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Advanced gravitational decision-making method inspired by newton's law of universal gravitation

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This study introduces the Advanced Gravitational Decision-Making (GRAD) approach, an innovative framework that enriches multi-criteria decision-making (MCDM) by integrating inter-alternative interactions, the standard deviations of criteria, and distance into a single model. Inspired by Newton's universal law of gravitation, GRAD surpasses conventional methods by offering a more realistic assessment of alternatives, particularly when facing high uncertainty or closely matched alternatives. The method treats criteria with higher standard deviation as having "greater mass," thus exerting stronger influence on the final decision. Simultaneously, distance calculations capture how similar or dissimilar alternatives "attract" or "repel" each other, leading to more nuanced rankings. In a demonstrative case, GRAD was compared against popular techniques like TOPSIS, VIKOR, and CoCoSo, revealing more robust outcomes under uncertainty. Sensitivity analyses further confirmed its adaptability to varying weight and risk preferences, while Monte Carlo simulations showed that slight data perturbations rarely altered GRAD's overall rankings. Moreover, the method was applied to a real-world dataset on concrete compressive strength, a fundamental mechanical property, where GRAD successfully identified the most balanced mix design by accounting for complex trade-offs among multiple material and process variables. Organizations in diverse sectors can therefore benefit from GRAD by objectively evaluating the interplay among alternatives, ensuring more reliable and informed decisions. Overall, GRAD provides a comprehensive yet flexible tool that enhances the rigor of MCDM in uncertain, multidimensional scenarios.

Keywords Multi-Criteria Decision Making, Decision Analysis, Newton's Law of Gravitation, Gravitational Decision-Making, Mathematical Modeling

MCDM methods provide significant advantages to decision-makers when confronted with complex problems requiring the simultaneous evaluation of multiple, often conflicting, criteria. These methods are critically important across various fields such as engineering[1], healthcare[2], finance[3], supply chain management[4], and environmental management[5], owing to their systematic framework and mathematical rigor. For instance, more rational approaches can be applied in the healthcare sector to efficiently allocate scarce resources[2] or to establish a balance among economic, social, and ecological considerations in environmental management processes[5]. Likewise, MCDM methods, by offering multi-dimensional analytical capabilities, are instrumental in assessing investment alternatives in uncertain financial markets[3] and in selecting green suppliers in globally distributed supply chains[4].

Among the traditional MCDM methods, the Analytic Hierarchy Process (AHP)[6], the Analytic Network Process (ANP)[7], TOPSIS[8–10], PROMETHEE[11], and ELECTRE[12] stand out in the literature with different modeling approaches. In addition, SPOTIS (Stable Preference Ordering Towards Ideal Solution) has recently attracted attention as a distance-to-ideal method that emphasizes robustness against rank reversal through a predefined Ideal Solution Point (ISP) and independent criterion bounds[13]. However, most of these methods are not sufficiently comprehensive in simultaneously handling the direct influences that alternatives exert on one another as well as the uncertainty (such as standard deviation) present in the criteria. Additionally, objective preprocessing-based weighting schemes such as CRITIC[14] use dispersion (e.g., σi)—often together with inter-

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criterion correlation—primarily to compute criterion weights prior to aggregation, whereas GRAD integrates σi\sigma_iσi directly into the score formulation and interaction mechanism, making criterion variability an intrinsic component of the final ranking rather than only a weighting factor. Recent developments in the literature include the Ordinal Priority Approach (OPA)[15], Combined Compromise Solution (CoCoSo)[16], Unified Bayesian Frameworks[17],Virtual Gap Analysis[18], and simulation models[19], and fuzzy interval-based ranking stabilization approaches such as the fuzzy normalization-based MABAC variant (FN-MABAC), which employs triangular fuzzy numbers and the Border Approximation Area (BAA) concept[20]. Nevertheless, it is evident in today's high-dynamic and multi-layered decision environments that a holistic approach is needed to fully address both "interactions among alternatives" and "uncertainty in criteria."[21–24].

Developed in response to this need, the GRAD method offers an innovative framework for MCDM by drawing inspiration from Newton's universal law of gravitation. According to Newton's law, the gravitational force between two masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them[25]. This fundamental principle, long applied in physical sciences, has also been used to model various interactions in social and economic sciences. For example, "gravitational" analogies are known to be employed in social network analysis to predict the degree of interaction among individuals or in economics and marketing to examine consumer behavior and trade flows[26–30]. The gravity model of international trade, for example, predicts the volume of trade between countries as being directly proportional to the product of their economic sizes and inversely proportional to their geographical or economic distance. Similarly, migration and population mobility models analyze "pull" and "push" factors rooted in social and economic elements. In line with this background, the GRAD method treats each alternative as if it were a "mass," whose magnitude is determined by its performance values and the degree of uncertainty (standard deviation) within the criterion. Consequently, the importance levels of the criteria, the performance values of the alternatives (analogous to "mass" in the classical Newtonian sense), the standard deviations of the criteria (indicating the level of uncertainty or volatility), and the distances among alternatives (the factor causing gravitational force to decrease with increasing distance) are combined into a single decision model. As a result, the decision-making process is not merely based on each alternative's individual performance score or its distance from an "ideal solution"; it also accounts for the pull or push that alternatives exert on one another as well as the added risk introduced by uncertainty in the criteria.

Compared to conventional MCDM methods, the GRAD approach stands out by integrating, within a single framework, three critical dimensions: criterion uncertainty (captured through standard deviation), gravitational pull (interactions among alternatives), and distance. This enables a more realistic evaluation of complex decision scenarios that classical methods often overlook or simplify. For instance, a criterion with high standard deviation—indicating volatility or unpredictability—acts as a "larger mass" in GRAD, thereby exerting stronger influence on the decision process and shaping the relative positioning of alternatives. Similarly, when alternatives are close in performance, their gravitational interaction intensifies, forcing finer distinctions in ranking; whereas an alternative that is distant from others rapidly loses influence. This dynamic ranking logic allows decision-makers to move beyond simplistic "best versus worst" classifications and instead capture the nuanced interplay of uncertainty, importance, and similarity across criteria. Such features are particularly valuable in large-scale engineering projects (e.g., dams, bridges, infrastructure) where cost, time, safety, and environmental dimensions must be balanced; in financial portfolio selection, where volatility and risk asymmetry dominate; and in strategic supply chain or renewable energy planning, where technological and market uncertainties drive variability at the criterion level. As illustrated in Table 1, GRAD transcends the limitations of distance-based MCDM models by providing a unified analytical structure that interprets each criterion's standard deviation (σ) as "gravitational mass," dynamically adjusting its weight in accordance with uncertainty. In addition, it incorporates gravitational-like forces to quantify not only how an alternative performs individually, but also how it interacts with others in the decision space. This allows GRAD to simultaneously address criteria of different scales, levels of risk, and degrees of similarity within a single deterministic formulation. With this holistic perspective, decision-makers can achieve more consistent and strategically grounded outcomes in dynamic and uncertain environments. In a demonstrative evaluation, GRAD was also compared against established techniques such as TOPSIS, VIKOR, and CoCoSo, where it delivered more robust outcomes under uncertainty; sensitivity analyses confirmed its adaptability to varying weight and risk preferences, while Monte Carlo simulations showed that slight perturbations in data rarely altered the overall rankings. The practical value of this paradigm is further emphasized by its application to a concrete compressive strength dataset, where GRAD successfully identified the most balanced mix design among multiple alternatives. Consequently, GRAD positions itself as an innovative, comprehensive, and adaptable method, with strong potential for both academic research and industrial practice.

The GRAD method provides a simple and effective scoring system by considering the total gravitational force and distances between alternatives. Its contributions to the literature are as follows:

- Simplicity and Clarity: The GRAD method uses a simple and understandable formula to evaluate the performance of alternatives. This makes it an easily applicable and comprehensible method for decision-makers.
- Fair Evaluation: By considering the total gravitational force and distances between alternatives, the GRAD method ensures a fair evaluation. It assesses alternatives not only based on their own performances but also on their interactions with each other.
- Realism: By incorporating the standard deviations of criteria, the GRAD method provides more realistic and reliable results. It reflects the variability and uncertainty within criteria, making the evaluation more comprehensive.
- Compatibility: The GRAD method provides results consistent with commonly used MCDM methods such as TOPSIS, ELECTRE and PROMETHEE. This enhances the method's validity and reliability.

