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Empirical Evidence for the New Definitions in Financial Markets and the Equity Premium Puzzle

Atilla ARAS *(https://orcid.org/0000-0002-7727-9797), Gazi University, Türkiye; aaras1974@gmail.com*
Duran TÜRKOĞLU *(https://orcid.org/0000-0002-8667-1432), Gazi University, Türkiye; dturkoglu@gazi.edu.tr*

Finansal Piyasalarda Yeni Tanımlar için Ampirik Kanıtlar ve Hisse Senedi Primi Bulmacası

Abstract

This study aims to present empirical evidence supporting the validity of new definitions of risk attitudes that may find a place in financial markets. A new method, partly based on the consumption capital asset pricing model, is developed to determine risk behaviour in financial markets. Additionally, the method specifies risk behaviours for unequal wealth values that differ in some respects from those of the standard method. Moreover, the method offers a solution to the equity premium puzzle. The results suggest that the new definitions accurately specify the risk behaviours and can be used in investment strategies.

Keywords : Equity Investor, Risk-Free Asset Investor, Coefficient of Relative Risk Aversion.

JEL Classification Codes : C30, D51, D53, G11, G12, G17.

Öz

Bu çalışma, finansal piyasalarda kendisine yer bulabilecek risk tutumları ile ilgili yeni tanımların doğruluğunu desteklemek için ampirik kanıtlar sunmayı amaçlamaktadır. Finansal piyasalardaki risk davranışlarını belirlemek için kısmen tüketime dayalı sermaye varlıkları fiyatlandırma modeline dayanan yeni bir yöntem geliştirilmiştir. Üstelik, yöntem standart yöntemden farklı olarak eşit olmayan servet değerlerinde risk davranışlarını belirtmektedir. Ayrıca, yöntem hisse senedi primi bulmacasına çözüm sunmaktadır. Sonuçlar, yeni tanımların risk davranışlarını doğru bir şekilde belirttiğini ve yatırım stratejilerinde kullanılması gerektiğini göstermektedir.

Anahtar Sözcükler : Hisse Senedi Yatırımcısı, Risksiz Varlık Yatırımcısı, Göreceli Riskten Kaçınma Katsayısı.

1. Introduction

This study examines new definitions of investors' risk attitudes in financial markets. Investors compare the utilities of certain and uncertain outcomes and then make financial investment decisions at specific points in time. This situation leads investors' risk attitudes to emerge in financial markets. Standard theory states that strict concavity, strict convexity, and linearity of certain utility curves indicate risk-averse, risk-loving, and risk-neutral behaviour, respectively. This characterisation is insufficient for most financial models. Hence, there is a research gap in the literature regarding alternative definitions of investors' risk attitudes in financial markets. The research gap in this area is the lack of a variety of alternative definitions of risk attitudes that can be easily used in economic and financial models or to address open problems in finance. This study aims to fill this gap by providing empirical evidence to support the new definitions of investors' risk attitudes proposed by Aras (2022) and additional new definitions introduced in this article. The study analyses data from US financial markets from 1889 to 1978. It applies the new definitions to equity investors in the S&P 500 composite index and to risk-free asset investors in US Treasury bills. The sufficiency factor of the model (SFOM), which is a crucial variable in the new definitions, is taken from Aras. The period 1889-1978 is selected because only one article calculates the SFOM for this period. The new definitions differ from standard definitions of investors' risk attitudes in their reliance on the SFOM variable. Moreover, these new definitions provide an opportunity to compare the certain and uncertain utility curves of investors at different wealth levels.

The estimations show that equity investors who invested in the composite S&P 500 index are risk-averse, while risk-free asset investors who invested in US Treasury bills are found to exhibit not enough (insufficient) risk-loving behaviour, which can be considered a type of risk-averse behavior, when utility curve $v(c_t) = \frac{{c_t}^{1-\rho}-1}{1-\rho}$ is selected for investors.

