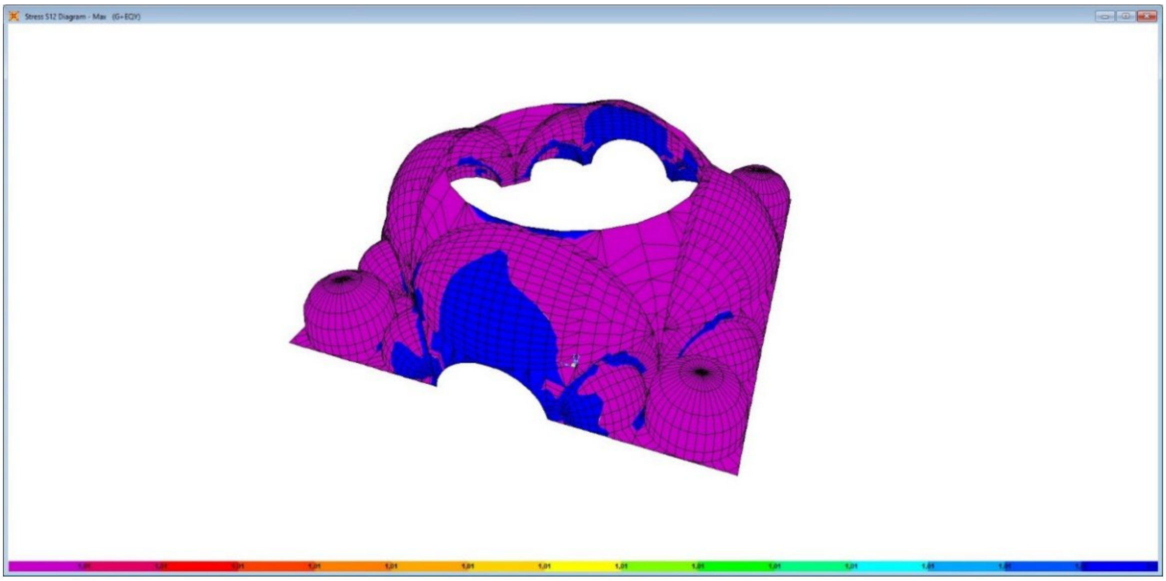

Figure 5.35. Shear forces (S12) occurred in the second layer due to G+EQy load

5.6.10. Limit compressive and tensile stresses occurring in the first layer due to G+EQx load

Figure 5.36 shows the S22 tensile stresses in the first layer due to G+EQx load. The areas that are colored dark blue indicate the tensile stresses larger than 0.135 MPa, which is the limit tensile stress for stone masonry (f_m = 0.135 MPa). Analysis shows that like the second layer, the majority of the first layer fails under the loads conceived in the structure due to given earthquake too.