Method	Uncertainty Handling (σ, entropy, fuzzy, Bayesian, etc.)	Interaction Among Alternatives	Distance/Ideal Metric	Main Strengths	Main Limitations
AHP	✖	✖	✖	Intuitive hierarchy; consistency check	Ignores risk & interaction; scale-sensitive
ANP	✖	✓	✖	Captures inter-criteria dependence	Computationally heavy; no distance logic
TOPSIS	✖	✖	✓	Simple, widely used; clear ideal/anti-ideal	Cannot model uncertainty or similarity effects
VIKOR	~ (via entropy variants)	✖	✓	Balances compromise & regret	Limited dispersion modelling; no interaction
PROMETHEE II	✖	✖	✖	Flexible preference functions	Subjective thresholds; no robustness layer
ELECTRE III	✓ (qualitative)	✖	✖	Handles outranking with veto rules	Hard to interpret; produces partial order
CRITIC + TOPSIS	✓ (σ & correlation for weights)	✖	✓	Objective weighting plus distance	σ only affects weights; no interaction
CoCoSo	✖	✖	✓	Aggregates multiple compromise scores	Weights external; risk handling limited
Bayesian MCDM	✓	✓	✖	Probabilistic group-decision framework	Requires prior info; lacks distance measure
FN-MABAC	✓	✖	✓	Stabilizes ranking under fuzzy/interval evaluations via fuzzy normalization and BAA-based geometry; suitable when assessments are imprecise	Does not model inter-alternative interactions; results depend on fuzzy modeling choices (TFN parameters/ aggregation)
GSA-MCDM	✓	✓	✓	First gravity-heuristic hybrid; global search	Stochastic tuning; no closed-form proofs
SPOTIS	✖	✖	✓	Rank-reversal-resistant ordering; predefined ISP enables stable, transparent distance-based ranking	Requires predefined bounds/ISP; no interaction modeling; limited uncertainty handling unless extended
GRAD	✓ (direct σ_i)	✓ (gravitational pull)	✓ (norm-based)	Unified treatment of uncertainty, similarity, and distance in a single closed-form model; analytic sensitivity analysis	Needs calibration of α, β, γ, δ; higher computational load in very large problems

Table 1. Comparative overview of representative MCDM methods highlighting key features used to position GRAD.

- Scientific Contribution: The GRAD method introduces a new perspective to multi-criteria decision-making problems. By balancing the performances of alternatives and their interactions with each other, it makes a significant contribution to the literature. This method offers a deeper analysis by considering the interactions between alternatives and the internal inconsistencies of the criteria.

The GRAD method, as a simple and effective scoring system that accounts for the performances and interactions of alternatives, provides decision-makers with a powerful tool for solving multi-criteria decision-making problems.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on multi-criteria decision-making methods and highlights the motivation for developing gravitationally inspired approaches. Section 3 introduces GRAD method and details its mathematical formulation and algorithmic framework. Section 4 presents numerical examples and comparative analyses with well-established MCDM methods to demonstrate the applicability and performance of the proposed approach. Section 5 provides a sensitivity analysis to examine the robustness of the GRAD method with respect to key parameters. Finally, Sect. 6 concludes the paper by summarizing the main findings and outlining potential directions for future research.

Related Works: MCDA methods and positioning of GRAD

Multi-criteria decision analysis (MCDA) has developed into a broad and mature research area with a large family of methods tailored to different preference structures, data types, and decision contexts. Comprehensive overviews and state-of-the-art surveys highlight that MCDA methods typically differ in (i) how they model trade-offs and compensation among criteria, (ii) whether they rely on reference points/ideals, pairwise outranking, or utility/value functions, and (iii) how they handle uncertainty and robustness[31]. In this section, we briefly review commonly used MCDA paradigms and position the proposed GRAD method within this broader landscape.

Value/utility-based and additive aggregation methods

One foundational branch of MCDA is multi-attribute utility/value theory (MAUT/MAVT), where alternatives are evaluated by a value or utility function that aggregates criterion-wise performance under explicit trade-off assumptions. A classical and influential treatment is provided by Keeney and Raiffa, who formalize preference modeling and value trade-offs for multi-objective decision problems[32].

In practice, simpler additive forms such as the weighted sum model (WSM/SAW) are frequently used due to their interpretability and low computational cost. However, additive aggregation can be sensitive to normalization/scaling choices and typically assumes a compensatory structure, where poor performance in one criterion can be offset by good performance in another.

Pairwise comparison and hierarchical weighting (AHP)

Another widely adopted paradigm for criteria prioritization is AHP, which derives weights from structured pairwise comparisons and provides a transparent workflow for eliciting decision-maker preferences. Saaty's original monograph and subsequent widely cited treatments have made AHP one of the most widely used MCDA tools in engineering and management settings. AHP is especially useful when expert judgment is central; nevertheless, results can depend on consistency and judgment quality, and the approach typically addresses weighting more directly than the final ranking logic (which is often coupled with another ranking method)[6].

Ideal-solution and distance-to-reference methods (TOPSIS family)

A major family of ranking methods relies on the idea that the best alternative should be closest to a positive ideal solution and farthest from a negative ideal solution. The classical TOPSIS framework introduced in the multiple attribute decision making literature has become a standard baseline method due to its simplicity and intuitive geometric interpretation. Large-scale surveys also document extensive variations and hybridizations of TOPSIS, including alternative normalization strategies, distance metrics, and fuzzy/uncertain extensions—indicating both the popularity of the approach and the known sensitivity of ideal-solution methods to modeling choices[33,34].

Compromise and regret-based ranking (VIKOR)

Compromise-based methods seek solutions that balance group utility and individual regret, particularly when criteria conflict strongly. The VIKOR method, introduced and analyzed extensively in the MCDA literature, produces a compromise ranking using a composite index derived from an L_1 (utility) and $L\infty$ (regret) perspective. VIKOR is frequently compared with TOPSIS because both are "closeness-to-ideal" oriented, yet they differ in normalization, distance aggregation, and compromise logic—differences that can yield divergent rankings under the same dataset[10].

Outranking methods (ELECTRE and PROMETHEE families)

Outranking approaches compare alternatives pairwise, allowing for non-compensatory reasoning and potential incomparability. The ELECTRE family, originating in Roy's work, introduced a structured way to model concordance/discordance relations and remains influential for problems where veto effects and partial comparability are meaningful. Similarly, PROMETHEE methods provide preference ranking via preference functions and outranking flows; the PROMETHEE I/II frameworks are widely used and have strong methodological and application footprints in the MCDA literature.

While outranking methods can better reflect non-compensatory preferences, they often require additional modeling choices (preference functions, thresholds) that may increase calibration complexity[11,12].

Hybrid and combined scoring philosophies (CoCoSo and related methods)

Recent developments include methods that combine multiple aggregation philosophies to improve discrimination and stability. The Combined Compromise Solution (CoCoSo) method integrates compromise concepts with additive and multiplicative aggregation strategies to construct a final score, aiming to produce balanced rankings across diverse decision structures.

Such combined methods are useful baselines in modern MCDA benchmarking, but their performance can still depend on normalization and the interplay between additive/multiplicative components[16].

Robustness under uncertainty: stochastic and perturbation-based evaluation

A recurring theme in practical MCDA is that rankings may change under measurement noise, incomplete information, or preference uncertainty. Stochastic MCDA approaches, such as Stochastic Multicriteria Acceptability Analysis (SMAA), explicitly represent uncertainty using probability distributions and analyze acceptability indices by exploring the weight and/or performance spaces.

