➢ *ORAL PRESENTATION*

Fuzzy backstepping control of quarter car suspension system

Meral ÖZARSLAN YATAK[1*] (*https://orcid.org/0000-0002-1091-1647*),

[*1]Gazi University, Faculty of Technology, Department of Electrical-Electronic Engineering, Ankara, Türkiye.

*Corresponding author e-mail: ozarslanm@gazi.edu.tr

Abstract
This paper presents a fuzzy backstepping controller of a quarter vehicle suspension system for a comfortable and safe ride with a high road holding feature. Active suspension systems are essential for autonomous and electric vehicles as well as classical vehicles. These nonlinear systems require a nonlinear controller for ride comfort, road holding, and low power consumption. Backstepping controller provides a globally asymptotic stability with Lyapunov design for nonlinear and systems with uncertain parameters. Additionally, it integrates control law into the controller structure through the virtual control variables at each step. The practical implementation of the backstepping controller requires constant closed-loop gains for the chosen Lyapunov function at each step. The system's dynamic response depends on these values at a high rate. In this study, the fuzzy logic controller determines these values. This approach also reduces the steady-state error occurring in the natural behavior of the classical backstepping algorithm. The results show that the fuzzy backstepping approach reduces the sprung mass displacement, acceleration, RMS values of dynamic tire load, and suspension deflection according to the classical control approach.
Keywords: Quarter vehicle model, Fuzzy backstepping, Ride comfort, Suspension deflection

INTRODUCTION

Improving ride comfort for passenger convenience and road holding for safe riding are the primary purposes of the automotive suspension system. The performance of the suspension has been improved by adding active elements to the passive suspension systems.

Road holding is obtained by supplying the contact of the road and the tire as much as possible. Additionally, ride comfort is supplied by isolating the road disturbances from the body of the vehicle. The road handling is improved with an active suspension system, too.

The system parameters have constant values for passive suspension systems, so the suspension stroke is limited (Ahmed 2021). Semi-active suspension systems usually have adjustable damping ratios through magnetorheological (MR), electrorheological, or variable orifice-type damping elements (Ergin and Yatak 2023). The active suspension system is the most enhanced suspension system among the other types of suspension systems. With an active suspension system, ride comfort and road holding can be improved concurrently (Yatak and Şahin 2021). The controllable part of the suspension system is the force actuator for arranging the ride comfort and suspension deviations.

The performance of the active suspension system strongly depends on the control technique. However, the complexity of the control technique, the cost, and the power consumption are the negative features of active suspension systems; the dynamic performance improvement on ride comfort and road holding makes it the most advanced suspension type.

Researchers have conducted many studies on the variety of control techniques for various vehicles, such as quarter, half-, or full-vehicle suspension systems. Linear quadratic regulator and static output feedback were compared in (Yim 2025). In (Tang and Ahmad 2025), enhanced fuzzy logic and two types of optimization methods were analyzed for active suspension systems. An adaptive integral terminal sliding mode active suspension system was implemented in (Liang, Gao et al. 2025). Passive suspension system, backstepping control, and sliding mode backstepping control were analyzed in (Li, Du et al. 2025). These control techniques are for improving the performance of the suspension systems. Nonlinear control algorithms are more appropriate for nonlinear systems. Zhang and Chen improved the body lateral vibration of high-speed trains with a backstepping semi-active control (Zhang and Chen 2025). They controlled the MR damper for this reason. In (Feng, Zhou et al. 2025), adaptive backstepping control was improved for

MR damper-based seat suspension. They reduced the frequency-weighted root mean square of acceleration, the vibration dose, and the root mean square of relative displacement effectively.

In this paper, the quarter vehicle suspension system has been controlled with a fuzzy backstepping controller (FBC). The controller performance has been compared with a passive suspension system (PSS) and a classical controller, a PID controller. The nonlinear structure of the backstepping control is acceptable for the nonlinear suspension system. The actuator force is determined with the FBC. The performance parameters of the controller are chosen as the acceleration of the sprung mass for ride comfort, tire deflection for road holding, suspension deflection for ride comfort, road holding, and the structural characteristics of the suspension system.

