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APPROXIMATION PROCESS OF BERNSTEIN TYPE RATIONAL FUNCTIONS

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By
Esma Yıldız Özkan

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Preface

In mathematical analysis, the history of the approximation theory is based on the Weierstrass approximation theorem established by K. Weierstrass in which stated that there exists a polynomial function which converges uniformly to each continuous function on a closed interval. A constructive proof of this theorem was proved by S.N. Bernstein in 1912.

K. Balázs defined in 1975 a rational function, investigated the convergence theorems of which and presented the relation between this rational function and the Bernstein polynomial. Afterwards, in 1982, K. Balázs and J. Szabados improved the results under some more restrictive conditions. K. Balázs and J. Szabados called their study as Bernstein type rational functions. In subsequent studies, the Bernstein type rational functions were called Balázs operators or Balázs-Szabados operators.

This present monograph is a collection of research devoted to the approximation process of the Bernstein type rational functions.

In all the chapters, the results appeared from 1975 to the present are compiled. The approximation of the Bernstein type rational functions was interested in for the one and two variable functions. Moreover, some generalizations of them were presented by means of both the quantum calculus (q-calculus) and the post quantum calculus ((p,q)-calculus). The first chapter contains the mainly classical results of the Bernstein type rational functions. The second chapter is devoted to some generalizations in the quantum calculus of the one variable Bernstein type rational functions. The third chapter is deal with the two variable Bernstein type functions by means of both the classical calculus and the quantum calculus. The fourth chapter is interested in the one and two variable post-quantum ((p,q)-calculus) generalization of the Bernstein type rational functions. Wheares the results placed in this monograph is presented with their proof, some results are given without the proofs. In the proofs of some results, it is used the crucial theorems of approximation theory such as Korovkin theorem (1953), Volkov theorem (1957), Vitali theorem, etc. Because some proofs are given briefly or without the proofs, the detailed proofs of the results can be reached from their first sources in the references.

Unfortunately, the limited content of this monograph did not allow to author to include the classical results of the approximation theory studied before. The author suggests to readers the references (Devore&Lorentz, 1993), (Altomare&Campiti, 1994), (Andrews, Askey&Roy, 1999), (Anastassiou&Gal, 2000), (Gal, 2009) etc…

The author is deeply grateful to all mathematicians because of their contributions to mathematical science and leading to the writing this monograph.

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2021

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