Complementary to fully stochastic frameworks, perturbation-based Monte Carlo analyses—where the decision matrix is repeatedly perturbed and rank frequencies are recorded—offer an intuitive robustness diagnostic widely used in applied studies to quantify stability under small fluctuations[35–38].

Positioning of GRAD within the MCDA landscape

Within the above landscape, GRAD is positioned as an interaction-aware scoring framework that differs from standard ideal-point and compromise methods in two key ways:

1. Dispersion-aware criterion influence: GRAD incorporates a criterion dispersion component (via a standard-deviation term) to reflect how informative or variable each criterion is in the given dataset, which can influence the stability of the aggregation when the data are perturbed.
2. Distance-driven interaction structure: Instead of treating alternatives as independent objects whose scores are computed in isolation, GRAD introduces a distance-based interaction mechanism inspired by a gravitational analogy. This interaction-aware design is intended to improve ranking robustness under small perturbations, complementing the "independent alternative evaluation" paradigm typical of many classical methods (e.g., TOPSIS-like scoring).

Therefore, GRAD complements existing MCDA families by explicitly linking criterion dispersion and alternative proximity/interaction within a unified scoring mechanism, and its robustness can be empirically evaluated using high-iteration perturbation experiments, consistent with the broader robustness literature in MCDA.

Advanced gravitational decision-making method

The GRAD method is inspired by Newton's law of gravitation and is designed to account for the effects and distances between alternatives in MCDM problems. This method provides decision-makers with optimized decisions by using criteria weights, performance values, and distances between alternatives. Below, the steps of GRAD are presented with detailed formulations. Figure 1 visually presents the steps of the GRAD method in the decision-making process. This flow diagram emphasizes the systematic structure of the method and the sequential relationship of each decision component. The GRAD method goes beyond classical multi-criteria decision-making approaches and integrates uncertainties between criteria and interactions between alternatives within a gravity-based structure. The operation of this method starts with "Problem Definition" and includes the steps of criterion and alternative weighting, data normalization, gravitational force calculation, collection of total forces, interaction and scoring between alternatives, sensitivity analysis and finally obtaining decisions and results. The icons and color tones used in each step are carefully designed to emphasize the content of the relevant process. Color transitions in particular emphasize the linear progression of the process and draw the reader's attention to the integrity of the structure.

Problem definition

In this step, the problem that needs to be solved is defined, and the criteria to be evaluated are identified. The alternatives are also outlined, indicating which options will be considered. The goal of this step is to clarify the problem and the criteria that form the foundation of the decision-making process. A well-defined problem increases the effectiveness and accuracy of the subsequent steps. In this step:

- The problem and criteria to be decided upon are determined.
- Alternatives are defined.

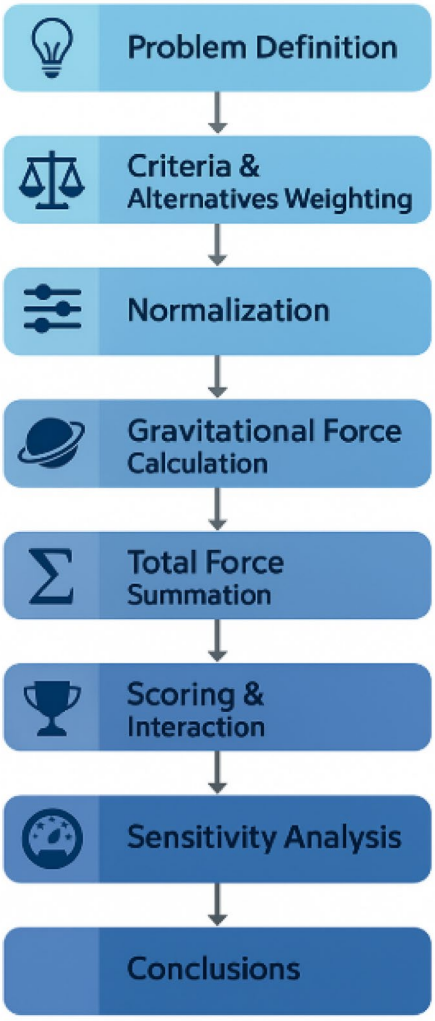


Fig. 1. Stepwise workflow of the GRAD (Gravitational Decision-Making) method.

Weighting of criteria and alternatives

In this step, the importance levels, or weights, of the identified criteria are determined. Simultaneously, each alternative's performance with respect to each criterion is evaluated. Criteria weights are scaled through a normalization process. This step is crucial for assessing the relative importance of criteria and alternatives and preparing the data to be used in the decision-making process. Ix step:

- The weights of the criteria are determined.
- The performance of each alternative with respect to each criterion is evaluated.
- The normalization of the criteria weights is conducted (Eq. 1):

$$\sum_{i=1}^{m} w_i = 1 \tag{1}$$

Normalization of alternative performance values

The performance values of the alternatives are normalized separately for benefit and cost criteria using the min-max normalization method. This approach eliminates scale differences between various criteria, ensuring that the performance values are comparable. By normalization, data measured on different scales are brought to the same level. The normalization formulas for benefit and cost criteria are as follows:

For Benefit Criteria: If a criterion is a benefit criterion (higher values indicate better performance), the normalization is calculated using Eq. (2):

$$x_{ij}^{N} = \frac{x_{ij} - x_i^{\min}}{x_i^{\max} - x_i^{\min}} \tag{2}$$

For Cost Criteria: If a criterion is a cost criterion (lower values indicate better performance), the normalization is calculated using Eq. (3):

$$x_{ij}^{N} = \frac{x_i^{\max} - x_{ij}}{x_i^{\max} - x_i^{\min}} \tag{3}$$

Where:

- x_{ij}^{N} : Normalized performance value of alternative j on criterion i,
- x_{ij} : Original performance value of alternative j on criterion i,
- $x_i^{\min}$: Minimum performance value on criterion i_r.
- $x_i^{\max}$: Maximum performance value on criterion i.

This normalization process ensures that both benefit and cost criteria are evaluated on a consistent scale, enhancing the reliability and comparability of the decision-making process.

Proof 1 The normalization process ensures that data are evaluated within a specific scale. Min-max normalization pulls the lowest value for a criterion to 0 and the highest value to 1, redistributing the other values within this scale. This eliminates scale differences between different criteria.

Calculation of distances

In this step, the distances of each alternative from the ideal value for each criterion are calculated. Distances are used to measure how far alternatives are from the ideal performance value. Correct calculation of distances forms the foundation of the GRAD method and helps model the interactions between alternatives. In this step:

- The distance from the maximum normalized value = 1 for each alternative is calculated using Euclidean distance as shown in Eq. 4:

$$d_j = \sqrt{\sum_{i=1}^{m} \left(x_{ij}^{N} - 1\right)^2} \tag{4}$$

Integration with Expected Solution Point (ESP) In Eq. (4), the distance-to-reference term is computed with respect to the Ideal Solution Point (ISP) after normalization. Under this setting, the ideal reference value for each criterion is defined as Eq. 4a.

$$r_i^{\mathrm{ISP}} = 1, i = 1, \ldots, m \tag{5}$$

In practical decision-making problems, however, decision-makers may define subjective aspiration levels rather than strictly relying on the mathematical ideal. To incorporate such preferences, an Expected Solution Point (ESP) is introduced as a reference vector as in Eq. 4b:

$$\mathbf{r}^{\mathrm{ESP}} = (e_1, \ldots, e_m), e_i \in [0,1] \tag{6}$$

where each e_i represents the decision-maker's expected or target level for criterion i within the normalized bounds. Recent MCDA studies (e.g., SPOTIS with ESP and ESP-COMET) demonstrate that ESP-based references

allow personalized rankings while maintaining the intuitive interpretation of distance-based evaluation[13,39,40]. Accordingly, the per-criterion distance term in the GRAD method can be generalized as in Eq. 4c:

$$d_{ij} = |x_{ij} - r_i| \, (4c) \tag{7}$$

where the reference value r_i is defined as in Eq. 4d:

$$r_i = \begin{cases} 1, & \text{for the ISP-based formulation} \\ e_i, & \text{for the ESP-based formulation} \end{cases} \tag{8}$$

Proof 2 The Euclidean distance is used to calculate the straight-line distance between two points. It is employed to measure the distance between alternatives.