The organization of the paper is as follows: The second section, Materials and Methods, contains the mathematical model of the quarter vehicle suspension system as a subsection. The other subsection is for the backstepping controller with fuzzy logic adaptation. The third section is for comparison of the controller simulations. The last section covers the results and the discussions of the work.

MATERIALS AND METHODS

Quarter Vehicle Suspension System

A quarter vehicle active suspension system, as seen in Figure 1, was used to reveal the superiority of FBC. The sprung mass (m_s) for representing the quarter of the vehicle body, unsprung mass (m_u) for representing one tire, k_s for suspension spring, b_s for suspension damping, f for actuator force, b_u for tire damping, and k_u for tire spring are the basic components of a quarter vehicle active suspension system. Table 1 shows the parameters for the steady-state model of the system.

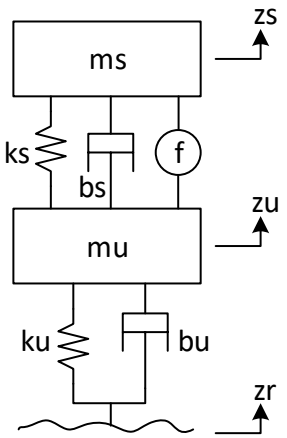


Figure 1. Quarter vehicle active suspension system model

The motion equations are derived from Newton's laws of motion, and they can be written as follows:

$$m_s\ddot{z}_s = k_s(z_u - z_s) + b_s(\dot{z}_u - \dot{z}_s) + f \quad (1)$$

$$m_u\ddot{z}_u = k_u(z_r - z_u) + b_u(\dot{z}_r - \dot{z}_u) - k_s(z_u - z_s) - b_s(\dot{z}_u - \dot{z}_s) - f \quad (2)$$

Table 1. Parameters and values of the quarter car suspension system

System	Parameter	Definition
Inputs	$(u_1 = f)$	Force actuator
	$(u_2 = z_r)$.	Road input
Outputs	$(y_1 = x_1)$	Position of the sprung mass
	$(y_2 = x_3)$	Position of the unsprung mass
State Variables	$(x_1 = z_s)$	Position of the sprung mass
	$(x_2 = \dot{z}_s = \dot{x}_1)$	Velocity of the sprung mass
	$(x_3 = z_u)$	Position of the unsprung mass
	$\left(x_4 = \dot{x}_3 - \frac{b_u}{m_u} u_2\right)$	

The steady state model and the coefficient matrices are given in (3) and (4).

$$\dot{x} = Ax + Bu_y = Cx + Du \tag{3}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & \frac{k_s}{m_s} & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_u+k_s}{m_u} & -\frac{b_u+b_s}{m_u} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_s} & \frac{b_s b_u}{m_s m_u} \\ 0 & \frac{b_u}{m_u} \\ -\frac{1}{m_u} & \frac{k_u}{m_u} - \frac{b_s b_u + b_u^2}{m_u^2} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = 0 \tag{4}$$

Backstepping Control

The backstepping approach is a systematic control structure for nonlinear systems (Kumar, Rana et al. 2021, Şahin and Yatak 2024). It derives stable control laws with a step-by-step approach without the knowledge of all model states. This method is especially effective for the strict-feedback formed nonlinear systems. This controller scheme is based on an iterative design in which the system states are accepted as virtual control inputs. In each step, a Lyapunov function candidate is selected, and a virtual control law is derived for the current sub-dynamics of the system. This virtual control law is treated as a real state variable for the following step.

The control aim in this study is to find such a control law $u_1 = f$ for the suspension system using the backstepping method that the suspension deflection of the quarter vehicle system follows the zero-reference trajectory. The suspension deflection is given as follows:

$$z_{sref} = z_s - z_u = 0 \tag{5}$$

Suspension deflection is a vital performance parameter that is related to nearly all aspects of the performance of the suspension system, as ride comfort, road holding, and handling. If it is not kept in control, an increase in suspension system wear and fatigue occurs (Ergin and Yatak 2023). So, this parameter is chosen as the reference parameter for FBC.

The control aim is to provide the suspension deflection to be zero. The system state variables in accordance with the backstepping approach are determined as follows:

The reference input, suspension deflection $z_{sref} = 0$:

$$x_1 = z_s - z_u \tag{6}$$

The design for the first sub-system (x_1):

The error variable: $e_1 = x_1 - z_{sref} = x_1$ (7)

The Lyapunov function candidate: $V_1 = \frac{1}{2}e_1^2$ (8)

Time derivative: $\dot{V}_1 = e_1.\dot{e_1} = e_1.x_2$ (9)

x_2 is virtual control input.