The motivation for the study is that the relevant technique in standard theory is insufficient in most cases. For instance, the standard theory cannot determine investors' risk attitudes in financial markets at any given point in time, leaving the equity premium puzzle unresolved. Additionally, certain and uncertain utilities are compared at equal values in the standard theory. However, there is no need to compare utilities at equal wealth values in the new method. A concave utility curve is assumed throughout the period in the unsolved version of the equity premium puzzle. This practice demonstrates that investors are risk-averse for the whole period according to standard theory, and standard theory does not specify risk behaviours, for example, in the year 1977 or the year 1978, etc. In contrast, Aras (2022) determines these attitudes at specific points in time (i.e., comparing certain and uncertain utilities in 1977 and determining whether the investor is risk-averse in 1977). Hence, this new method is developed by formulating alternative definitions in the financial markets. The study contributes to the existing literature by proposing new methods and alternative definitions of investors' risk attitudes in financial markets, the validity of which is empirically supported. The new method draws in part on the consumption capital asset

pricing model (CCAPM), and alternative definitions differ from the standard ones in that they use a coined variable and do not require specific shapes of utility curves. Moreover, the logic behind the alternative definitions provides a solution to the equity premium puzzle.

Literature Review, Materials and Methods, Results and Discussion, Conclusions, and Statements and Declarations are presented in subsequent sections.

2. Literature Review

The literature covers a range of studies and methodologies investigating risk attitudes and behaviour. Some studies explore alternative risk measures, such as downside variance, while others investigate the shape of the utility function and the degree of risk aversion of decision-makers. Many studies focus on estimating the coefficients of relative risk aversion (CRRAs) using different data sources and methodologies, including experimental gambling approaches, labour supply data, and asset pricing models. Some studies also examine the effects of constraints on risk-averting behaviour and the determinants of risk attitude. The literature review shows that ongoing research in this field includes recent studies exploring new formulas for calculating relative risk aversion and estimating attitudes towards risk using hypothetical games and non-fair lotteries.

The literature on risk attitudes is vast and varied. Some researchers propose alternative methods for calculating risk preferences or alter the assumptions of the expected utility model to account for behavioural anomalies. Leibowitz (1986) argues for a new risk tool that accounts for the liability side of investment decisions. Szpiro (1986a) develops an alternative method for calculating the CRRA using data on property/liability insurance, while Sortino and van der Meer (1991) propose using downside variance as a better risk tool for many investment decisions. Over the last five years, researchers have proposed new ways to assess risk preferences, rather than relying on traditional models. Zeytoon-Nejad (2022) uses indirect utility functions in a laboratory environment to examine the circumstances dependency of risk preferences. Li et al. (2022) randomised the spectral risk measure as a new structure to investigate risk preferences. Cheng et al. (2023) make use of interactive questioning instead of a random one to highlight the agent's risk aversion. They decide in the end that interactive questioning is more effective than a random one. Luo et al. (2023) use Gini deviation rather than variance in risk-averse reinforcement learning to achieve a more effective risk-aversion measure. Fairchild et al. (2023) propose a new approach to studying the simultaneous maximum acceptable risk threshold in discrete choice experiments when risks are dependent, and agents' preferences are nonlinear.

Other researchers investigate the risk attitudes of different groups of people. For example, Binici et al. (2003) examine the risk attitudes of farmers in Türkiye and find that they are generally risk averse. Yesuf and Bluffstone (2009) estimate risk aversion in Ethiopia and find high levels of risk aversion, as well as significant effects of constraints on risk-averse behaviour. Picazo-Tadeo and Wall (2011) estimate risk aversion coefficients for farmers and find that they exhibit risk aversion. Researchers continued to highlight the risk

behaviours of different groups in the last five years. Chan et al. (2020) examine how risk attitudes influence mobility during the COVID-19 pandemic. Meissner et al. (2023) conducted a multi-country survey to investigate how individual characteristics are connected with risk aversion. Kavas and Medetoğlu (2023) examine how demographic differences affect financial risk tolerance in Türkiye.

Some studies explore the determinants of risk attitudes. Diaz and Esparcia (2019) review the most novel methodologies and perspectives and identify the main determinants of risk attitude. Jineakoplos and Bernasek (2007) find that women are generally more risk-averse than men in financial decision-making, and Schechter (2007) combines experimental and survey data to estimate CRRAs using expected-utility maximisation. Scientists have continued to examine risk determinants of various groups over the last five years. Rahman et al. (2023) explore psychological determinants of financial risk tolerance in Malaysia. Suherman et al. (2023) investigate the factors of financial risk tolerance among Indonesian millennials. Quiroga and Tanner (2024) examine how risk choices change after the 2023 Türkiye Earthquake. Rodrigues and B.V. (2024) adopt a multigenerational perspective on how the Big Five personality traits influence financial risk tolerance.