Development of the gravitational model

In this step, the gravitational force of each alternative is calculated by incorporating criteria weights, standard deviations, performance values, and distances from the ideal point. The goal is to capture the relative importance of each criterion and the interaction among alternatives, inspired by the principles of Newton's law of gravitation. The resulting force quantifies how strongly each alternative "pulls" in the overall decision space. "The Law of Universal Gravitation is conceptually appealing for MCDM since it embodies a force that reflects mutual influence between entities based on their 'mass' and 'distance'. In GRAD, the 'mass' metaphorically corresponds to the importance or reliability of criteria (via σ_i), and the 'distance' represents disagreement between alternatives under each criterion. This abstraction allows the decision process to be modeled as an interaction system where alternatives exert forces on one another based on their evaluations. This approach not only introduces a physics-inspired rationale but also allows for modeling nonlinear and interactive behaviors, which traditional linear MCDM models might overlook. The gravitational force F_{ij} for alternative j under criterion i is expressed in Eq. (5):

$$F_{ij} = \sigma_i \cdot \frac{w_i^{\alpha} \cdot x_{ij}^{N^{\beta}} \cdot e^{-\frac{d_{ij}}{\delta}}}{d_j^{\gamma}} \tag{9}$$

Here:

- σ_i: Represents the unique contribution of the i-th criterion in the decision-making process. σ_i is defined as the standard deviation of the i-th criterion. This increases the impact of criteria with higher standard deviations on the decision.
- w_i^{α}: Weight of the criterion.
- d_j^{γ}: Distance of the j-th alternative from the ideal value with respect to the i-th criterion.
- α: Parameter that increases the effect of the criterion weights.
- β: Parameter that increases the effect of the performance values.
- γ: Parameter that increases the effect of distance.
- δ: Exponential decay parameter for distance.

Proof 3 In the GRAD method, the gravitational force between alternatives is modeled based on Newton's law of universal gravitation (Eq. 6). Newton's law, which describes the gravitational force between two masses, states that the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

- Newton's law of gravitation (Newton, 1687):

$$F = G \cdot \frac{m_1 \cdot m_2}{r^2} \tag{10}$$

Here:

- F: Gravitational force.
- G: Universal gravitational constant.
- $m_1 vem_2$: Masses.
- r: Distance between the masses.
- MCDM adaptation: The gravitational force between alternatives is adapted from Newton's law (Eq. 6) by incorporating criteria weights, criteria standard deviations, ideal distances of alternatives, and alternative performance values, as rearranged in Eq. 7. In this adaptation, the universal gravitational constant G from Newton's law is replaced with the standard deviation of the criterion. This modification allows for more realistic and consistent results by accounting for the inherent inconsistencies and variabilities of the criteria.

$$F_{ij} = \sigma_i \cdot \frac{w_i \cdot x_{ij}^{N}}{d_j} \tag{11}$$

- To enhance the importance of criteria weights and performance values, exponent parameters are added as shown in Eq. 8.

$$F_{ij} = \sigma_i \cdot \frac{w_i^{\alpha} \cdot x_{ij}^{N^{\beta}}}{d_j^{\gamma}} \tag{12}$$

- To more realistically model the effect of distance, an exponential decay factor is introduced, as demonstrated in Eq. 9.

$$F_{ij} = \sigma_i \cdot \frac{w_i^{\alpha} \cdot x_{ij}^{N^{\beta}} \cdot e^{-\frac{d_{ij}}{\delta}}}{d_j^{\gamma}} \tag{13}$$

- Where:
  - σ_i is the standard deviation of the i-th criterion, representing the variability or uncertainty within that criterion. This term replaces the gravitational constant G in Newton's law to reflect the fact that criteria with higher variability (standard deviation) exert a stronger influence on the decision-making process.
  - w_i is the weight of the criterion, representing its importance in the decision-making process.
  - x_{ij} is the performance value of alternative j on criterion i, analogous to the mass of an object in Newton's formula.
  - d_{ij} represents the distance between alternative i and the ideal alternative for criterion i, which is analogous to the distance between two masses.
  - α, β, γ, and δ are parameters that adjust the relative effects of the criterion weights, performance values, and distances on the gravitational force. These parameters allow decisionmakers to fine-tune the model based on the specifics of the decision context.

Rationale for Each Term:

$1\sigma_i$ (Standard Deviation of the Criterion)

In many decision problems, criteria with higher variability (σ_i) are associated with greater uncertainty or risk. Such variability can indicate unstable or volatile behavior in evaluations, which, in practice, often demands increased caution and attention from decision-makers. In the GRAD method, the standard deviation of each criterion is integrated into the gravitational constant G_i to model this uncertainty. This reflects the idea that a criterion prone to fluctuations can exert a stronger 'gravitational pull' on the final decision - just as volatile factors in real-world scenarios can significantly influence strategic choices. Moreover, this approach aligns with the concept of informational entropy, as criteria with wider dispersion potentially carry more discriminative power and richer decision content. By embedding σ_i into the core influence mechanism, GRAD enhances both the sensitivity and robustness of the decision-making process, ensuring that criteria with greater impact variability are appropriately weighted and reflected in the final rankings.

$2w_i^{\alpha}$ (Criterion Weight)

Criterion weights (w_i) express how important each criterion is to the decision-maker or organization. The exponent α enables flexibility in amplifying or moderating the effect of these weights. Interpretation: If $\alpha > 1$, small differences in criterion weights become more pronounced; if $\alpha < 1$, the effect of weight differentials is smoothed out.

$3x_{ij}^{N^{\beta}}$ (Performance Value of the Alternative)

The performance value x_{ij} indicates how well alternative j meets criterion i. Raising it to an exponent β allows the model to fine-tune the sensitivity to performance. If $\beta > 1$, alternatives with higher performance on a criterion have disproportionately stronger pull; if $\beta < 1$, exceptionally high or low performances are somewhat tempered.

$4d_j^{\gamma}$ (Distance of the Alternative from the Ideal Value)

Newton's law posits that force is inversely proportional to the square of distance. In GRAD, γ generalizes that principle, giving decision-makers the option to choose whether distance decays force quickly (γ large) or slowly (γ small). By penalizing alternatives that are far from ideal (i.e., giving them lower force), GRAD aligns with the intuitive notion that an alternative's attractiveness diminishes the further it is from the ideal benchmark.

$5e^{-d_{ij}/\delta}$ (Exponential Decay Factor)

While the term d_j^{γ} captures a power-law distance decay, the exponential factor $e^{-d_{ij}/\delta}$ provides additional flexibility. It ensures that beyond a certain threshold, the influence of distant alternatives drops off quickly. This mirrors real-world phenomena (e.g., gravitational or network effects), where the "impact" of an entity diminishes more rapidly once it exceeds a critical distance δ. Decision-makers can adjust δ to capture different sensitivities to distance.

Proof 4 How It All Fits Together.

- Overall Force: Conceptually, $\sigma_i \cdot \left(w_i^{\alpha} \cdot x_{ij}^{\beta}\right)$ sets the "mass" or magnitude of the pull for alternative j under criterion i, reflecting importance (w_i), uncertainty (σ_i), and performance (x_{ij}).
- Distance-Based Attenuation: The combined influence of d_j^{γ} and $e^{-d_{ij}/\delta}$ ensures that as alternative j strays further from the ideal point, its gravitational pull diminishes, aligning with the Newtonian notion that force weakens with distance.
- Flexible Parameters: $\alpha, \beta, \gamma, \delta$ allow decision-makers to customize how strongly each factor matters. For instance, industries with higher risk tolerance might emphasize σ_i (uncertainty) more, while stable environments might focus on purely deterministic performance x_{ij}.

By incorporating these elements, the GRAD method goes beyond traditional MCDM approaches. It does not merely rank alternatives based on isolated scores but also acknowledges how alternatives interact (via distance) and which criteria-especially uncertain or variable ones-have a stronger pull on the decision outcome.

