Ricerche di Matematica
https://doi.org/10.1007/s11587-026-01124-6

Corrigendum to Nonexistence of perfect locally finite minimal non-FC and CC-groups, and a characterization of perfect locally finite barely transitive groups

Ahmet Arikan[1]

Received: 26 March 2026 / Accepted: 7 May 2026
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Abstract
We provide a corrigendum to Nonexistence of perfect locally finite minimal non-FC and CC-groups, and a characterization of perfect locally finite barely transitive groups published in Ricerche di Matematica https://doi.org/10.1007/s11587-025-01048-7 ([1]).

Keywords FC-group · CC-group · Minimal non-FC-group · Minimal non-CC-group · Barely transitive group · Locally finite group · Perfect group

Mathematics Subject Classification 20B07 · 20B35 · 20F24 · 20F50

1 Introduction

We have found a fatal error in Theorem 2.2 in the second paragraph of the the proof "Let $\alpha \in \Delta_j$ and $\gamma \in \Delta_k \setminus \Delta_j$ for some $k > j$. So there is $y \in X$ such that $\alpha^y = \gamma$ (false argument) as $\Delta_k^y = \Delta_k$. But since $\gamma = \alpha^y \in \Delta_j = \Delta_j^X$, we reach a contradiction."

In the next section we fallow another strategy to obtain the main result.

2 Proof of the new strategy

Proof Recall that

$$\mathcal{F}(G) = \{g \in G : |\mathrm{supp}(g)| < \omega\} \trianglelefteq G \text{ and } X = \langle x \in \mathcal{F}(G) : |x| = p \rangle,$$

✉ Ahmet Arikan
arikan@gazi.edu.tr

[1] Department of Mathematics and Science Education, Gazi University, Gazi Education Faculty, 06500 Teknikokullar, Ankara, Turkey

Published online: 22 May 2026

it follows that X is a non-trivial normal subgroup of G. We will show that $X = 1$. Suppose, for a contradiction, that $X \neq 1$.

Let $\{\Delta_i\}_{i\in\omega}$ be an ascending sequence of finite G-blocks such that $\Omega = \bigcup_{i\in\omega} \Delta_i$. By hypothesis, X fixes each G-block setwise, $\Delta_i^X = \Delta_i$. For any finite G-block Δ with $\Delta^X = \Delta$, the pointwise stabilizer

$$X_{(\Delta)} = \{x \in X : \alpha^x = \alpha \ \text{ for all } \alpha \in \Delta\}$$

is normal in X. Indeed, if $h \in X$ and $x \in X_{(\Delta)}$, then for every $\alpha \in \Delta$ we have $\alpha^h \in \Delta$, since h stabilizes Δ setwise. Hence

$$\alpha^{h^{-1}xh} = ((\alpha^h)^x)^{h^{-1}} = (\alpha^h)^{h^{-1}} = \alpha.$$

Thus $h^{-1}xh \in X_{(\Delta)}$, so $X_{(\Delta)} \trianglelefteq X$.

Similarly, for each $g \in G$, the conjugate block Δ^g is again a finite G-block stabilized setwise by X, and therefore $X_{(\Delta^g)} \trianglelefteq X$.

Now the quotient $X/X_{(\Delta_i)}$ is finite, since $\Delta_i^X = \Delta_i$. Fix $i \in \omega$. By hypothesis, $X_{(\Delta_i)} \trianglelefteq G$, so the conjugation action of G on X induces a well-defined homomorphism

$$\psi_i : G \longrightarrow \mathrm{Aut}(X/X_{(\Delta_i)}),$$

given by

$$\psi_i(g)\big(xX_{(\Delta_i)}\big) = gxg^{-1}X_{(\Delta_i)}.$$

Since $X/X_{(\Delta_i)}$ is finite, its automorphism group

$$\mathrm{Aut}(X/X_{(\Delta_i)})$$

is finite. Therefore,

$$|G : \ker(\psi_i)| < \infty.$$

Because G is a locally finite p-group, every subgroup of finite index has p-power index. In particular, a proper finite-index subgroup would yield a nontrivial finite transitive permutation representation, contrary to the barely transitive hypothesis. Hence G has no proper subgroup of finite index, and so

$$\ker(\psi_i) = G.$$

Therefore ψ_i is trivial, which means that for every $g \in G$ and every $x \in X$,

$$gxg^{-1}X_{(\Delta_i)} = xX_{(\Delta_i)}.$$

Equivalently,

$$gxg^{-1}x^{-1} \in X_{(\Delta_i)}.$$

Thus

$$[X, G] \leq X_{(\Delta_i)} \qquad \text{for every } i \in \omega.$$

Intersecting over all i, we obtain

$$[X, G] \leq \bigcap_{i \in \omega} X_{(\Delta_i)}.$$

Now let $z \in \bigcap_{i \in \omega} X_{(\Delta_i)}$. Then z fixes every point of every Δ_i. Since

$$\Omega = \bigcup_{i \in \omega} \Delta_i,$$

it follows that z fixes every point of Ω. Because the action of G on Ω is faithful,

$$\bigcap_{i \in \omega} X_{(\Delta_i)} = 1.$$

Hence

$$[X, G] = 1.$$

Therefore $X \leq Z(G)$. Since G is centerless,

$$Z(G) = 1,$$

and thus

$$X = 1,$$

contrary to assumption.

This contradiction proves that $X = 1$.

The contradiction $X = 1$ proves that the initial assumption $X \neq 1$ is false. Consequently, for any ascending sequence of finite G-blocks $\{\Delta_i\}_{i \in \omega}$ such that $\Omega = \bigcup_{i \in \omega} \Delta_i$, we may assume that the blocks are not setwise invariant under X. That is, $\Delta_i^X \neq \Delta_i$ for all $i \in \omega$. □

The rest of the proof follows exactly from the proof Theorem 2.2 in [1].

Author Contributions There is single author contributed to the main results and writing of the article.

Funding Open access funding provided by the Scientific and Technological Research Council of Türkiye (TÜBİTAK). No funding received for now.

Data Availability Not applicable.

Declarations

Competing interests Not applicable.

Ethical Approval Not applicable.

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