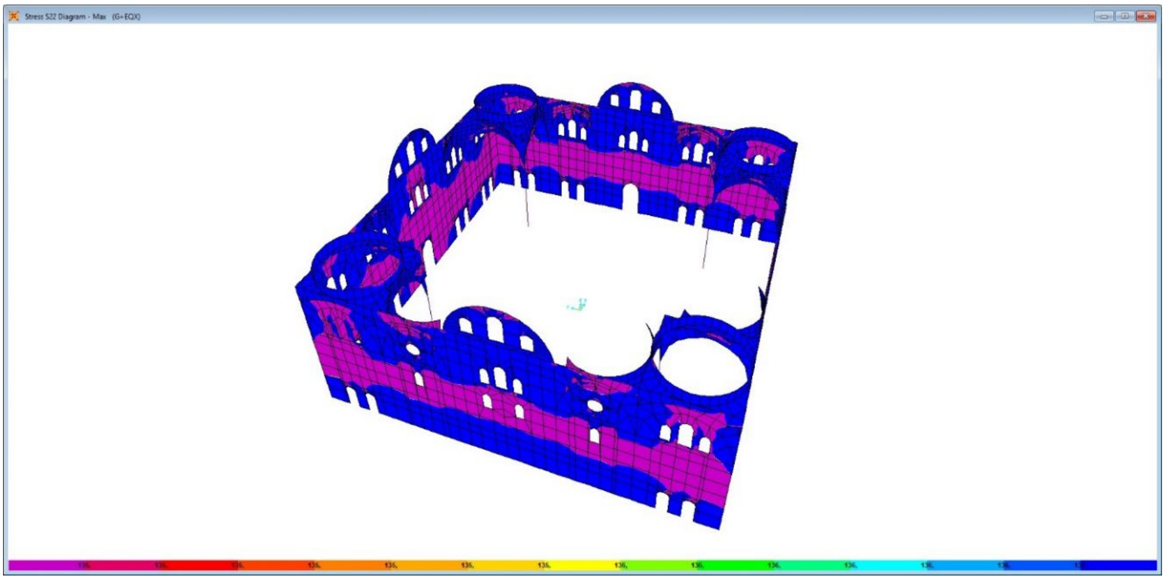

Figure 5.36. S22 tensile stresses in the first layer due to G+EQx load

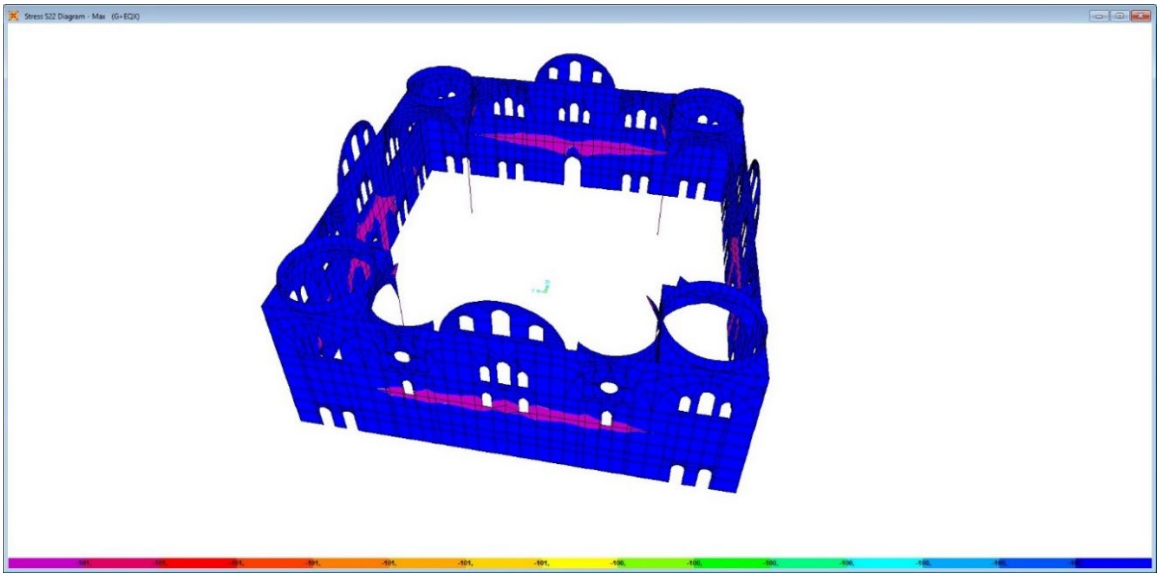

Figure 5.37. S22 compressive stresses in the first layer due to G+EQx load

As shown in Figure 5.37, the value of 0,9 MPa, the limit compressive stress for the stone masonry (f_m = 0,9 MPa), is not exceeded at any point of the first layer due to G+EQx load. The biggest compressive stress in the layer is around 0.2 MPa. The areas that are colored pink in the figure indicate the areas with compressive stress larger than 0.1 MPa.

5.6.11. Limit compressive and tensile stresses occurring in the first layer due to G+EQy load

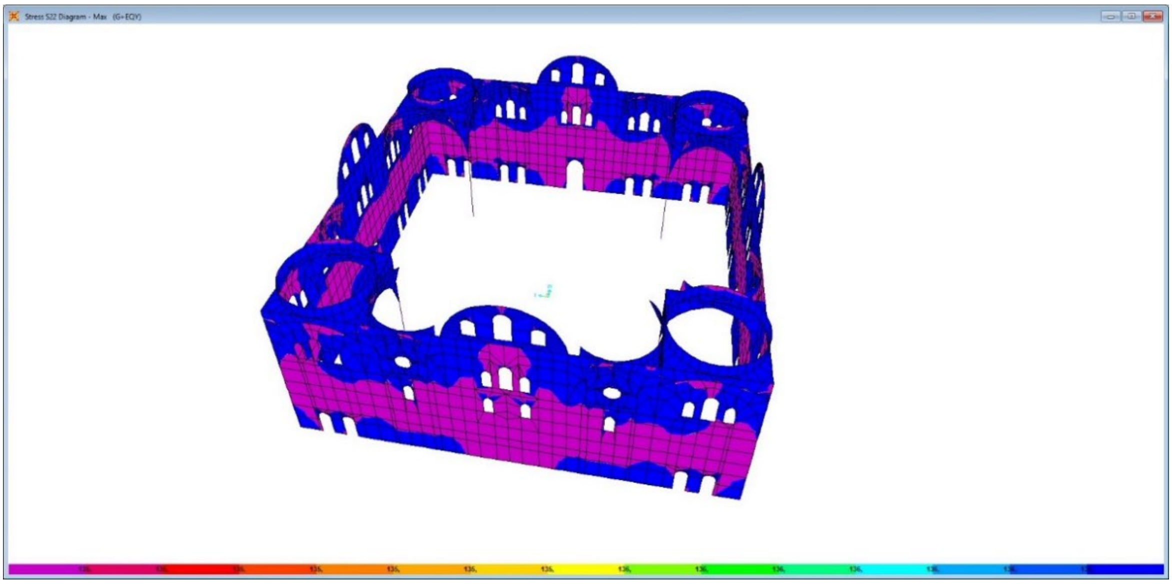

Figure 5.38. S22 tensile stresses in the first layer due to G+EQy load

Figure 5.38 shows the S22 tensile stresses in the first layer due to G+EQy load. The areas that are colored dark blue indicate the tensile stresses larger than 0.135 MPa, which is the limit tensile stress for stone masonry (f_m = 0.135 MPa). Due to the structure's symmetry, it shows similar failure patterns in every layer regardless of the earthquake's direction.