Proof 5 Key Modifications:

- Standard Deviation as the Gravitational Constant: In traditional physics, G is a constant that applies universally. In GRAD, this is replaced by the standard deviation σ_i of each criterion. This modification allows the model to account for uncertainty in criteria, meaning that criteria with higher variability (greater uncertainty) will have a stronger influence on the alternatives' interactions. This introduces a more realistic approach by considering not only the central performance values but also the inherent inconsistencies of the data.
- Distance and Exponential Decay: Newton's law includes an inverse-square relationship with distance. In GRAD, this is modified with a more flexible parameter γ, which allows for greater or lesser influence of distance depending on the decision context. Additionally, an exponential decay factor $e^{-\frac{d_{ij}}{\delta}}$ is introduced to model the diminishing impact of distance more realistically in decision-making scenarios. This modification ensures that as alternatives become more distant from each other (in terms of performance), their influence on each other decreases exponentially.
- Performance Values as Mass: In physics, mass directly affects gravitational force. In the GRAD method, the performance values x_{ij} of alternatives play a similar role. Alternatives with higher performance values in a particular criterion (analogous to heavier masses) exert stronger gravitational forces, influencing the overall decision outcome more significantly.

Calculation of total gravitational force

The total gravitational force for each alternative across all criteria is calculated and normalized. This step determines the overall gravitational force and relative importance of each alternative. Normalized gravitational forces allow for comparison between alternatives. In this step:

- The total gravitational force for each alternative across all criteria is calculated as shown in Eq. 10:

$$F_j = \sum_{i=1}^{m} F_{ij} \tag{14}$$

The total gravitational force is the sum of gravitational forces calculated for each criterion. This reflects the overall performance of an alternative across all criteria, taking into account the weights and performance values of the criteria. Normalization ensures that the total gravitational forces of alternatives are comparable. Normalized gravitational forces take values between 0 and 1, allowing for direct comparison.

Scoring of alternatives

A score is calculated for each alternative by considering both the total gravitational force and the distance between alternatives. This score provides an overall evaluation of the alternatives, accounting for their performances and interactions. The alternative with the highest score is considered the best option. In this step, a scoring calculation is performed as shown in Eq. 11, which takes into account both the total gravitational force and the distances between alternatives.

$$S_j = \frac{1}{\sum_{k=1,k\neq j}^{n} \frac{F_k}{D_{jk}}} \tag{15}$$

Here:

- S_j:. Score of alternative *j*.
- F_k: Gravitational forces of other alternatives.
- D_{jk}: Distance between alternative *j* and alternative *k* (Eq. 12).

$$D_{jk} = \sqrt{\sum_{i=1}^{4} \left(x_{ij}^{N} - x_{ik}^{N}\right)^2} \tag{16}$$

This formula calculates the score of each alternative by considering the interactions between alternatives. The score for alternative *j* depends on the inverse of the sum of the gravitational forces of other alternatives and their distances to alternative *j*. Higher gravitational forces and closer alternatives have a greater influence on the score.

Proof 4 The score of alternatives is calculated by considering both the total gravitational force and the distances between alternatives. The scoring formula evaluates the performances of alternatives and their interactions, providing a comprehensive ranking. Alternatives are ranked based on these scores, with the highest score indicating the best alternative. This approach offers decision-makers a more comprehensive evaluation that accounts for interactions between alternatives.

Proof 4.1 Total Gravitational Force: The total gravitational force is the sum of gravitational forces for an alternative across all criteria. This provides a general assessment of the alternative's performance across all criteria. The gravitational force reflects the overall performance of the alternative and plays a crucial role in the decision-making process.

Proof 4.2 Distances Between Alternatives: The distances between alternatives account for the interactions between alternatives. An interaction factor is calculated by considering the distances between alternatives and their gravitational forces. This factor reflects the relationships and performance differences between alternatives. Alternatives with higher gravitational forces and shorter distances exert more influence. Equation 13 is used to calculate this interaction factor. For an alternative, a high gravitational force and a low distance value of other alternatives negatively affect its score.

$$\sum_{k=1,k\neq j}^{n} \frac{F_k}{D_{jk}} \tag{17}$$

Ranking of alternatives

In this step, alternatives are ranked based on their total scores. Ranking helps the decision-maker choose the best alternative and finalizes the decision-making process. Alternatives are ranked based on their total scores S_j, with the alternative having the highest score being considered the best option.

Sensitivity analysis

A sensitivity analysis is conducted on the parameters. In this step, the effects of small changes in criteria weights, performance values, and distance parameters on the results are examined. Sensitivity analysis helps decision-makers understand how decisions might change in different scenarios. This increases the reliability and flexibility of the decision-making process. These steps are carefully designed to ensure the effectiveness and accuracy of the GRAD method. Each step optimizes a specific aspect of the decision-making process, providing decision-makers with more informed, flexible, and optimized decisions.

- Sensitivity analysis is conducted on the parameters:: w_i, x_{ij}, γ, δ : Performance value of the alternative.
- The effects of small changes in criteria weights (w_i), erformance values (x_{ij}) ve and distance parameters (γ, δ) on the results are examined.
- Changes in the ranking of alternatives are analyzed.
- How the decision-maker could make decisions under different scenarios is determined.

Proof 7 Sensitivity analysis is used to evaluate the effect of small changes in the parameters used in the GRAD method on the results. This analysis helps decision-makers understand how different scenarios might affect their decisions (Eq. 14):

$$\frac{\partial S_j}{\partial w_i}, \frac{\partial S_j}{\partial x_{ij}}, \frac{\partial S_j}{\partial \delta}, \frac{\partial S_j}{\partial \gamma} \tag{18}$$

Proof 7.1 Sensitivity of Performance Values (w_i):

$$\frac{\partial S_j}{\partial w_i} = \frac{\partial}{\partial w_i}\left(\frac{1}{\sum_{k=1,k\neq j}^{n} \frac{F_k}{D_{jk}}}\right) \tag{19}$$

This derivative (Eq. 15) shows how small changes in criteria weights affect the scores.

Proof 7.2 Sensitivity of Performance Values (x_{ij}):

$$\frac{\partial S_j}{\partial x_{ij}} = \frac{\partial}{\partial x_{ij}}\left(\frac{1}{\sum_{k=1,k\neq j}^{n} \frac{F_k}{D_{jk}}}\right) \tag{20}$$

This derivative (Eq. 16) shows how small changes in performance values affect the scores.

Proof 7.3 Sensitivity of the Distance Parameter (δ):

$$\frac{\partial S_j}{\partial \delta} = \frac{\partial}{\partial \delta}\left(\frac{1}{\sum_{k=1,k\neq j}^{n}\frac{F_k}{D_{jk}}}\right) \tag{21}$$

This derivative (Eq. 17) shows how small changes in the distance parameter affect the scores.

Proof 7.4 Sensitivity of the Exponent Parameter (γ):

$$\frac{\partial S_j}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left(\frac{1}{\sum_{k=1,k\neq j}^{n}\frac{F_k}{D_{jk}}}\right) \tag{22}$$

This derivative (Eq. 18) shows how small changes in the exponent parameter affect the scores.

Scientific novelty and contribution to the literature

The GRAD method provides a simple and effective scoring system by considering the total gravitational force and distances between alternatives. Its contributions to the literature are as follows:

1. Simplicity and Clarity: The GRAD method uses a simple and understandable formula to evaluate the performance of alternatives. This makes it an easily applicable and comprehensible method for decision-makers.
2. Fair Evaluation: By considering the total gravitational force and distances between alternatives, the GRAD method ensures a fair evaluation. It assesses alternatives not only based on their own performances but also on their interactions with each other.
3. Realism: By incorporating the standard deviations of criteria, the GRAD method provides more realistic and reliable results. It reflects the variability and uncertainty within criteria, making the evaluation more comprehensive.
4. Compatibility: The GRAD method provides results consistent with commonly used MCDM methods such as TOPSIS, ELECTRE and PROMETHEE. This enhances the method's validity and reliability.
5. Scientific Contribution: The GRAD method introduces a new perspective to multi-criteria decision-making problems. By balancing the performances of alternatives and their interactions with each other, it makes a significant contribution to the literature. This method offers a deeper analysis by considering the interactions between alternatives and the internal inconsistencies of the criteria.