Other researchers develop new methods for calculating risk preferences. For example, Chetty (2006) develops a new method for calculating the CRRA using data from labour supply. Samartsiz and Pittis (2022) derive an expression for the relative risk aversion of non-fair lotteries, arguing that their formula is better than previously proposed methods. Aras (2022) offers a solution to the equity premium puzzle by introducing a variable, the "sufficiency factor of the model". This coined variable helps determine investors' risk attitudes. Aras (2024) again shows that a utility curve need not assume specific shapes to capture individuals' risk demeanours.

Researchers agree that CRRA varies due to psychological factors, past outcomes, time preferences, life-cycle ageing factors, and market conditions. When people consider themselves richer, they take more risks. Volatility cycles are an additional factor. High market volatility increases risk aversion. Researchers explain that as previous profits lower risk aversion, prior losses increase it. Steffensen (2011) finds that CRRA increases with age. Berardi (2016) states that CRRA is sensitive to unanticipated returns. Li et al. (2021) find that investors exhibit less risk aversion during bull markets and more during bear markets. Finally, Levy (2024) shows that CRRA values outside 0,75-1,15 yield paradoxical preferences; hence, he concludes that CRRA should be about 1.

Overall, the literature on risk attitudes in financial markets is extensive and has produced many insights into how different factors affect individuals' risk preferences.

3. Materials and Methods

3.1. Data

The study is based on a dataset used by Mehra and Prescott (1985), a dataset from the Survey Research Centre, Institute for Social Research, The University of Michigan (1978), and data from the United States Census Bureau (1975).

The dataset of Mehra and Prescott (1985) covers US financial markets data for the 1889-1978 period. The dataset from the Survey Research Centre, Institute for Social Research, The University of Michigan (1978) contains projected per capita real consumption of non-durables and services for 1978. Lastly, data from the United States Census Bureau (1975) is data from the projected population of the USA in 1978. The last two datasets are listed in Table 1.

Table: 1
Sample Statistics Projections for the US Economy for the Year 1978

Projected Population mid-year (as of July 1)	Projected Nominal Consumption on Non-durables (Billions of Dollars)	Projected Nominal Consumption on Services (Billions of Dollars)	Projected Nominal Consumption on Non-durables and Services (Billions of Dollars)	Projected GNP Deflator (1972=100)	Projected Per Capita Real Consumption on Non-durables and Services (Dollars)
219441872	515,4	613,7	1129,1	150	≅ 3430

Source: Institute for Social Research, The University of Michigan, Economic Outlook USA, Vol. 5, No.1, Winter 1978 and United States Census Bureau, Projections of the Population of the United States: 1977 to 2050.

3.2. Research Methodology

In this study, the new definitions of investor risk attitudes proposed by Aras (2022), as well as additional definitions, are tested using data from the US financial markets to assess their validity using a new method.

The descriptive research design is used to assess the validity of new definitions (e.g., by calculating the conditional expectation value of an uncertain utility). The quantitative approach is followed for data analysis (i.e., comparing certain and uncertain utilities quantitatively in the new definitions).

To determine investors' risk attitudes, the study includes equity and risk-free asset investors who invested in the composite S&P 500 index and US Treasury bills, respectively.

Alternative definitions of risk behaviours are tested by following the new method. The steps of the new method are as follows:

1) Formulate the problem of the typical agent (i.e., the problem of the equity premium puzzle),

2) Solve the problem using dynamic programming,

3) Derive the fundamental equation of CCAPM and other equations (i.e., equations 18-23 in Aras, 2022) from the problem,

4) Find the values of CRRA and SFOMs for the equity and risk-free asset by calibrating STDF value as 0,99 for the period from the derived group of equations,

5) Test the new definitions by using the calculated SFOM and CRRA values and calibrated STDF value.