If $x_2 = -c_1 e_1$, $(c_1 > 0)$, $\dot{V}_1 = -c_1\, e_1^2 < 0$ and $e_1 \to 0$ can be obtained. This control law is determined as $\alpha_1 = -c_1 e_1$.

The design for the second sub-system (x_2):

The second error variable is described as the difference between x_2 and the virtual control α_1:

$e_2 = x_2 - \alpha_1 = x_2 + c_1 e_1$ (10)

The expanded Lyapunov function candidate:

$V_2 = V_1 + \frac{1}{2}e_2^2 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2$ (11)

Time derivative: $\dot{V}_2 = \dot{V}_1 + e_2\dot{e}_2 = e_1(e_2 - c_1 e_1) + e_2\left(\dot{x}_2 + c_1(e_2 - c_1 e_1)\right)$

$\dot{V}_2 = -c_1 e_1^2 + e_1 e_2 + e_2(\dot{x}_2 + c_1 e_2 - c_1^2\, e_1)$ (12)

A control law u is chosen that makes e_2 term to zero and provides error damping to make the Lyapunov derivative negative semi-definite:

$e_1 - c_1^2 e_1 + \dot{x}_2 + c_1 e_2 = -c_2 e_2$ (13)

Here c_2 is a positive design gain.

Fuzzy Backstepping Control

The backstepping approach is a strong controller for nonlinear systems. In spite of this, it is insufficient for competing with the steady-state error. The reason for this, it generally provides asymptotic stability. When the state is reached, that place is not the exact point; it is balanced at the nearest point to the target point. To overcome this disadvantage, a robust control strategy can be provided by utilizing fuzzy logic (Şahin and Yatak 2024).

In this study, c_2, a positive design gain is determined with fuzzy logic. It is the most effective gain to make the error zero. The first input is $e_1 = x_1 - z_{sref} = x_1$ and the membership functions for this input is seen in Figure 2a. The second input is a derivative of e_1, and the membership functions are as in Figure 2b. The output is c_2 gain. The singleton membership functions for this output is seen in Figure 2c.

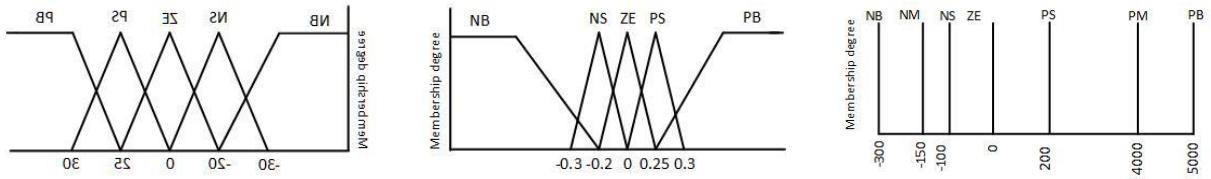


Figure 2. Membership functions for the FBC **a.** Input e_1 **b.** Input $\dot{e_1}$ **c.** Output

RESULTS and DISCUSSION

Vehicle suspension systems are evaluated over various performance parameters. These parameters are improved by various controllers. In this study, a nonlinear and iterative controller, the fuzzy backstepping controller, was analyzed to put forward its superiority over the classical PID controller and passive suspension system.

The system was tested for two different road types. The results were evaluated from three critical parameters. The first parameter is suspension deflection as shown in Eq. 14. This is the relative vertical displacement between the sprung and unsprung masses and is related to ride comfort and road handling.

$$s_d = z_s - z_u \tag{14}$$

The second parameter is tire deflection as shown in Eq. 15. Tire deflection is the displacement between the unsprung mass and the road input. It indicates whether the tire contacts the road surface and determines the road holding feature.

$$t_d = z_u - z_r \tag{15}$$

The last parameter is the acceleration of the unsprung mass. It is an important ride comfort parameter that shows how much road disturbances are transmitted to the vehicle.