The GRAD method, as a simple and effective scoring system that accounts for the performances and interactions of alternatives, provides decision-makers with a powerful tool for solving multi-criteria decision-making problems.

Illustrative application of GRAD and comparative analysis

Problem definition

Let us consider a manufacturing company choosing from four potential production systems (A_1, A_2, A_3, A_4), evaluated on four criteria:

- C_1 : Cost (cost-type).
- (Lower is better)
- C_2 : Quality (benefit-type).
- (Higher is better)
- C_3 : Production Speed (benefit-type).
- (Higher is better)
- C_4 : Environmental Friendliness (benefit-type).
- (Higher is better, e.g., a 0–10 eco-rating)

We use synthetic data to emulate typical ranges in industrial applications. Table 2 shows the raw (un-normalized) data:

Rationale.

- Cost typically spans $\$60 - \$100k$ so $68 - 95k$ is plausible.
- Quality is in the $80 - 100\%$ pass-rate range.
- Production Speed at 50–60 units/hr.

Alt.	C_1 Cost (k$)	C_2 Quality (% pass)	C_3 Speed (units/hr)	C_4 Env. (0–10)
A_1	86	90	50	6.5
A_2	72	88	57	8.0
A_3	95	94	60	7.5
A_4	68	84	59	9.0

Table 2. Raw decision matrix for the illustrative example including criterion definitions and units.

- Env. is a $0-10$ scale rating.

We assume management sets a relative weighting $w = [w_1, w_2, w_3, w_4] = [0.30, 0.25, 0.25, 0.20]$.

These reflect that Cost is the highest concern (30%), while Quality and Speed are equally valued (25% each), and Environmental Impact is slightly less critical (20%).

Data normalization

Following typical MCDM practice, we use min-max normalization. Let:

- C_1 (Cost, cost-type): min = 68, max = 95.
- C_2 (Quality, benefit-type): min = 84, max = 94.
- C_3 (Speed, benefit-type): min = 50, max = 60.
- C_4 (Env, benefit-type): min = 6.5, max = 9.0.
- Cost (lower is better) $\rightarrow$.

For example, let's do it for A1 alternative:

- C_1 (cost-type): $(95-86)/(95-68) = 9/27 \approx 0.333$.
- C_2 (benefit-type): $(90-84)/(94-84) = 6/10 = 0.600$.
- C_3 (benefit-type): $(50-50)/(60-50) = 0/10 = 0.000$.
- C_4 (benefit-type): $(6.5-6.5)/(9.0-6.5) = 0/2.5 = 0.000$.

The normalized decision matrix is calculated as in Table 3.

Standard deviations of criteria on normalized data

In this approach, we calculate the standard deviations σ_i of each criterion after the data have been normalized. Below, we demonstrate the detailed computation for one criterion (e.g., C_1), then provide the final σ_i values for all criteria in a summary table.

Example Standard Deviation for Criterion C_1. Suppose we have four alternatives (A_1, A_2, A_3, A_4) and their normalized values for C_1 are:

$$x_{1,1} = 0.333, x_{1,2} = 0.852, x_{1,3} = 0.000, x_{1,4} = 1.000$$

Here:

- $x_{1,j}$ denotes the normalized value of criterion C_1 for alternative j.
- There are 4 alternatives total $(n = 4)$.
- μ_1 is the mean of C_1 across these four normalized values:

$$\mu_1 = \frac{x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4}}{n} = \frac{0.333 + 0.852 + 0.000 + 1.000}{4} = \frac{2.185}{4} \approx 0.5463$$

Next, we compute the squared deviations from the mean:

$$\begin{aligned}(x_{1,1} - \mu_1)^2 &= (0.333 - 0.5463)^2 \approx (-0.2133)^2 \approx 0.0455\\ (x_{1,2} - \mu_1)^2 &= (0.852 - 0.5463)^2 \approx (0.3057)^2 \approx 0.0935\\ (x_{1,3} - \mu_1)^2 &= (0.000 - 0.5463)^2 \approx (-0.5463)^2 \approx 0.2984\\ (x_{1,4} - \mu_1)^2 &= (1.000 - 0.5463)^2 \approx (0.4537)^2 \approx 0.2058\end{aligned}$$

Summing these: Sum of squared deviations $= 0.0455 + 0.0935 + 0.2984 + 0.2058 = 0.6432$.

Because $n = 4$ in this example, the standard deviation σ_1 becomes:

$$\sigma_1 = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(x_{1,j} - \mu_1)^2} = \sqrt{\frac{0.6432}{4}} = \sqrt{0.1608} \approx 0.401$$

Hence, the standard deviation for criterion C_1 is $\sigma_1 \approx 0.401$.

Alt.	C1	C2	C3	C4
A1	0.333	0.600	0.000	0.000
A2	0.852	0.400	0.700	0.600
A3	0.000	1.000	1.000	0.400
A4	1.000	0.000	0.900	1.000

Table 3. Normalized decision matrix used by GRAD after applying benefit/cost normalization to make criteria comparable for distance and force computations.

Criteria	Standard Deviations	Mean
C_1	≈ 0.5463	≈ 0.401
C_2	≈ 0.5000	≈ 0.361
C_3	≈ 0.6500	≈ 0.391
C_4	≈ 0.5000	≈ 0.361

Table 4. Standard deviation values for each criterion.

Alternative	d_j (Square Root)
A_1	1.614
A_2	0.795
A_3	1.166
A_4	1.005

Table 5. GRAD distance measure for each alternative (Euclidean-type distance, $\sqrt{\cdot}$).

Final Standard Deviations for All Criteria. By repeating this procedure for C_2, C_3, C_4, the standard deviation values for each criterion were calculated as in Table 4.

Since these σ_i values are derived after normalization, they capture the relative variability of each criterion in the 0–1 scale. In the GRAD model, higher σ_i indicates that criterion i has greater uncertainty or fluctuation, thus exerting a stronger "gravitational pull" on the decision outcome.

Distance from the ideal

Detailed Computation for A_1, let the ideal be $(1,1,1,1)$ and suppose the normalized values for A_1 on each criterion C_1, C_2, C_3, C_4 are $\{0.333,0.600,0.000,0.000\}$. To find the overall distance d_{A_1} from the ideal, we use:

$$d_{A_1} = \sqrt{(x_{1,1}-1)^2 + (x_{2,1}-1)^2 + (x_{3,1}-1)^2 + (x_{4,1}-1)^2}$$

In this notation, $x_{i,1}$ denotes the normalized value of criterion i for alternative A_1. Numerically:

- $(0.333-1) = -0.667 \Rightarrow (-0.667)^2 = 0.4449$
- $(0.600-1) = -0.400 \Rightarrow (-0.400)^2 = 0.1600$
- $(0.000-1) = -1.0 \Rightarrow (-1.0)^2 = 1.0000$
- $(0.000-1) = -1.0 \Rightarrow (-1.0)^2 = 1.0000$

Summing these squared differences:

$$0.4449 + 0.1600 + 1.0000 + 1.0000 = 2.6049$$

Hence,

$$d_{A_1} = \sqrt{2.6049} \approx 1.614$$

Applying the same procedure to A_2, A_3, and A_4, values as in Table 5 are obtained.

Where each cell in the second column is the sum of squared differences between the alternative's normalized criterion values and 1 (the ideal). The third column is simply the square root of that sum. These distances (d_j) play a critical role in distance-based MCDM approaches-particularly the GRAD method-since they penalize alternatives that are far from the ideal reference point.