As shown in Figure 5.39, the value of 0,9 MPa, the limit compressive stress for the stone masonry (f_m = 0,9 MPa), is not exceeded at any point of the first layer due to G+EQy load. The biggest compressive stress in the layer is around 0.2 MPa. The areas that are colored pink in the figure indicate the areas with compressive stress larger than 0.1 MPa.

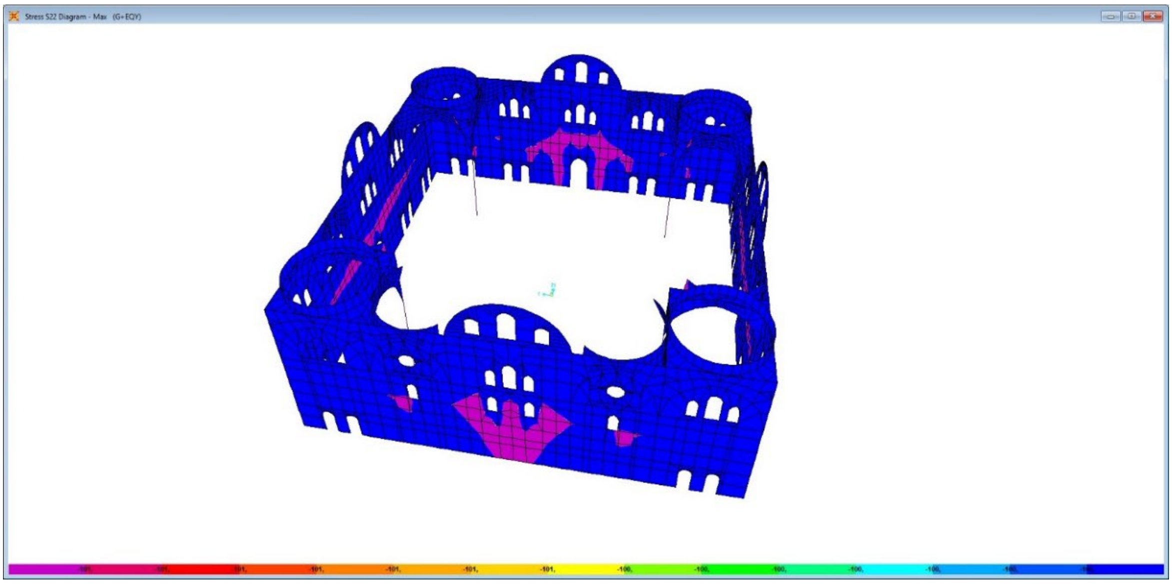

Figure 5.39. S22 compressive stresses in the first layer due to G+EQy load

5.6.12. Limit shear stresses occurring in the second first due to G+EQx and G+EQy load

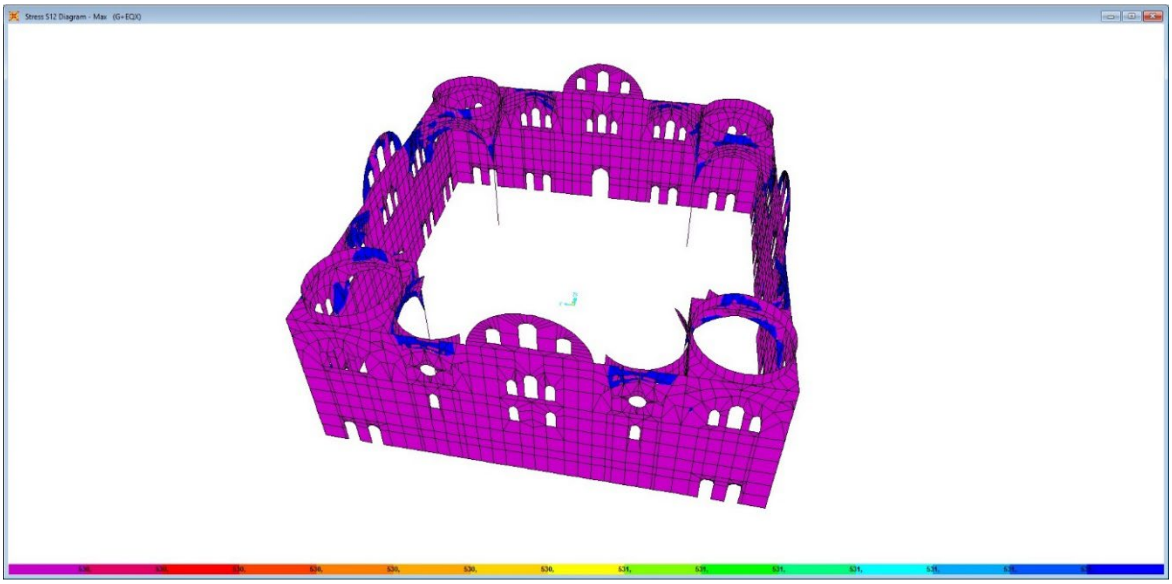

Figure 5.40. Shear forces (S12) occurred in the first layer due to G+EQx

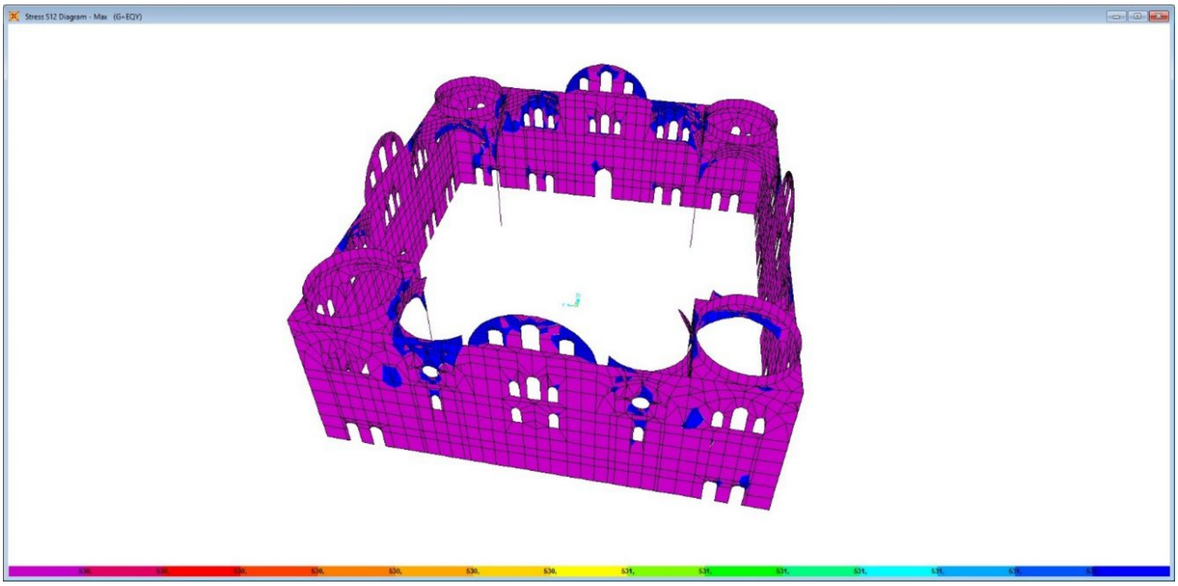

Figure 5.41. Shear forces (S12) occurred in the first layer due to G+EQy

Figures 5.40 and Figure 5.41 shows the shear forces (S12) occurred in the first layer due to G+EQx and G+EQy loads respectively. According to the results, the safety limit for the stone masonry domes (τm = 0,53MPa) has been exceeded in several points of the layer, which are colored dark blue. Even though they are not in the majority like in second layer, analysis shows that the first layer becomes vulnerable against shear forces under the earthquake too.

5.7. Introduction to The Results of Finite Element Analysis on The Şehzade Mehmet Mosque

The report in this section presents the results of the finite element analysis carried out on Şehzade Mehmet Mosque (located in Fatih/İstanbul) to determine its structural behavior in a possible earthquake.

Şehzade Mehmet Mosque has a square plan with 35 m x 35 m dimensions. The diameter that forms the third layer of the structure is 19 m. The 0,5 cm thick brick dome is supported by two tension rings and 24 thick stone buttresses that act as both windows and supports for the dome. The height of the dome from its supports is 6,75 cm. The total height of the building reaches up to 35 meters starting from the base level. The first two layers of the structure are carried by thick stone walls of the building which reach thicknesses up to 2,5 m.

The modeling approach in this case study only considers the main building, excluding the Avlu and minarets from the equation.