The first road input is shown in Figure 3. The suspension deflections for the three types of controllers are shown in Figure 4 for this road input.

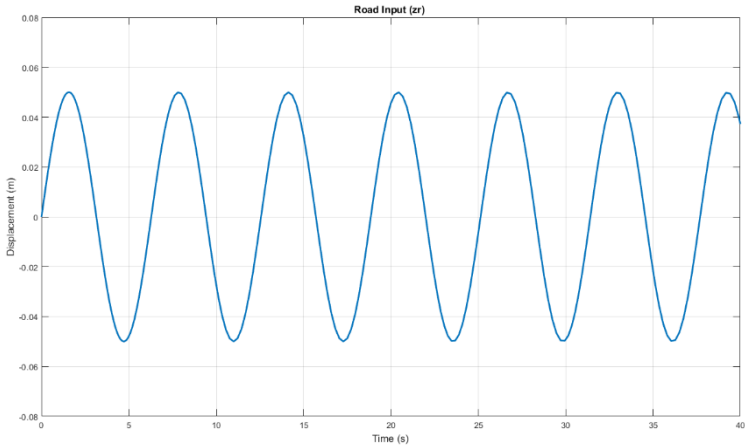


Figure 3. The first road input profile

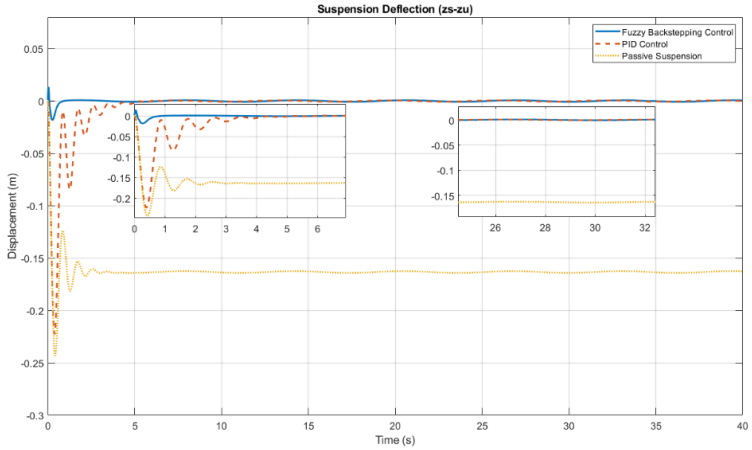


Figure 4. Suspension deflections for three types of suspension systems

Figure 5 is the tire deflection for three types of suspension systems, showing the road holding and handling. The acceleration of sprung mass is related to ride comfort, and it can be seen from Figure 6 for the sinus road input type.