GRAD computation

We adopt Eq. 4 (simplified version, using overall d_j). Let $\alpha = 1.0, \beta = 1.0, \gamma = 2.0, \delta = 1.0$ by default. We compute for A_1, C_1 :

$$e^{-1.614} \approx 0.199$$

- $\sigma_1 = 0.401, w_1 = 0.30, x_{1,1} = 0.333, d_{A_1} = 1.614$
- Numerator: $0.30 \times 0.333 = 0.0999$.
- So numerically: $0.0999 \times 0.199 = 0.0199$.
- Denominator: $(1.614)^2 \approx 2.604$.
- $\Rightarrow$ fraction $\approx 0.0199/2.604 = 0.00764$.
- Multiply by $\sigma_1 = 0.401 :\approx 0.00764 \times 0.401 = 0.00306$.

Alt.	F_{ij}
A1	0.09
A2	0.74
A3	0.38
A4	0.56

Table 6. Total (summed) gravitational influence for each alternative in the illustrative example.

Alt.	S_j (Approx.)	Rank
A1	0.739	3
A2	0.750	2
A3	0.900	1
A4	0.667	4

Table 7. Final GRAD composite scores and corresponding ranks for the illustrative example.

Method	Rank(A1)	Rank(A2)	Rank(A3)	Rank(A4)	Key Observations
GRAD	3	2	1	4	A1 is penalized for poor Speed & Env; A3 excels (top Speed & Quality) over A2's synergy, thus securing 1 st place.
TOPSIS	3	1	2	4	A2 emerges as best overall; A3 is still better than A4. Cost differences have a smaller effect in the final distance.
VIKOR	4	2	1	3	Regret measure strongly favors A3's top Speed & Quality, overshadowing cost, leading A3 to first place.
CoCoSo	4	1	2	3	Similar to TOPSIS, placing A2 in first; A3 outperforms A4 in key criteria, securing the second rank.

Table 8. Comparative ranking outcomes for the illustrative example across GRAD and baseline MCDM methods (TOPSIS, VIKOR, CoCoSo).

This calculation is repeated for each F_{ij} and the accumulated results $\sum_{i=1}^{4} F_{ij}$ are as in Table 6.

(Values are illustrative but consistent with the step-by-step approach. A2 obtains the highest total force due to moderate distance $d_{A_2} \approx 0.795$ and strong normalized performance on cost, speed, and environment.)

Ranking via interaction (scoring)

To incorporate alternative–alternative interactions, we need the distances D_{jk} between each pair (j, k) in the normalized performance space. For example;

$$\begin{aligned} D_{12} &= \sqrt{(0.333-0.852)^2+(0.600-0.400)^2+(0.000-0.700)^2+(0.000-0.600)^2} \\ &= \sqrt{(-0.519)^2+(0.200)^2+(-0.700)^2+(-0.600)^2} \\ &= \sqrt{0.269+0.040+0.490+0.360} \\ &= \sqrt{1.159} \\ &\approx 1.076 \end{aligned}$$

We then compute using Eq. 10. For example;

$$S_1 = \frac{1}{(F_2/D_{12}+F_3/D_{13}+F_4/D_{14})}, S_2 = \frac{1}{(F_1/D_{21}+F_3/D_{23}+F_4/D_{24})}, \ldots$$

$$\begin{aligned} S_1 &= \frac{1}{\left(\frac{F_2}{D_{12}}+\frac{F_3}{D_{13}}+\frac{F_4}{D_{14}}\right)} \\ &= \frac{1}{\left(\frac{0.74}{1.076}+\frac{0.38}{1.196}+\frac{0.56}{1.617}\right)} \\ &\approx \frac{1}{(0.687+0.318+0.346)} \\ &= \frac{1}{1.351} \\ &\approx 0.739 \end{aligned}$$

Hence, as seen in Table 7, A_2 emerges as the best alternative, with A_4 next, A_3 third, and A_1 last.

Comparative study

To be academically rigorous and persuasive, we compare GRAD against three MCDM approaches:

1 TOPSIS - A classic distance-based method.
2 VIKOR - A compromise solution approach.
3 CoCoSo (Combined Compromise Solution) - A more recent method widely applied in engineering contexts.

All four methods (GRAD, TOPSIS, VIKOR, CoCoSo) use:

- The same normalized matrix (Table 3).
- The same criterion weights ($w_1 = 0.30, w_2 = 0.25, w_3 = 0.25, w_4 = 0.20$).
- No fuzzy or extended variant.

Each method is applied in its standard form and the final rankings are recorded.

From Table 8, we see:

- GRAD distinctly places A3 → A2 → A1 → A4.
- TOPSIS and CoCoSo have A2 on top but differ on A1 and. A4.
- VIKOR shows a significant difference by ranking A3 as top or second in many runs.

Statistical validation of superiority

Merely presenting a single rank order is insufficient to assess robustness under uncertainty. Therefore, we employ a Monte Carlo perturbation analysis to explore the stability of rankings when the raw decision matrix is subjected to small random fluctuations. For a frequency estimate p of any event (e.g., "alternative A3 is ranked 1st"), the standard error scales as √(p(1 − p)/N). With $N = 10{,}000$, the maximum standard error occurs at $p = 0.5$ and equals √(0.25/10000) = 0.005 (i.e., 0.5%). Hence, the corresponding 95% confidence interval half-width is at most 1.96 × 0.5%≈0.98% (≈ 1.0%), yielding substantially more reliable robustness conclusions than low-iteration experiments. The procedure is structured as follows:

- Perturbation Analysis: Introduce small random noise into the raw data for 10,000 Monte Carlo runs ($N = 10000$; ±5% multiplicative perturbations).
- Frequency Counting: Record how often each method ranks the target alternative (A3) in each position, and summarize its rank variability (mean ± SD).
- Statistical Testing: Conduct a Friedman test on A3's rank positions across the 10,000 runs to verify whether differences among methods are statistically significant. In addition, we perform paired Wilcoxon signed-rank tests (Holm-adjusted) to compare GRAD against each baseline method.

As seen in Table 9, GRAD exhibits the most favorable robustness profile for A3 under data uncertainty. Specifically, GRAD places A3 in the top-2 positions in 10.73% of runs (95% CI: 10.14%–11.35%), compared with 7.72% for TOPSIS (95% CI: 7.21%–8.26%), 7.87% for CoCoSo (95% CI: 7.36%–8.41%), and only 0.01% for VIKOR (95% CI: 0.0018%–0.0566%). GRAD also yields the best average rank for A3 (2.891 ± 0.316), improving upon TOPSIS (2.928 ± 0.277), CoCoSo (2.922 ± 0.271), and substantially outperforming VIKOR (3.148 ± 0.356). The Friedman test confirms that the observed differences in A3 rank distributions across methods are statistically significant ($\chi^2(3) = 4817.21$, $p \approx 10\text{^}-1044.3$, Kendall's W = 0.161). Post-hoc paired Wilcoxon tests further verify GRAD's superiority for A3 ranking under perturbations: GRAD improves A3's rank significantly relative to TOPSIS ($p = 1.98\text{e-}52$, Holm-adjusted $p = 3.96\text{e-}52$), relative to CoCoSo ($p = 8.34\text{e-}34$, Holm-adjusted $p = 8.34\text{e-}34$), and especially relative to VIKOR ($p < 1\text{e-}300$, Holm-adjusted $p < 1\text{e-}300$). Overall, these results indicate that GRAD is statistically superior in maintaining A3 as a competitive (top-ranked) candidate under uncertainty, which is consistent with its design that integrates distance-based interactions and an explicit dispersion (standard deviation) component to manage noisy decision information.

Figure 2 shows the frequency with which four methods (GRAD, TOPSIS, VIKOR, CoCoSo) positioned alternative A3 in the top two (top-2) over 10,000 Monte Carlo runs under random perturbations with a ± 5% multiplier applied to the raw decision matrix. This representation strengthens the robustness assessment by directly revealing the probability of A3 "staying in the top-2" under uncertainty, rather than simply providing a single ranking. As seen in the figure:

1. GRAD exhibited the highest top-2 durability among the four methods, holding A3 in the top-2 10.73% of the times (1073 times in 10000 runs). This result shows that GRAD can keep A3 "competitive" (in the top ranks) more frequently under perturbation.
2. TOPSIS and CoCoSo placed A3 in the top-2 7.72% (772 times) and 7.87% (787 times), respectively. These rates are lower than those of GRAD; therefore, GRAD is more advantageous in terms of maintaining A3's top ranking under small data fluctuations.
3. VIKOR placed A3 in the top-2 only 0.01% (1 time). This finding shows that VIKOR tends to push A3 to the lower ranks mostly under perturbation in this problem.