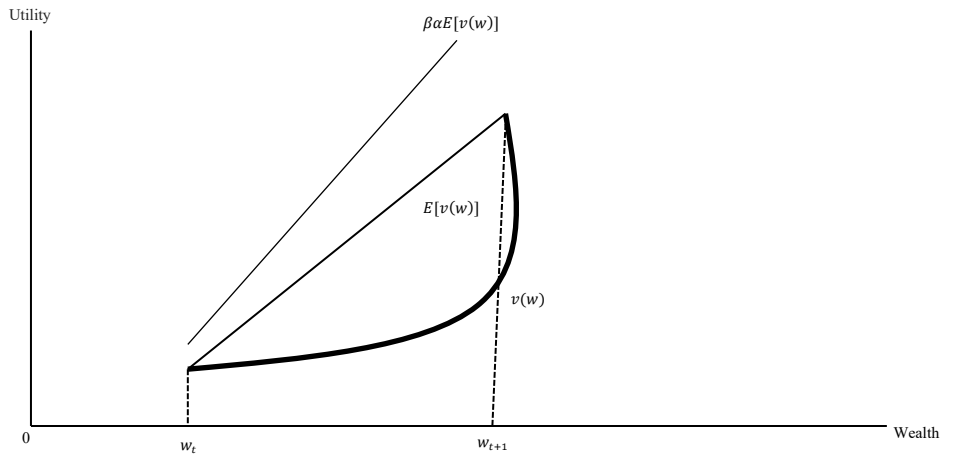
The new method partly depends on the basic equations of CCAPM and partly on new definitions. Dynamic programming transforms the equity premium puzzle into a two-period problem. Hence, this transformation enables the use of new definitions. Moreover, the alternative definitions allow determining individuals' risk attitudes at unequal wealth levels at specific points in time, whereas the standard method identifies these behaviours at equal wealth levels and cannot specify them at specific points in time.

3.2.1. A New Method

Aras (2024) proves that his definitions show strict concavity or strict convexity, or linearity in a way that is not the same as the standard theory (i.e., the new definitions possess a broader content than the standard definitions when shape is considered and the new definitions provide strict concavity or strict convexity, or linearity one by one). Detailed analysis of this issue is in Aras.

One figure (i.e., Figure 1) for only one definition is exhibited to ease understanding.

Figure: 1
Risk-Loving Investor



Note. The figure demonstrates the certain utility curve v(w), the uncertain utility curve Ev(w) and the uncertain utility curve $\beta\eta E[v(w)]$ that includes the SFOM (α) and the STDF (β) of a risk-loving investor who assigns extra positive utility to the uncertain wealth values.

Investors assign extra positive utility to uncertain wealth values and uncertain utility $\beta\alpha E[v(w_{t+1})]$ is larger than certain utility $[v(w_t)]$ according to Figure 1. Hence, this characterisation is labelled as a risk-loving attitude in the alternative definitions. $\beta\alpha E[v(w_{t+1})]$ shifts up because agents assign extra positive utility to uncertain values.

It is necessary to demonstrate that the inequalities below are also valid when the following conditions are met.

1) Investors can be either risk-averse or not sufficiently risk-loving if a continuous concave utility curve exists.

2) Investors can be either risk-loving or not sufficiently risk-averse when the certain utility curve is continuous and convex.

3) Lastly, investors can be risk-neutral when a certain utility curve is a continuous linear curve.

Assuming that a certain utility curve is a continuous concave curve on the given interval for all types of investors, as proposed by Aras (2022), here are the following definitions.

According to Aras (2022), an investor who allocates at time t extra negative utility to uncertain wealth values owing to future uncertainties and an insufficient predictive model is risk-averse, and the following inequality holds true for him/her.

$$v(w_t) > \beta\alpha_t E_t[v(w_{t+1})] \tag{1}$$

By contrast, an investor who allocates at time t extra positive utility to uncertain wealth values owing to future uncertainties and an insufficient predictive model is a risk-loving investor, and the following inequality holds true for him/her.

$$v(w_t) < \beta\alpha_t E_t[v(w_{t+1})] \tag{2}$$

Additionally, an investor who assigns at time t extra positive utility to uncertain wealth values due to future uncertainties and an insufficient predictive model is not sufficiently risk-loving, and the following inequality holds true for him/her.

$$v(w_t) > \beta\alpha_t E_t[v(w_{t+1})] \tag{3}$$

Moreover, an investor who allocates at time t extra negative or extra positive utility for uncertain wealth values owing to the future uncertainties and an insufficient model used for prediction is a risk-neutral investor, and the following inequality holds true for him/her when the certain utility curve is a continuous concave, convex, or linear curve.

$$v(w_t) = \beta\alpha_t E_t[v(w_{t+1})] \tag{4}$$

Finally, an investor who assigns at time t extra negative utility to uncertain wealth values due to future uncertainties and an insufficient predictive model is not sufficiently risk-averse, and the following inequality holds true for him/her when the certain utility curve is continuous and convex.

$$v(w_t) < \beta\alpha_t E_t[v(w_{t+1})] \tag{5}$$

Zero utility allocation is excluded from the definitions in this paper.

