


# VARYANSLARIN HOMOJENLİĞİ İÇİN TESTLER

BÖLÜM 4

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## Varyansların Homojenliği İçin Testler

- Levene Testi
- Brown-Forsythe Testi
- Layard Ki-kare Testi



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# Varyansların Homojenliği İçin Testler

$H_0: \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_k^2 = \sigma^2$

$H_1: \sigma_j^{2\prime}$lerin en az biri farklıdır

| Örnek no | | | |
|---|---|---|---|
| **1** | **2** | **…** | ***k*** |
| $X_{11}$ | $X_{12}$ | | $X_{1k}$ |
| $X_{21}$ | $X_{22}$ | | $X_{2k}$ |
| ⋮ | ⋮ | | ⋮ |
| $X_{n_1 1}$ | $X_{n_2 2}$ | | $X_{n_k k}$ |



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# Levene Testi

$$Z_{ij} = |X_{ij} - \bar{X}_j| \qquad \overline{Z}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} Z_{ij} \qquad \overline{Z} = \frac{1}{n}\sum_{j=1}^{k}\sum_{i=1}^{n_j} Z_{ij}$$

$$GAKT = \sum_{j=1}^{k} n_j(\overline{Z}_j - \overline{Z})^2 \qquad GIKT = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(Z_{ij} - \overline{Z}_j)^2$$

$$GAOK = \frac{GAKT}{k-1} \qquad GIOK = \frac{GIKT}{n-k}$$

$$\frac{GAOK}{GIOK} \sim F_{k-1,n-k}$$



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# Örnek

| İlaç Türü | | | |
|---|---|---|---|
| **A** | **B** | **C** | **D** |
| 20 | 16 | 24 | 17 |
| 24 | 19 | 24 | 22 |
| 20 | 15 | 30 | 17 |
| 18 | 15 | 21 | 15 |
| 18 | 15 | 21 | 14 |
| **100** | **80** | **120** | **85** |

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = \sigma^2$

$H_1: \sigma_j^{2\prime}$lerin en az biri farklıdır



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# Örnek

| $Z_{i1}$ | $Z_{i2}$ | $Z_{i3}$ | $Z_{i4}$ |
|---|---|---|---|
| \|20–20\|=0 | \|16–16\|=0 | \|24–24\|=0 | \|17–17\|=0 |
| \|24–20\|=4 | \|19–16\|=3 | \|24–24\|=0 | \|22–17\|=5 |
| \|20–20\|=0 | \|15–16\|=1 | \|30–24\|=6 | \|17–17\|=0 |
| \|18–20\|=2 | \|15–16\|=1 | \|21–24\|=3 | \|15–17\|=2 |
| \|18–20\|=2 | \|15–16\|=1 | \|21–24\|=3 | \|14–17\|=3 |
| **8** | **6** | **12** | **10** |

$$\overline{Z}_1 = \frac{8}{5} = 1.6, \quad \overline{Z}_2 = \frac{6}{5} = 1.2, \quad \overline{Z}_3 = \frac{12}{5} = 2.4, \quad \overline{Z}_4 = \frac{10}{5} = 2$$

$$\overline{Z} = \frac{8 + 6 + 12 + 10}{20} = 1.8$$



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# Örnek

| $Z_{i1} - \overline{Z}_1$ | $(Z_{i1} - \overline{Z}_1)^2$ | $Z_{i2} - \overline{Z}_2$ | $(Z_{i2} - \overline{Z}_2)^2$ |
|---|---|---|---|
| –1.6 | 2.56 | –1.2 | 1.44 |
| 2.4 | 5.76 | 1.8 | 3.24 |
| –1.6 | 2.56 | –0.2 | 0.04 |
| 0.4 | 0.16 | –0.2 | 0.04 |
| 0.4 | 0.16 | –0.2 | 0.04 |
| **0** | **11.20** | **0** | **4.80** |

| $Z_{i3} - \overline{Z}_3$ | $(Z_{i3} - \overline{Z}_3)^2$ | $Z_{i4} - \overline{Z}_4$ | $(Z_{i4} - \overline{Z}_4)^2$ |
|---|---|---|---|
| –2.4 | 5.76 | –2 | 4 |
| –2.4 | 5.76 | 3 | 9 |
| 3.6 | 12.96 | –2 | 4 |
| 0.6 | 0.36 | 0 | 0 |
| 0.6 | 0.36 | 1 | 1 |
| **0** | **25.20** | **0** | **18** |