5.7.1. Outlines of the finite element model and calculations

The finite element model of Şehzade Mehmet Camii used in the analysis is prepared according to the modeling features and proper modeling practices of the SAP2000 program. All geometric dimensions and measurements required to create the mathematical model are obtained from technical drawings. The parameters of modeling and calculation are presented below:

- Most of the structure is capable of being modeled using solely shell elements. Because of this, the meshes of the main dome, greater and lesser half domes, corner domes, pendentives, and all the walls are created with shell elements.
- The finite element model used in the analysis consists of 7130 joints, 873 frame elements, and 7842 shell elements.
- The arches and the columns that bear the load of the domes are modeled with frame elements. In the analysis the free shear potential of these frame element’s joins are neglected.
- The material properties of the masonry structural elements were selected by considering the values recommended for masonry structures in the Earthquake Specification, which were previously used in the research that was published in international journals and already consists in the literature.
- Brick and stone masonry structural elements are considered a unique assembly of masonry units and mortar, and their modulus of elasticity and unit weight were input to the analysis accordingly.
- Two different loading cases were applied to the model, considering constant loads and ground motion-induced stress defined by the given earthquake spectrum. The spectrum is applied individually in both directions at EQx and EQy loading.
- For easier interpretation of the results, two load combinations were defined as G + EQx (Dead loads + earthquake loading along the x-axis) and G + EQy (Dead loads + earthquake loading along the y-axis).
- The first 4 modes are taken into account in the spectral calculation where earthquake effects are determined.

Some of the material properties that were used in the finite element analysis of the Şehzade Mehmet Mosque are given in the table below.

Table 5.1. Material properties used in the finite element analysis of Şehzade Mehmet Mosque

Element Type	Modulus of Elasticity (kN/m^2)	Unit Weight (kN/m^3)	Unit Mass (t/m3)
Brick Domes and Pendentives (including mortar)	1200000 (1200 MPa)	24	2.45
Stone walls (including mortar)	450000 (450 MPa)	24	2.45
Columns	2000000 (2000 MPa)	24	2.45

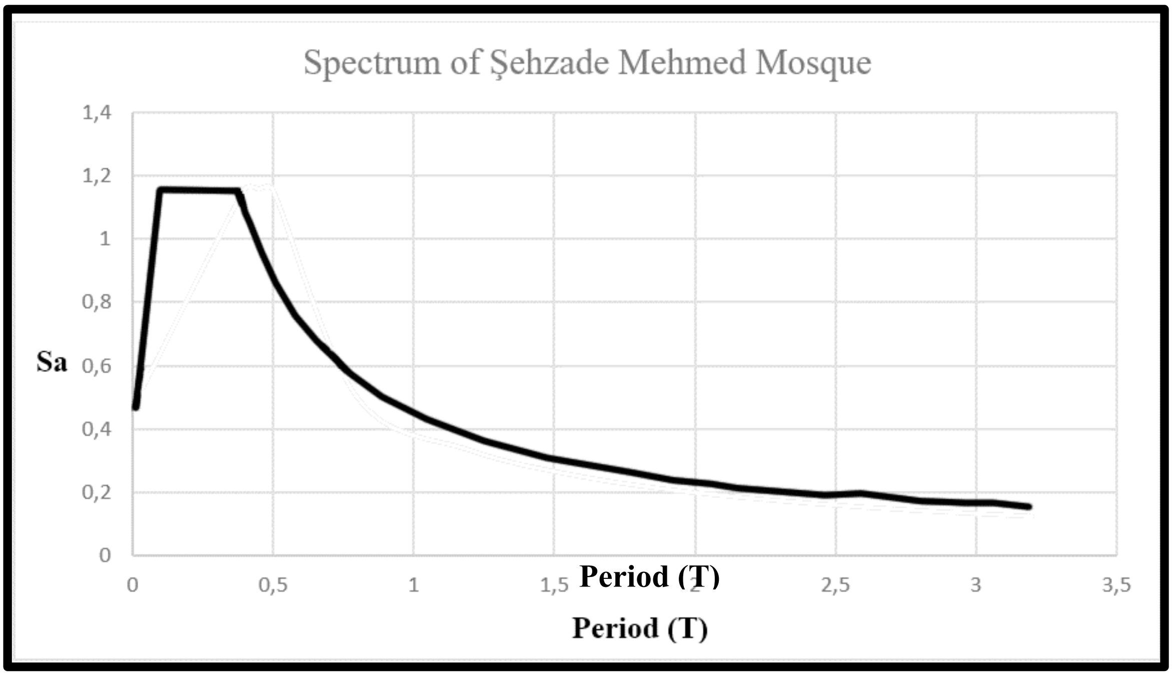

Figure 5.42. Spectrum of Şehzade Mehmet Mosque

5.7.2. Evaluation of the finite element analysis results

In current earthquake regulation, the recommended compressive strength for brick masonry walls is f_{sf}= 0.8 MPa, and the accepted compressive strength for stone masonry walls is f_{sf}= 0.3 MPa. To evaluate the earthquake resistance of Şehzade Mehmet Mosque, the resultant stresses from the analysis can be compared to the recommended values. During the evaluation of the results, no reduction was made in the resultant forces (R=1). In return, the stresses that were calculated in the analysis were compared with the limit values that were attained by multiplying the safety stresses by 3. As a result, limit stress for the domes becomes;

$$f_m = 0.8 \times 3 = 2.4 \text{ MPa}$$

while the limit for masonry walls and arches becomes;

$$f_m = 0.3 \times 3 = 0.9 \text{ MPa}$$

as the accepted values.