In these inequalities, w_t, v, t, $v(w_t)$, $E_t[v(w_{t+1})]$, β and α_t are wealth value at time t, utility curve, the time the comparison is made between the utilities of certain and uncertain wealth values, the certain utility, the projected uncertain utility conditional on information at time t, the subjective time discount factor (STDF) and SFOM at time t, respectively.

α_t is a coefficient that is deemed appropriate for the uncertain utility curve, that is, $E_t[v(w_{t+1})]$. According to Aras (2022), it is as follows:

"$\alpha_t E_t[v(w_{t+1})] = E_t[v(w_{t+1})]$ + extra negative utility allocated by the investor at time t owing to the future uncertainties and insufficient model used for prediction, $\alpha_t E_t[v(w_{t+1})] = E_t[v(w_{t+1})]$ + extra positive utility allocated by the investor at time t because of the future uncertainties and insufficient model used for prediction and $\alpha_t E_t[v(w_{t+1})] = E_t[v(w_{t+1})]$ + extra zero utility allocated by the investor at time t because of the future uncertainties and insufficient model used for prediction" (Aras, 2022: 616).

A calculation example can be found in Aras (2022), where STDF and SFOM are used together for future utilities.

Unconditional and conditional expectation operators are assumed to be equivalent in determining individuals' risk attitudes, thereby enabling an analytically tractable solution.

The same logic in the definitions can be applied when certain utility curves are horizontal in the x-y axis. However, the definition "a not enough (insufficient) risk-averse investor" is not valid if a certain utility curve is horizontal.

The SFOM is a crucial variable as investors often make incorrect predictions owing to insufficient models and future uncertainties. As a result, the magnitudes of future utilities may differ from those of present utilities, depending on wealth values, STDFs, and uncertainty. The magnitude of the uncertain utility determined by investors' methods cannot account for all future uncertainties, and their methods may be insufficient to predict this variable. Hence, an additional amount must be considered for the uncertain utility at present to account for the uncertainty. This extra amount, reflected in the uncertain utility, is called SFOM. Depending on their allocation of extra positive or negative utility to future utilities, investors are said to be risk-averse, risk-loving, or risk-neutral. For instance, the current utility of 30 stocks is not equal to the future utility of 30 stocks, not only because of the STDF but also because the risk-averse investor assigns extra negative utility to the future.

In a sense, just as STDF indicates agents' impatience for consumption, SFOM indicates the adjustment of future utility to uncertainty in financial markets.

Definition. The investor is a forward-looking investor if the investor compares the future utility (v_f) with the current utility (v_c) and then decides at the current time.

Definition. The investor is backwards-looking if the investor compares the past utility (v_p) with the current utility (v_c) and then decides at the current time.

Definition. The investor is divergent if the decision differs between the forward-looking and backwards-looking investors at the current time.

Definition. The investor is convergent if the decision is the same for the forward-looking and the backwards-looking investor at the current time.

Theorem 1. Assume that the investor's decision is either to sell or to hold the security at the current time. If $v_p < v_c < v_f$ or $v_f < v_c < v_p$ holds true for the investor, the investor is said to be a divergent investor at the current time. By contrast, the investor is said to be a convergent investor at the current time for all remaining alternative sequences of v_p, v_c, and v_f.

Proof. There are 6 different sequences of v_p, v_c, and v_f. For $v_p < v_c < v_f$, the backwards-looking investor will sell the security at the current time because $v_p < v_c$ holds true at the current time. By contrast, the forward-looking investor will continue to hold the security at the current time because $v_c < v_f$ holds true. Hence, for $v_p < v_c < v_f$, the investor is said to be a divergent investor.

For $v_f < v_c < v_p$, the backwards-looking investor will continue to hold the security at the current time because $v_c < v_p$ holds true at the current time. By contrast, the forward-looking investor will sell the security at the current time because $v_f < v_c$ holds true at the current time. Hence, for $v_f < v_c < v_p$, the investor is said to be a divergent investor.

The investor is a convergent investor for all remaining sequences of v_p, v_c, and v_f.

Q.E.D.

Theorem 2. Assume that the investor's decision is either to buy or not to buy the security at the current time. If $v_p < v_c < v_f$ or $v_f < v_c < v_p$ holds true for the investor, the investor is said to be a divergent investor at the current time. By contrast, the investor is said to be a convergent investor at the current time for all remaining alternative sequences of v_p, v_c, and v_f.