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# Örnek

$$GAKT = 5(1.6 - 1.8)^2 + 5(1.2 - 1.8)^2 + 5(2.4 - 1.8)^2 + 5(2 - 1.8)^2$$
$$= 5(0.04 + 0.36 + 0.36 + 0.04)$$
$$= 4$$

$$GIKT = 11.20 + 4.80 + 25.20 + 18$$
$$= 59.2$$

$$GAOK = \frac{4}{4-1} = 1.333 \qquad GIOK = \frac{59.2}{20-4} = 3.7$$

$$F_h = \frac{1.333}{3.7} = 0.360$$

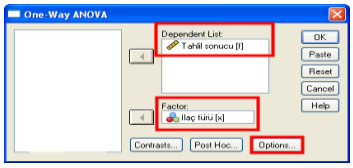
$$p\text{-değeri} = P(F_{3,16} > 0.360) = \int_{0.360}^{\infty} f(F)dF$$

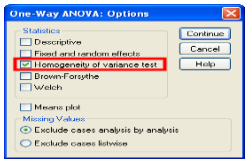


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# SPSS Uygulama

| | x | f |
|---|---|---|
| 1 | 1 | 20 |
| 2 | 1 | 24 |
| 3 | 1 | 20 |
| 4 | 1 | 18 |
| 5 | 1 | 18 |
| 6 | 2 | 16 |
| 7 | 2 | 19 |
| 8 | 2 | 15 |
| 9 | 2 | 15 |
| 10 | 2 | 15 |
| 11 | 3 | 24 |
| 12 | 3 | 24 |
| 13 | 3 | 30 |
| 14 | 3 | 21 |
| 15 | 3 | 21 |
| 16 | 4 | 17 |
| 17 | 4 | 22 |
| 18 | 4 | 17 |
| 19 | 4 | 15 |
| 20 | 4 | 14 |




**Test of Homogeneity of Variances**

| Levene Statistic | df1 | df2 | Sig. |
|---|---|---|---|
| ,360 | 3 | 16 | ,782 |



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# Brown-Forsythe Testi

$$m_j = Medyan\left(X_{1j}, X_{2j}, \dots, X_{n_j j}\right) \qquad j = 1,2,\dots,k$$

$$Z_{ij} = |X_{ij} - m_j| \qquad \overline{Z}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} Z_{ij} \qquad \overline{Z} = \frac{1}{n}\sum_{j=1}^{k}\sum_{i=1}^{n_j} Z_{ij}$$

$$GAKT = \sum_{j=1}^{k} n_j(\overline{Z}_j - \overline{Z})^2 \qquad GIKT = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(Z_{ij} - \overline{Z}_j)^2$$

$$GAOK = \frac{GAKT}{k-1} \qquad GIOK = \frac{GIKT}{n-k} \qquad \frac{GAOK}{GIOK} \sim F_{k-1,n-k}$$



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# Örnek

| İlaç Türü | | | |
|---|---|---|---|
| **A** | B | C | D |
| 20 | 16 | 24 | 17 |
| 24 | 19 | 24 | 22 |
| 20 | 15 | 30 | 17 |
| 18 | 15 | 21 | 15 |
| 18 | 15 | 21 | 14 |
| **100** | **80** | **120** | **85** |

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = \sigma^2$

$H_1: \sigma_j^{2\prime}$lerin en az biri farklıdır



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# Örnek

| Sıralanmış ölçüm değerleri | | | |
|---|---|---|---|
| **A** | **B** | **C** | **D** |
| 18 | 15 | 21 | 14 |
| 18 | 15 | 21 | 15 |
| 20 | 15 | 24 | 17 |
| 20 | 16 | 24 | 17 |
| 24 | 19 | 30 | 22 |
| $m_1 = 20$ | $m_2 = 15$ | $m_3 = 24$ | $m_4 = 17$ |