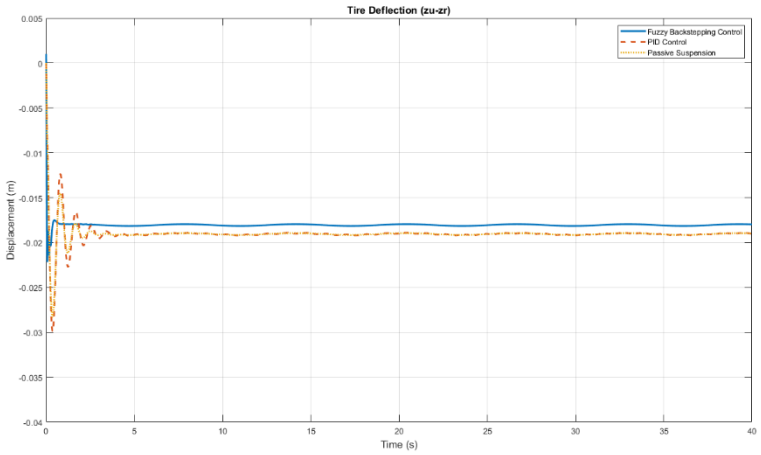


Figure 5. Tire deflection three types of suspension systems

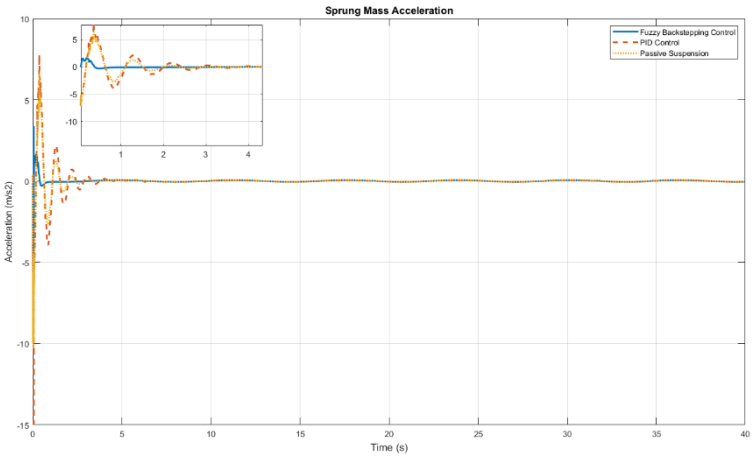


Figure 6. Sprung mass accelerations

The displacements of sprung mass and unsprung mass are shown in Figures 7 and 8, respectively.

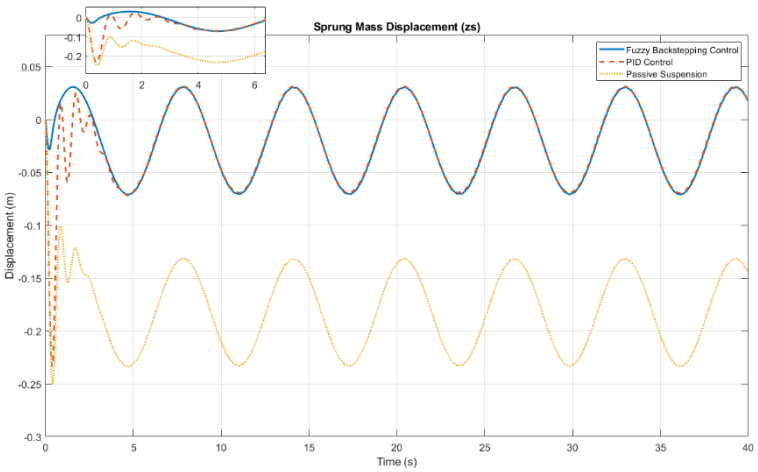


Figure 7. Sprung mass displacements

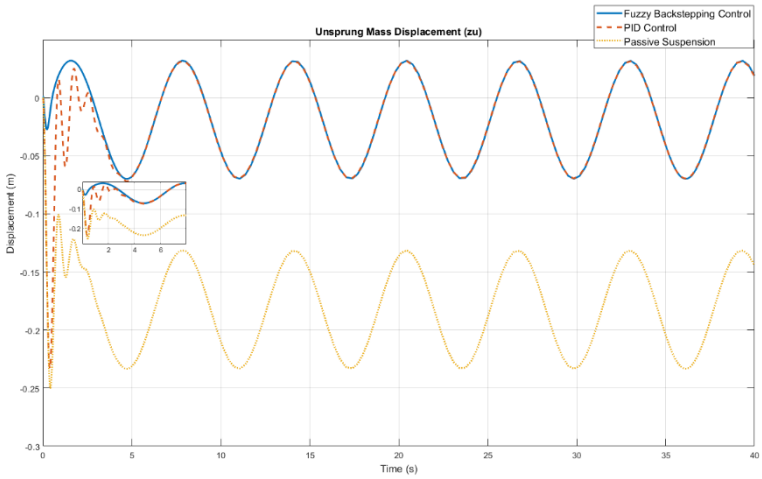


Figure 8. Unsprung mass displacements

The quarter car suspension system model was evaluated for a bump input as a different type of road disturbance, as seen in Figure 9.

