Approximation by Some Kantorovich Type Max-Product Operators

Şule Yüksel Güngör [1]
1 Gazi University, Faculty of Sciences, Department of Mathematics, 06500, Ankara, Turkey
sulegungor@gazi.edu.tr

ABSTRACT

In order to draw attention to a possible interesting new direction in the constructive approximation theory, an open problem was presented by S.G. Gal in [1]. According to this problem a sequence of discrete max-product approximation operators $L_n^{(M)}$ attached to $f: I \subseteq \mathbb{R} \to \mathbb{R}^+$ with the operations "$\vee$" (maximum) and "."(product), can be defined by

$$L_n^{(M)}(f)(x) = \frac{\vee_{k\in I_n} p_{n,k}(x).f(x_{n,k})}{\vee_{k\in I_n} p_{n,k}(x)}, x \in I, n \in \mathbb{N}$$

where $p_{n,k}(x)$ are various kinds of function basis on I, I_n are finite or infinite families of indices and $\{x_{n,k}: k \in I_n\}$ represents a division of I. These operators are nonlinear positive operators having pseudo-linearity property. Moreover, the max-product operators are subadditive, positive homogenous and monotone. In [2], to each max-product operator $L_n^{(M)}$, the authors defined its Kantorovich variant by

$$LK_n^{(M)}(f)(x) = \frac{\vee_{k\in I_n} p_{n,k}(x).\left(1/(x_{n,k+1}-x_{n,k})\right).\int_{x_{n,k}}^{x_{n,k+1}} f(t)dt}{\vee_{k\in I_n} p_{n,k}(x)}, x \in I, n \in \mathbb{N}$$

with $\{x_{n,k}: k \in I_n\}$ a division of the finite or infinite interval I. They studied uniform and pointwise approximation properties, shape preserving properties and localization results for certain well-known operators.

Based on this study we introduce Kantorovich variants of some max-product operators and give quantitative estimates with the help of the quantitative estimates of the original operators. Compared with the approximation order by the original operators, it follows that the approximation order by its Kantorovich variants is no worse. Also, we investigate shape preserving properties for Kantorovich type operators.

Key Words: Max-product operators, max-product operators of Kantorovich type, shape preserving properties.