In masonry analysis, the limit tensile strength of the masonry can be considered as 15% of its limit compressive strength. With this the limit tensile strength for the domes becomes;

$$f_{m(tensile)} = 2.4 \times 0.15 = 0.36 \text{ MPa}$$

while for walls and arches it becomes;

$$f_{m(tensile)} = 0.9 \times 0.15 = 0.135 \text{ MPa}$$

as the accepted values.

The shear stress that occurs due to earthquake (S12 stresses in the SHELL elements), will be compared to the limit shear strength (τm), which is calculated according to the following equation.

$$\tau_m = \tau_o + \mu_\sigma$$

In this equation τ_m = limit stress for masonry wall, τ_o = wall cracking limit stress, μ= friction coefficient (can be taken as 0,5), and σ indicates vertical stress of the masonry wall. For main dome, pendentives, arches, lesser domes, and greater domes, which are all made from brick masonry, the limit cracking stress is;

$$\tau_o = 0.15 \times 3 = 0.45 \text{ MPa}$$

while for stone walls this value becomes;

$$\tau_o = 0.10 \times 3 = 0.30 \text{ MPa}$$

as accepted.

Table 5.2. Material types and strengths of structural elements in Şehzade Mehmet Mosque

Material Type	Compressive Strength (MPa)	Tensile Strength (MPa)	Shear Strength (MPa)
Brick domes, arches, and pendentives	2.4	0.36	1.05
Stone walls	0.9	0.135	0.53

The structural calculations of Şehzade Mehmet Mosque have been done according to the load combinations given in the Table 5.2 on SAP2000 finite element software. Logically it is impossible to interpret the displacements, forces, and stresses in every joint or structural element individually. For this reason, interpretation of the results is done with the help of heat maps of stress provided by SAP2000 software. Only the most unfavorable values are considered in the interpretation of the given results. Several base shear forces obtained from various load conditions and combinations along with the reactions are provided in Table 5.3. The total weight of the structure is 133162 kN, while the base shear force occurring due to the earthquake in X direction is 49899 kN, and the base shear force occurring due to the earthquake in Y direction is 49246 kN. According to these results, the base shear force structure is subjected corresponds to 37% of its total weight in X direction and Y direction.

Table 5.3. Base shear forces and axial forces

Loading Type	Analysis Type	Stage	Base Shear force in X direction (kN)	Base Shear force in Y direction (kN)	Vertical Reactions (kN)
G (Lateral load)	Linear-Static		0	0	133162
EQx	Linear-Response Spectrum	Max	49899	45,8	61,3
EQy	Linear-Response Spectrum	Max	45,6	49246	2,4
G+EQx	Combination	Max	49899	45,8	133224
G+EQx	Combination	Min	-49899	-45,8	133101
G+EQy	Combination	Max	45,7	49246	133165
G+EQy	Combination	Min	-45,7	-49246	133160

As seen in the figures, the biggest displacement in the structure along the X direction due to earthquake is $\Delta_x = 116$ mm while the biggest displacement along the Y direction due to earthquake is $\Delta_y = 116$ mm. Displacement values are taken from the top of the dome, which represents the highest point of the numerical model.