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# Örnek

| $Z_{i1}$ | $Z_{i2}$ | $Z_{i3}$ | $Z_{i4}$ |
|---|---|---|---|
| \|20–20\|=0 | \|16–15\|=1 | \|24–24\|=0 | \|17–17\|=0 |
| \|24–20\|=4 | \|19–15\|=4 | \|24–24\|=0 | \|22–17\|=5 |
| \|20–20\|=0 | \|15–15\|=0 | \|30–24\|=6 | \|17–17\|=0 |
| \|18–20\|=2 | \|15–15\|=0 | \|21–24\|=3 | \|15–17\|=2 |
| \|18–20\|=2 | \|15–15\|=0 | \|21–24\|=3 | \|14–17\|=3 |
| **8** | **5** | **12** | **10** |

$$\overline{Z}_1 = \frac{8}{5} = 1.6, \quad \overline{Z}_2 = \frac{5}{5} = 1, \quad \overline{Z}_3 = \frac{12}{5} = 2.4, \quad \overline{Z}_4 = \frac{10}{5} = 2$$

$$\overline{Z} = \frac{8 + 5 + 12 + 10}{20} = 1.75$$



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# Örnek

| $Z_{i1} - \overline{Z}_1$ | $(Z_{i1} - \overline{Z}_1)^2$ | $Z_{i2} - \overline{Z}_2$ | $(Z_{i2} - \overline{Z}_2)^2$ |
|---|---|---|---|
| –1.6 | 2.56 | 0 | 0 |
| 2.4 | 5.76 | 3 | 9 |
| –1.6 | 2.56 | –1 | 1 |
| 0.4 | 0.16 | –1 | 1 |
| 0.4 | 0.16 | –1 | 1 |
| **0** | **11.20** | **0** | **12** |

| $Z_{i3} - \overline{Z}_3$ | $(Z_{i3} - \overline{Z}_3)^2$ | $Z_{i4} - \overline{Z}_4$ | $(Z_{i4} - \overline{Z}_4)^2$ |
|---|---|---|---|
| –2.4 | 5.76 | –2 | 4 |
| –2.4 | 5.76 | 3 | 9 |
| 3.6 | 12.96 | –2 | 4 |
| 0.6 | 0.36 | 0 | 0 |
| 0.6 | 0.36 | 1 | 1 |
| **0** | **25.20** | **0** | **18** |



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# Örnek

$$GAKT = 5(1.6 - 1.75)^2 + 5(1 - 1.75)^2 + 5(2.4 - 1.75)^2 + 5(2 - 1.75)^2$$
$$= 5(0.0225 + 0.5625 + 0.4225 + 0.0625)$$
$$= 5.35$$

$$GIKT = 11.20 + 12 + 25.20 + 18$$
$$= 66.4$$

$$GAOK = \frac{5.35}{4 - 1} = 1.783 \qquad GIOK = \frac{66.4}{20 - 4} = 4.15$$

$$F_h = \frac{1.783}{4.15} = 0.4296$$



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# Örnek

3 ve 16 serbestlik dereceleri ve $\alpha = 0.05$ ile $F_{3,16,0.05} = 3.24$ değeri T3'den bulunur. Bu sonuçlara göre %95 güven düzeyinde,

$$F_h < F_{3,16,0.05}$$

olduğundan, varyansların homojenliği hipotezi kabul edilir.



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# Örnek

Bu teste ilişkin *p-değeri*,

$$p\text{-değeri} = P(F_{3,16} > 0.4296) = \int_{0.4296}^{+\infty} f(F)dF$$

olur.

Bu *p-değeri* T3'den kabaca bulunabilir. 3 ve 16 serbestlik dereceli $F$ istatistiği için T3'de $P(F_{3,16} > 3.24) = 0.05$ olduğu görülmektedir. Bu nedenle $P(F_{3,16} > 0.4296) > 0.05$, yani *p-değeri*>0.05, olduğu açıktır.





# Layard Ki-Kare Testi

$$\hat{\gamma} = \frac{n\sum^k\sum^{n_j}(X_{ij} - \overline{X}_j)^4}{\left(\sum^k\sum^{n_j}(X_{ij} - \overline{X}_j)^2\right)^2} - 3$$

$$\hat{T}^2 = 2 + \left(1 - \left(\frac{k}{n}\right)\right)\hat{\gamma}$$

$$L = \frac{\sum_{j=1}^{k}(n_j - 1)\left[logS_j^2 - \frac{1}{\sum^k(n_j - 1)}\sum^k(n_j - 1)logS_j^2\right]^2}{\hat{T}^2}$$

$$L \sim \chi^2_{k-1}$$

# Örnek

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|---|---|---|---|
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| 24 | 19 | 24 | 22 |
| 20 | 15 | 30 | 17 |
| 18 | 15 | 21 | 15 |
| 18 | 15 | 21 | 14 |
| **100** | **80** | **120** | **85** |