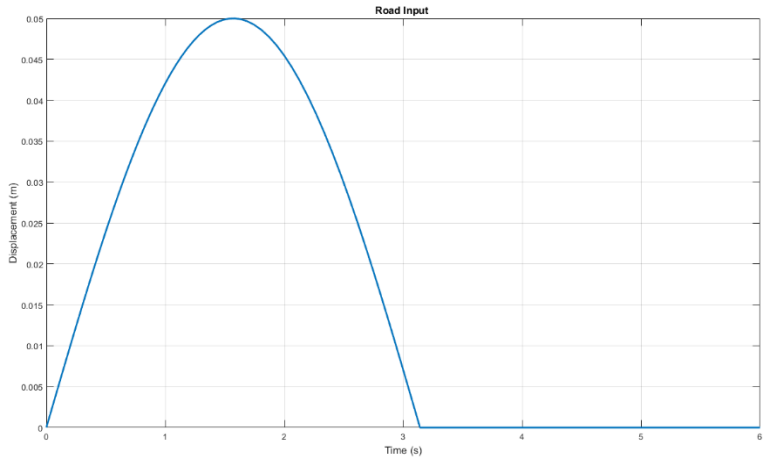


Figure 9. Bump road input

The suspension deflections and tire deflections for bump input are shown in Figures 10 and 11, respectively.

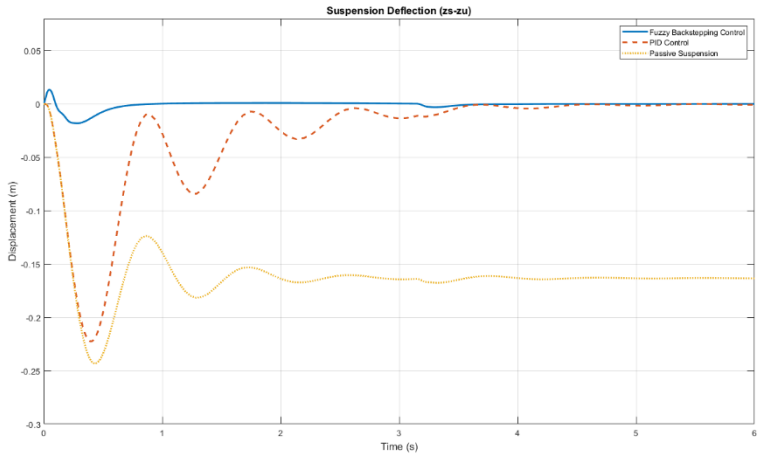


Figure 10. Suspension deflections

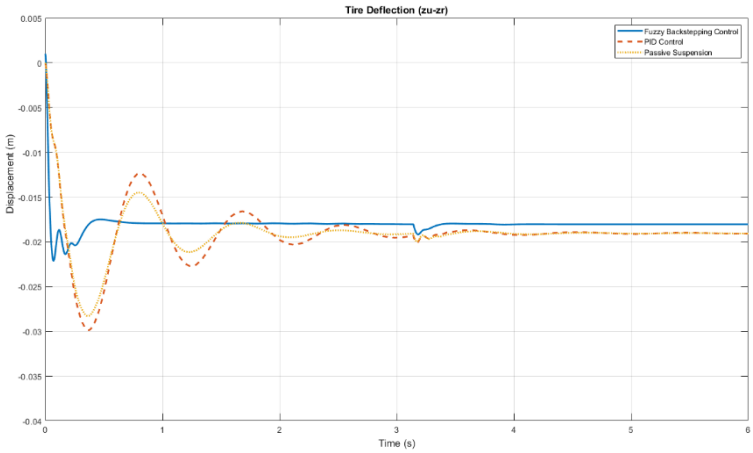


Figure 11. Tire deflections

The acceleration of sprung mass is shown in Figure 12.

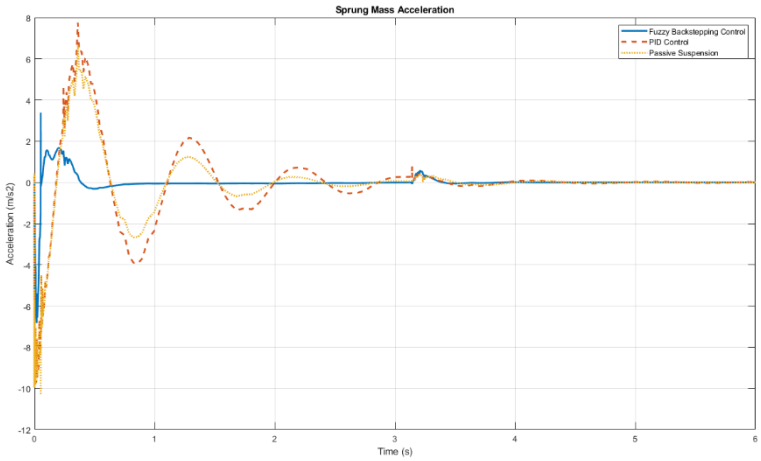


Figure 12. Sprung mass accelerations

The displacements of sprung mass and unsprung mass are shown in Figures 13 and 14, respectively.