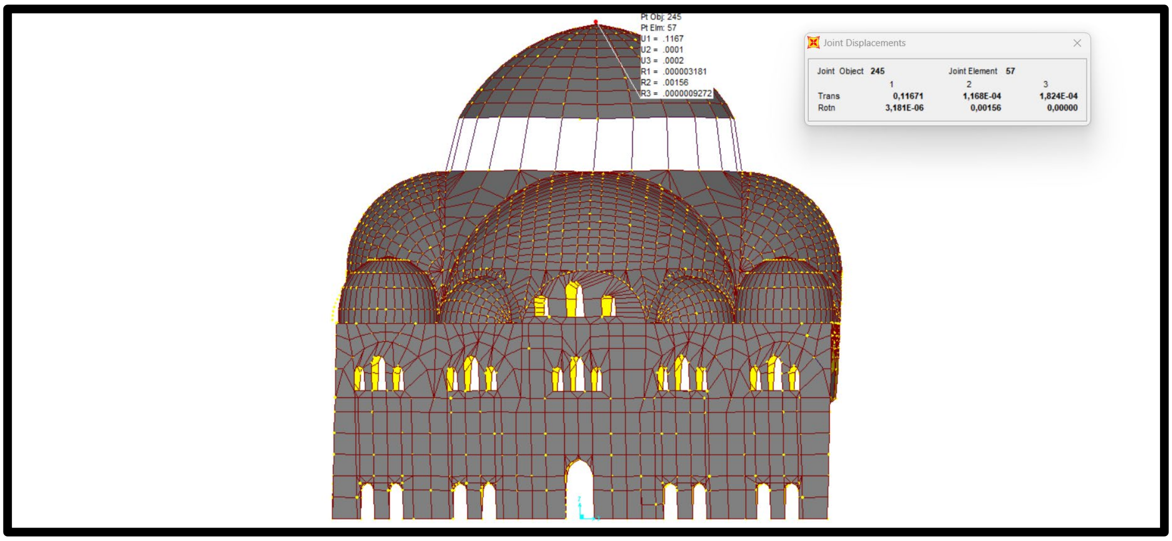

Figure 5.43. Biggest displacement occurring in the X direction due to earthquake

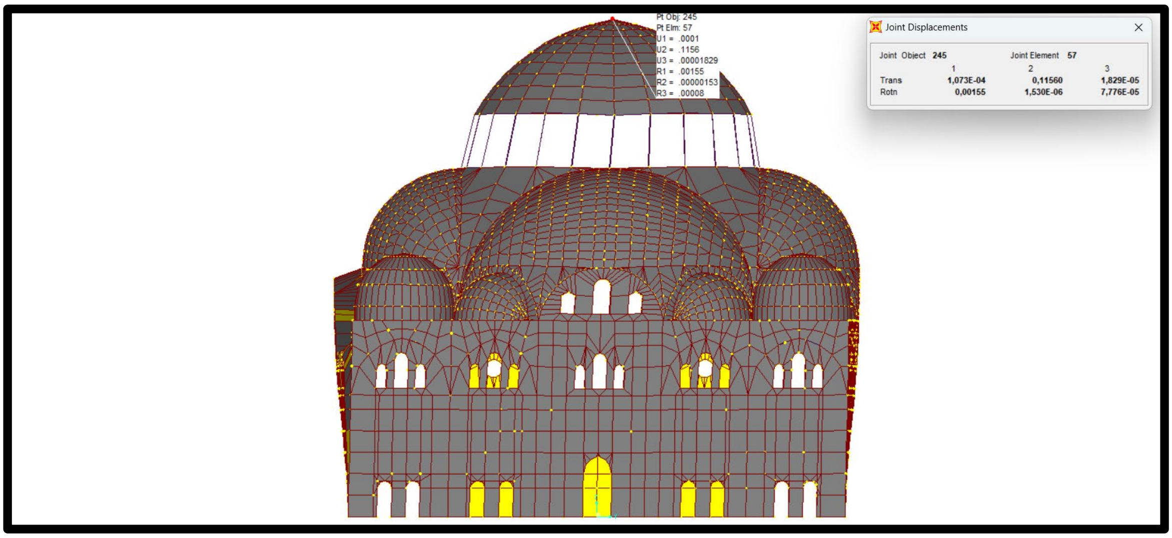

Figure 5.44. Biggest displacement occurring in the Y direction due to earthquake

By evaluating and interpreting the stress values of the SHELL elements in both S22 (in the vertical direction relative to the element's local axis which corresponds to the compressive and tensile stresses) and S12 directions (corresponds to the shear stresses) (formats of the directions were given according to the format of SAP2000 software), a descriptive conclusion can be drawn about the earthquake resistance of the structure.

The characteristic structural elements that affect the structures behavior the most are shown in Table 5.4 as well as their S22 stresses, which is obtained from stress heat maps for G + EQx and G + EQy load combinations.

The structural elements of the building are separated into 8 categories. These are “domes”, “greater half domes”, “pendentives”, “corner domes”, “lesser half domes”, “main arches”, “smaller arches that support small domes”, and “walls”. S22 stresses are investigated in detail for every category stated previously. Table 5.4 shows the biggest tension and compression in every category and for both top and bottom faces of the SHELL elements. The SHELL elements that possess the biggest stress in every category are shown in the figures individually.

Table 5.4. Biggest stress values obtained from various element categories (S22)

Element category			G + EQx Load MPa	G + EQy Load MPa
Dome	Top Face	Compression	-0,12	-0,12
		Tension	1,46	1,44
	Bottom Face	Compression	-0,14	-0,19
		Tension	0,86	0,82
Greater Half Domes	Top Face	Compression	-1	-1
		Tension	4,7	4,7
	Bottom Face	Compression	-0.5	-0,5
		Tension	4,7	4,9
Pendentives	Top Face	Compression	-0,38	-0,49
		Tension	2.3	2.4
	Bottom Face	Compression	-0,56	-0,56
		Tension	5,1	5,1
Corner Domes	Top Face	Compression	-0,08	-0,08
		Tension	0.4	0.4
	Bottom Face	Compression	-0,18	-0,18
		Tension	0,3	0,3
Lesser Half Domes	Top Face	Compression	-0,88	-0,88
		Tension	4	4
	Bottom Face	Compression	-0,34	-0,34
		Tension	6,4	6,4
Walls	Top Face	Compression	-0,6	-0,6
		Tension	1,3	1,3
	Bottom Face	Compression	-0,47	-0,47
		Tension	1,75	1,75

5.8. Overall evaluation of analysis results

In this section, the results of the stress calculations based on the finite element method are evaluated to investigate the structural behaviour of Şehzade Mehmet Mosque against a possible earthquake and thus to determine its earthquake resistance. As a result of the analysis, the following observations were made regarding the behavior of Şehzade Mehmet

Mosque during a possible earthquake and the resistance of its structural elements to the effects they encounter.