Proof. There are 6 different sequences of v_p, v_c, and v_f. For $v_p < v_c < v_f$, the backwards-looking investor will buy the security at the current time because $v_p < v_c$ holds true at the current time. By contrast, the forward-looking investor will not buy the security at the current time because $v_c < v_f$ holds true. Hence, for $v_p < v_c < v_f$, the investor is said to be a divergent investor.

For $v_f < v_c < v_p$, the backwards-looking investor will not buy the security at the current time because $v_c < v_p$ holds true at the current time. By contrast, the forward-looking investor will buy the security at the current time because $v_f < v_c$ holds true at the current time. Hence, for $v_f < v_c < v_p$, the investor is said to be a divergent investor.

The investor is a convergent investor for all remaining sequences of v_p, v_c, and v_f.

Q.E.D.

In Aras (2022), the SFOMs and the CRRA for the year 1977 are estimated. The new model, presented as a solution to the equity premium puzzle, estimates the CRRA, the SFOM for equity investors, and the SFOM for risk-free asset investors. The calculated values of the CRRA and STDF in the article are compatible with the existing empirical literature (Arrow, 1971; Friend & Blume, 1975; Hildreth & Knowles, 1982; Kydland & Prescott, 1982; Kehoe & Richardson, 1983; Szpiro, 1986a, 1986b; Ljungquist & Sargent, 2000; Azar, 2007; Barro & Jin, 2011; Levy, 2024).

Aras (2022) formulates the problem of the typical agent to possess the solution for the equity premium puzzle as follows:

$$max_{\{\theta_{t+1}, z_{t+1}, c_t\}} \{v(c_t) + \eta E_t[\sum_{s=t}^{\infty} \beta^{s+1-t} v(c_{s+1})]\}$$

s.t.

$$z_{t+1} q_t + \theta_{t+1} p_t + c_t \leq \theta_t y_t + \theta_t p_t + z_t \quad (6)$$

$$c_t \geq 0, 0 \leq \theta_t \leq 1 \text{ for each t}$$

where $\beta, c, v, \eta, z_t, \theta_t, q_t, p_t$ and y_t denote the STDF, per capita consumption, utility, SFOM of investors, amount of risk-free asset, amount of equity, price of risk-free asset, price of equity and dividend, respectively.

Aras (2022) solves the typical agent problem by dynamic programming. He calculates the empirically correct values of CRRAs and STDF from his group of equations in the article (i.e., equations 18-23). This programming technique also transforms the problem into a two-term recurrence. Hence, the transformation enables the use of alternative definitions in the problem and the specification of risk behaviours.

The SFOM for risk-free asset investors exists at time t for two reasons:

1) Taking into account of the uncertainty of $c_{t+1} = \theta_{t+1}y_{t+1} + \theta_{t+1}p_{t+1} + z_{t+1} - z_{t+2}q_{t+1} - \theta_{t+2}p_{t+1}$,

2) Given that risk-free asset investors may not wait until the bond matures, and instead, could trade with the FED (The Federal Reserve System) in the open market. This probable situation gives way to uncertainty at time t.

Market clearing exists for equity and risk-free asset investors with z_{t+1} = 0 and $\theta_t = \theta_{t+1}\ldots$ = 1. The first no-trade equilibrium for risk-free assets occurs at time t+1 when the uncertainty of $c_{t+1} = \theta_{t+1}y_{t+1} + \theta_{t+1}p_{t+1} + z_{t+1} - z_{t+2}q_{t+1} - \theta_{t+2}p_{t+1}$ is taken into account.

If the Table 4 in Mehra (2003) that demonstrates the sample statistics of US Economy for the 1889-1978 period is used, the solution of this system of equations is as follows when the uncertainty of $c_{t+1} = \theta_{t+1}y_{t+1} + \theta_{t+1}p_{t+1} + z_{t+1} - z_{t+2}q_{t+1} - \theta_{t+2}p_{t+1}$ is taken into account (Aras, 2022).

$\zeta \cong 0.961745$

$\xi \cong 1.019392$

$\rho \cong 1.033526$

If only the realised value of real per capita consumption of 1978 in the dataset used by Mehra and Prescott (1985) is changed with the projected one of the same year, the solution set of this system of equations is as follows when the uncertainty of c_{t+1} is taken into account.