$H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = \sigma^2$

$H_1: \sigma_j^{2\prime}$lerin en az biri farklıdır



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# Örnek

| $\bar{X}_{i1} - \bar{X}_1$ | $(\bar{X}_{i1} - \bar{X}_1)^2$ | $(\bar{X}_{i1} - \bar{X}_1)^4$ | $\bar{X}_{i2} - \bar{X}_2$ | $(\bar{X}_{i2} - \bar{X}_2)^2$ | $(\bar{X}_{i2} - \bar{X}_2)^4$ |
|---|---|---|---|---|---|
| 20–20=0 | 0 | 0 | 16–16=0 | 0 | 0 |
| 24–20=4 | 16 | 256 | 19–16=3 | 9 | 81 |
| 20–20=0 | 0 | 0 | 15–16=-1 | 1 | 1 |
| 18–20=-2 | 4 | 16 | 15–16=-1 | 1 | 1 |
| 18–20=-2 | 4 | 16 | 15–16=-1 | 1 | 1 |
| **0** | **24** | **288** | **0** | **12** | **84** |

| $\bar{X}_{i3} - \bar{X}_3$ | $(\bar{X}_{i3} - \bar{X}_3)^2$ | $(\bar{X}_{i3} - \bar{X}_3)^4$ | $\bar{X}_{i4} - \bar{X}_4$ | $(\bar{X}_{i4} - \bar{X}_4)^2$ | $(\bar{X}_{i4} - \bar{X}_4)^4$ |
|---|---|---|---|---|---|
| 24–24=0 | 0 | 0 | 17–17=0 | 0 | 0 |
| 24–24=0 | 0 | 0 | 22–17=5 | 25 | 625 |
| 30–24=6 | 36 | 1296 | 17–17=0 | 0 | 0 |
| 21–24=-3 | 9 | 81 | 15–17=-2 | 4 | 16 |
| 21–24=-3 | 9 | 81 | 14–17=-3 | 9 | 81 |
| **0** | **54** | **1458** | **0** | **38** | **722** |



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# Örnek

$$\hat{\gamma} = \frac{20(288 + 84 + 1458 + 722)}{(24 + 12 + 54 + 38)^2} - 3$$
$$= 0.115$$

$$\hat{T}^2 = 2 + \left(1 - \left(\frac{4}{20}\right)\right)(0.115)$$
$$= 2.092$$

$$S_1^2 = \frac{24}{4} = 6, \qquad S_2^2 = \frac{12}{4} = 3, \qquad S_3^2 = \frac{54}{4} = 13.5, \qquad S_4^2 = \frac{38}{4} = 9.5$$





# Örnek

$$\frac{1}{\sum^k (n_j - 1)} \sum^k (n_j - 1) log S_j^2 = \frac{13.4534}{16}$$
$$= 0.8408$$

| $n_j$ | $n_j - 1$ | $S_j^2$ | $logS_j^2$ | $(n_j - 1)logS_j^2$ | $(n_j - 1)[logS_j^2 - 0.8408]$ |
|---|---|---|---|---|---|
| 5 | 4 | 6 | 0.7782 | 3.1126 | $4(0.7782–0.8408)^2$=0.0157 |
| 5 | 4 | 3 | 0.4771 | 1.9085 | $4(0.4771–0.8408)^2$=0.5288 |
| 5 | 4 | 13.5 | 1.1303 | 4.5213 | $4(1.1303–0.8408)^2$=0.3352 |
| 5 | 4 | 9.5 | 0.9772 | 3.9110 | $4(0.9772–0.8408)^2$=0.0744 |
| **20** | **16** | | **3.3628** | **13.4534** | **0.9541** |

$$L = \chi_h^2 = \frac{0.9541}{2.092} = 0.4561 \qquad \chi_h^2 < \chi_{3,0.05}^2$$

# Örnek

Bu test için p-değeri aşağıdaki gibi ifade edilebilir.

$$p\text{-değeri} = P(\chi_3^2 > 0.4561) = \int_{0.4561}^{+\infty} f(\chi^2)d\chi^2$$

Bu p-değeri T4'den yaklaşık olarak bulunabilir. 3 serbestlik derecesi için T4'de $P(\chi_3^2 > 0.352) = 0.95$ ve $P(\chi_3^2 > 6.251) = 0.10$ olduğu görülmektedir. Bu nedenle $0.10 < P(\chi_3^2 > 0.4561) < 0.95$, yani 0.10<p-değeri<0.95 olduğu söylenebilir.