- It is seen that Şehzade Mehmet Mosque has a rigid structure. The largest displacement values obtained from the analysis due to earthquake, whose calculation spectrum is given in Fig. 5.40, are 116 mm in both X and Y directions, which further shows the structure's rigidity and symmetry. Considering that the elevation of the point that these displacements has occurred is 27 m relative to the ground, it exceeds the acceptable limits since it does not correspond to a relative displacement ratio of 0.003. Due to this collapse can be considered as a likely possibility.

- In an evaluation like this, it is important to know not only the largest displacement in the structure but displacement of all points. Because in finite element analysis, performance of a building's numerical model can only be fully obtained by converting all the displacements into local stresses and examining these stress maps of the structural elements, as shown in section 5.6.

- As a result of the structural analysis, it was observed that the compressive and shear stress limits recommended for masonry structures in Turkish Earthquake Code were exceeded in several points of the Şehzade Mehmet Mosque's structural elements.

- As shown in detail in section 5.6, the limit tensile stresses were exceeded in large areas in walls at the first layer, in domes and half domes at the second layer, and in the several parts of the main dome at the third layer.

- In this case, according to the accepted calculation principles, it can be said that the resistance of Şehzade Mehmet Mosque against earthquake will be insufficient. However, it must not be forgotten that during the stress calculations, the material properties of the elements constituting the structural system of the building are determined in accordance with the literature research and the values recommended in the current regulations. In this case, it can be said that possible material deterioration in certain parts of the structure or material losses that may occur in the structural elements may affect the behavior of the structure.

6. CONCLUSION

The Şehzade Mehmet Mosque holds a eminent place in the career of Sinan the Great Builder. The mosque itself was commissioned by Sultan Suleyman in memory of Şehzade Mehmet, his son. This was a breakpoint in Mimar Sinan's architectural career because the project was foned by the Padisah himself, which let Sinan showcase his architectural talent and structural knowledge.

Şehzade Mehmet Mosque is often denoted as first of Sinan's many masterpieces which displayed the evolution in his philosophy of design and technical finesse. Şehzade Mehmet Mosque's innovative use of space, harmony in its proportional structure, and decorations that are finest of its time demonstrates Mimar Sinan's ability to create a hybrid of aesthetic and structural integrity. It also influences many other of Sinan's later works, including eminent historical religious buildings like Süleymaniye and Selimiye Mosque.

The reason for choosing Şehzade Mehmet Mosque as a case study was its susceptibility to be modeled with frame and shell elements, its structural and aesthetic beauty, and its historical significance. The systemical breakdown method that used in the thesis was easily applied to the Şehzade Mehmet Mosque due to its symmetry and structural clarity, which also aided the modeling process that consisting of frame and shell elements.

In this thesis, it was aimed to develop a finite element model of Şehzade Mehmet Mosque, a historical masonry building, by using frame and shell elements, aiming to facilitate interdisciplinary data transfer and overall communication by streamlining the outputs of the finite element model of the building. The finite element modeling method used in the thesis can be characterized by its simplicity, efficiency, and macro-mechanical approach. These qualities enhance the accessibility of the model's outputs, thus improving the decisions that depend on finite element model during the restoration phase.

The primary objective of the thesis is to strengthen the clarity of the data coming from the numerical model's outputs, which is transferred between the disciplines that contribute to the restoration process. In traditional finite element models of historical masonry buildings, whole structure is often modeled with solid meshes, which tends to be complex and nontransparent. This potentially hinders the effectiveness of decision-making and directly affects the building’s health. The modeling process of using frame and shell element and constructing the model by hand reduces the complexity of the model significantly while increasing the awareness of the users who benefit from the model’s outputs. The proposed method does this by choosing the finite elements for each structural element that represent its real behavior as close as possible.

The application of the method to Şehzade Mehmet Mosque was to demonstrate the method’s efficacy in practice. The constructed model was efficient, and the results obtained from it were clear and interpretable. This case study highlights the potential of the proposed approach to improve the effectiveness of restoration decisions, ensuring that all participants of the preservation process can benefit meaningfully from the numerical model’s outputs.

In summary, this thesis presents a viable and practical finite element modeling method tailored for historical masonry buildings. The methodology's simplicity and clarity not only enhance communication among diverse disciplines but also support informed decision-making in restoration projects. Future works can broaden the application of this method to other historical structures, further testing its flexibility and utility across different buildings. Method represents a way to a simple effective preservation practice in hope maintaining the heritage that is held by historical masonry buildings for future generations.

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Gazili olmak ayrıcalıktır