$\zeta \cong 0.96232455$

$\xi \cong 1.01997145$

$\rho \cong 1.05803027$

The input values for the equations are supposed to have 8 decimal places of accuracy in the calculation. Here, ζ, ξ, and ρ denote the SFOM for equity investors, the SFOM for risk-free asset investors, and the CRRA.

Aras (2022) states that SFOMs are constant coefficients in his article. The investor's utility curve is presumed to be strictly concave, given by the equation $v(c_t) = \frac{{c_t}^{1-\rho}-1}{1-\rho}$. When the CRRA is greater than or equal to zero, an SFOM less than one implies that investors assign greater negative utility to uncertain wealth values. Conversely, when the CRRA is greater than or equal to zero, an SFOM larger than one implies that investors assign extra positive utility to uncertain wealth values.

The equation $c_t = \theta_t y_t + \theta_t p_t + z_t - z_{t+1}q_t - \theta_{t+1}p_t$ should be accepted to specify the risk attitudes of investors in financial markets, by referring to Equations 1-5. If

the trade with the FED occurs, SFOMs must be recalculated after the subjective time discount factor is redetermined based on the trade date.

When making financial investment decisions, a typical investor considers both certain and uncertain utilities. He/she assigns extra positive or extra negative utility to uncertain wealth values and is said to be risk-averse, risk-loving or risk-neutral. In this study, the certain utility for 1977 and the uncertain utility for 1978 were estimated.

The author selects a strictly concave utility curve $v(c_t) = \frac{{c_t}^{1-\rho}-1}{1-\rho}$ for the utility curve of the agents. To determine their risk attitudes, the following formula is used.

$$E(z^a) = E[(exp(alnz)] = \exp\left(a\mu_z + \frac{1}{2}\, a^2{\sigma_z}^2\right) \quad (7)$$

This general formula is used to calculate the uncertain utility where z^a is equal to $\frac{{c_t}^{1-\rho}-1}{1-\rho}$.

4. Results and Discussion

All calculation results with $v(c_t) = \frac{{c_t}^{1-\rho}-1}{1-\rho}$ and β= 0.99 are presented in from Tables 2 and 3.

The following inequalities hold true for both 1,033526 and 1,05803027 values of CRRA.

If the year 1977 is assumed as the current time and the utility of the year 1976 is calculated with $(\frac{{c_{1976}}^{1-\rho}-1}{1-\rho})$, $v_{1978} < v_{1976} < v_{1977}$ will be possessed for equity and risk-free asset investors. Hence, the equity and risk-free asset investors in 1977 are convergent investors.

When the 1978 (realised) per capita real consumption value is used to forecast uncertain utility, the following inequality holds true for equity investors.

$$v(c_{1977}) > \beta\alpha_{1977}E_{1977}[v(c_{1978\,(realized)})] \quad (8)$$

When the 1978 (projected) per capita real consumption value is used to forecast uncertain utility, the following inequality also holds true for equity investors.

$$v(c_{1977}) > \beta\alpha_{1977}E_{1977}[v(c_{1978(projected)})] \quad (9)$$

As the SFOMs for equity investors are 0,961745 and 0,96232455, the above inequalities imply that as equity investors allocate extra negative utility to uncertain wealth values, they are risk-averse for both realised and projected 1978 per capita real consumption values. Hence, Equation 1 is appropriate for equity investors in 1977.

The following equations hold for risk-free asset investors in 1977.

$$v(c_{1977}) > \beta\alpha_{1977}E_{1977}[v(c_{1978\ (realized)})] \tag{10}$$

$$v(c_{1977}) > \beta\alpha_{1977}E_{1977}[v(c_{1978(projected)})] \tag{11}$$

As the SFOMs for risk-free asset investors are 1,019392 and 1,01997145, the above inequalities imply that, as risk-free asset investors allocate extra positive utility to uncertain wealth values, they are not enough (insufficient) risk-loving for both realised and projected 1978 per capita real consumption values. Hence, Equation 3 is appropriate for risk-free asset investors in 1977.

Many investment professionals say that no asset, including US Treasury bills, can be considered risk-free in the investment world. Hence, our results are consistent with industry practice. Additionally, it is stated as a fact that equity investors whose portfolios track indexes (e.g., the S&P 500 Index) are risk-averse. Hence, our results for equity investors are compatible with the risk section of finance theory.