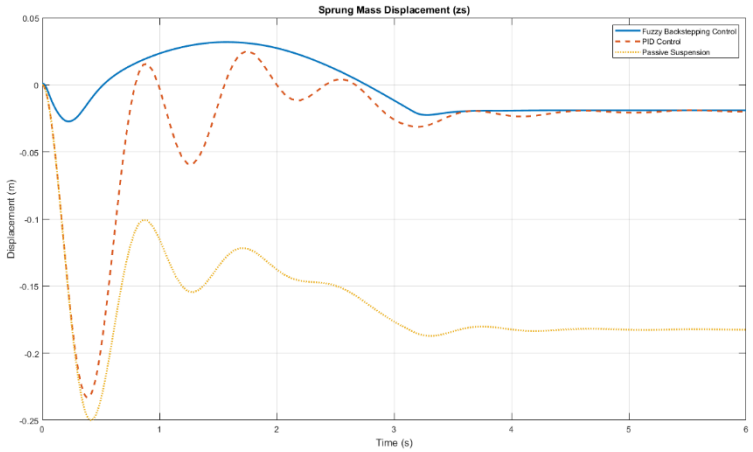


Figure 13. Sprung mass displacement

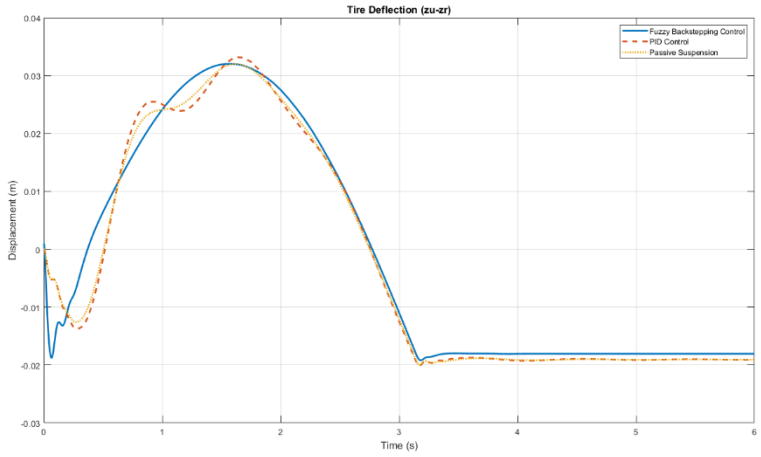


Figure 14. Unsprung mass displacement

The performance results of the passive suspension system and the active suspension systems with PID and FBC on the RMS values are seen Table 2. While ↑ shows the improvement of the value according to the passive system, ↓ indicates the decrease in the value. It can be seen from the table that FBC has superiority in all the parameters over the passive suspension system. These analytical results are consistent with the figures of the parameters. Especially the control variable, suspension deflection, has been made better with the controller, nearly 100%. Additionally, ride comfort is improved with FBC according to the acceleration improvement of nearly 100%.

Table 2. RMS values of the performance parameters for passive and active suspension systems

Parameter	Passive	PID	FBC
Suspension Deflection	0.1635	0.0008298 (↑99.49%)	0.0009724 (↑99.41%)
Tire Deflection	0.01907	0.0191 (↓1.6%)	0.01906 (↑0.5%)
Sprung Mass Acceleration	0.007385	0.02337 (↓2.2%)	0.0001465 (↑98%)

CONCLUSION

This paper presents a nonlinear controller for a nonlinear system. Backstepping controller uses Lyapunov functions for determining the control law for asymptotic stability. As a result of this natural structure, there is a bit of steady-state error in the dynamic response. To overcome this, fuzzy logic was used in this study. Sprung mass acceleration, suspension deflection, and tire deflection are chosen as performance parameters. Nearly 100 % optimization was obtained on the ride comfort with the proposed controller. Although the calculation and programming cost and the complexity of the controller are factors, the improvement in ride holding and a bit of road holding make the FBC an appropriate controller for a suspension system.

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