Empirically compatible values for CRRAs and STDF derived from the equity premium puzzle are used to test the alternative definitions (i.e., 1,033526 and 1,05803027 for CRRAs and 0,99 for STDF). This practice aligns with the empirical literature (e.g., Arrow, 1971; Levy, 2024).

Aras (2022) solved the equity premium puzzle by empirically calculating and calibrating compatible values for CRRAs and STDF. The correctness of alternative definitions from one point of view also supports the validity of the Aras solution.

The above findings suggest the validity of the alternative definitions. Moreover, these findings provide us with the opportunity to easily incorporate investors' risk attitudes into financial markets and economic and financial models.

5. Conclusion

It is now possible to specify the types of investors in financial markets using these alternative definitions under the new method. However, these determinations at certain points in time are impossible in standard theory because comparisons between certain and uncertain utilities are made at equal wealth levels. Hence, it may be impossible, under standard economic and financial models, to determine investors' risk attitudes at any given point in time.

Solving the equity premium puzzle, as Aras does, involves using alternative definitions. Dynamic programming reformulates the problem as a two-term recurrence. When comparing certain and uncertain utilities over two-term periods, alternative definitions are used to detect risk behaviours. Hence, it can be concluded that the validity of alternative definitions strengthens the solution to the equity premium puzzle.

Hence, the correctness of that solution suggests one consider these definitions in the investment strategies of financial institutions. Financial institutions identify the types of investors in the market using the new method and alternative definitions, and then invest accordingly. This will probably be the most correct investment strategy for them. Additionally, the new method and alternative definitions provide policy-makers with a link between the real and financial sectors, as the method's steps involve the CCAPM and per capita real consumption in terms of the amounts and prices of financial assets. CCAPM links consumption decisions to financial decisions, and alternative definitions can be stated in both real and financial terms. In this context, various relationships may emerge between the two sectors for researchers. Hence, the researchers can establish and investigate very rich relationships between the real and financial sectors.

The equity and risk-free asset investors were found to be convergent investors in 1977. Additionally, the study found that equity investors who invested in the S&P 500 composite index were risk-averse in 1977, while risk-free asset investors who invested in US Treasury bills were not sufficiently risk-loving (i.e., a form of risk-averse behaviour). These findings are consistent with finance theory and indicate the validity of the new definitions.

Specifically, the nature of the definition "a not enough (insufficient) risk-loving behaviour" is not known adequately. The researchers should study this behaviour. Hence, new definitions in the article may open new fields of study for the researchers.

Table: 2
Type of Equity Investors in the US Financial Markets Using Data from the Period of 1889-1978

Year	Per Capita Real Consumption (in dollars)	Certain Utility	Uncertain Utility	Utility Allocation	Type of CRRA Investor
1977 (realised)	3340	7,103787			1,033526
1978 (realised)	3450		6,192691	Equity investors allocate extra negative utility	Risk-averse
1977 (realised)	3340	6,47116728			1,05803027
1978 (projected)	3430		5,69098755	Equity investors allocate extra negative utility	Risk-averse

Source: 1977 (realised) and 1978 (realised) per capita real consumption values are taken from the data used in Mehra and Prescott (1985). 1978 (projected) per capita real consumption value is computed according to values in the Institute for Social Research, The University of Michigan, Economic Outlook USA, Vol. 5, No.1, Winter 1978 and those in the United States Census Bureau, Projections of the Population of the United States: 1977 to 2050.

Table: 3
Type of Risk-Free Asset Investors in US Financial Markets, Using Data from the Period of 1889-1978

Year	Per Capita Real Consumption (in dollars)	Certain Utility	Uncertain Utility	Utility Allocation	Type of CRRA Investor
1977 (realised)	3340	7,103787			1,033526
1978 (realised)	3450		6,563881	Risk-free asset investors allocate extra positive utility	Insufficient risk-loving
1977 (realised)	3340	6,47116728			1,05803027
1978 (projected)	3430		6,03189934	Risk-free asset investors allocate extra positive utility	Insufficient risk-loving

Source: 1977 (realised) and 1978 (realised) per capita real consumption values are taken from the data used in Mehra and Prescott (1985). 1978 (projected) per capita real consumption value is computed according to values in the Institute for Social Research, The University of Michigan, Economic Outlook USA, Vol. 5, No.1, Winter 1978 and those in the United States Census Bureau, Projections of the Population of the United States: 1977 to 2050.

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