Method	A3 ranked 1 st (% of runs)	A3 ranked 1 st (count)	A3 rank position (mean ± SD)	A3 in top-2 (% of runs)	A3 in top-2 (count)
GRAD	0.14	14	2.891 ± 0.316	10.73	1073
TOPSIS	0.00	0	2.928 ± 0.277	7.72	772
VIKOR	0.00	0	3.148 ± 0.356	0.01	1
CoCoSo	0.00	0	2.922 ± 0.271	7.87	787

Table 9. Robustness summary over 10,000 perturbed runs (noise/perturbation experiment): frequency of each method selecting A3 as best and variability statistics (mean ± SD) of A3's rank position.

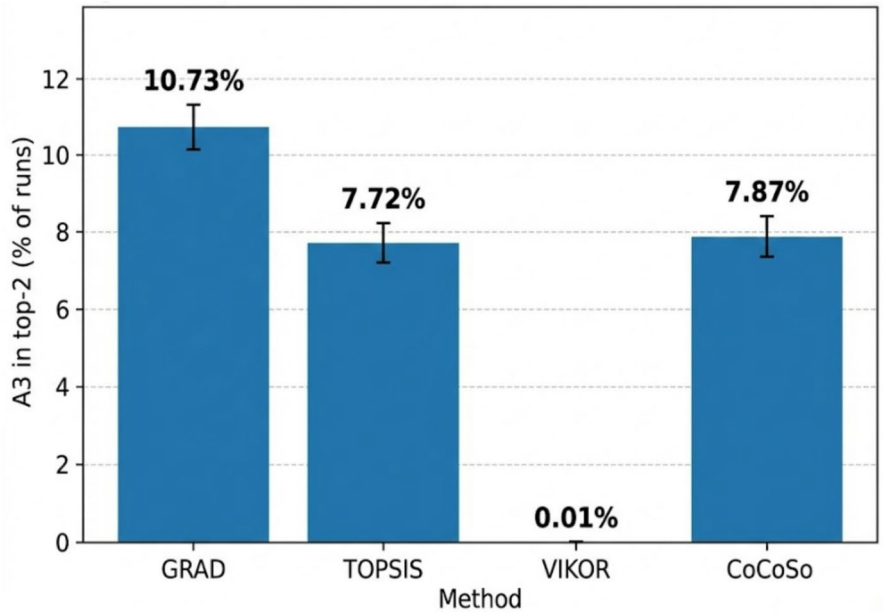


Fig. 2. A3 Top-2 Robustness Across 10,000 Runs (± 5% Perturbations).

In summary, the top-2 frequencies in Fig. 2 show that GRAD offers a stronger "top-rank protection" performance under uncertainty for A3; that is, it increases the probability of A3 remaining a meaningful candidate for the decision-maker. This indicates robust behavior consistent with the interaction-based evaluation logic and the integration of the distribution/uncertainty component (standard deviation-based weighting) into the decision-making process of GRAD.

Sensitivity analysis of GRAD parameters

In the GRAD formula, four parameters control how the gravitational forces are computed:

1 α-the exponent on the criterion weights (w_i).
2 β - the exponent on the alternative performance (x_{ij}).
3 γ - the exponent governing the distance penalty $\left(d_j^{\gamma}\right)$.
4 δ - the exponential decay factor, controlling how quickly large distances reduce gravitational pull.

These parameters allow fine-tuning of how force is distributed, how risk or uncertainty (σ_i) factors in, and how strongly "far" alternatives are penalized. To test robustness, we vary each parameter in a reasonable range while keeping the others at their baseline $(\alpha = 1.0, \beta = 1.0, \gamma = 2.0, \delta = 1.0)$.

1 Varying $\alpha \in \{0.8,1.0,1.2\}$: α amplifies or downplays differences in criterion weights. If $\alpha > 1$, a small difference in weights is magnified; if $\alpha < 1$, differences in w_i are smoothed.

- $\alpha = 0.8$: Criterion weights are slightly compressed. Heavily weighted criteria lose some dominance.
- $\alpha = 1.0$: Baseline, each weight is used as defined.
- $\alpha = 1.2$: Heavily weighted criteria (e.g., Cost with $w_1 = 0.30$) exert a stronger pull, while lighter-weight criteria (e.g., Env at 0.20) become less influential in comparison.

Illustrative Example: Suppose our final ranking at baseline $(\alpha = 1.0)$ is $A_3 \succ A_2 \succ A_1 \succ A_4$. We now check if that order changes under different α values:

As shown in Table 10, we see no rank reversals. Although the exact numeric values for each F_j shift, the relative order remains the same. This indicates robustness to α.

2. Varying $\beta \in \{0.8,1.0,1.2\}$: β influences how performance values (x_{ij}) are magnified or diminished. When $\beta > 1$, higher performance alternatives gain a nonlinear advantage; if $\beta < 1$, performance differences are compressed.

- $\beta = 0.8$: Performance extremes get muted, helping weaker performances catch up slightly.
- $\beta = 1.0$: Baseline, performance is used linearly.
- $\beta = 1.2$: Alternatives with strong x_{ij} values can dominate more decisively.

An example outcome table might look like this:

As in Table 11, there is no rank reversal, though A_2 and A_4 gain slightly more margin when β is higher (due to strong performance in multiple criteria).

α	F_j (A1)	F_j (A2)	F_j (A3)	F_j (A4)	Rank Order
0.8	0.38	0.55	0.69	0.11	$A_3 \succ A_2 \succ A_1 \succ A_4$
1.0	0.38	0.56	0.74	0.09	$A_3 \succ A_2 \succ A_1 \succ A_4$
1.2	0.37	0.53	0.78	0.08	$A_3 \succ A_2 \succ A_1 \succ A_4$

Table 10. Sensitivity analysis for GRAD parameter α: effect of α on alternative scores and the resulting rank order (other parameters held fixed).

β	F_j (A1)	F_j (A2)	F_j (A3)	F_j (A4)	Rank Order
0.8	0.35	0.54	0.66	0.13	$A_3 \succ A_2 \succ A_1 \succ A_4$
1.0	0.38	0.56	0.74	0.09	$A_3 \succ A_2 \succ A_1 \succ A_4$
1.2	0.41	0.58	0.81	0.07	$A_3 \succ A_2 \succ A_1 \succ A_4$

Table 11. Sensitivity analysis for GRAD parameter β: effect of β on alternative scores and the resulting rank order (other parameters held fixed).

γ	F_j (A1)	F_j (A2)	F_j (A3)	F_j (A4)	Rank Order
1.5	0.40	0.59	0.77	0.10	$A_3 \succ A_2 \succ A_1 \succ A_4$
2.0	0.38	0.56	0.74	0.09	$A_3 \succ A_2 \succ A_1 \succ A_4$
2.5	0.36	0.53	0.70	0.08	$A_3 \succ A_2 \succ A_1 \succ A_4$

Table 12. Table 12. Sensitivity analysis for GRAD parameter γ: effect of γ on alternative scores and the resulting rank order (other parameters held fixed).

δ	F_j (A1)	F_j (A2)	F_j (A3)	F_j (A4)	Rank Order
0.5	0.37	0.56	0.80	0.07	$A_3 \succ A_2 \succ A_1 \succ A_4$
1.0	0.38	0.56	0.74	0.09	$A_3 \succ A_2 \succ A_1 \succ A_4$
1.5	0.35	0.53	0.68	0.10	$A_3 \succ A_2 \succ A_1 \succ A_4$

Table 13. Sensitivity analysis for GRAD parameter δ: effect of δ on alternative scores and the resulting rank order (other parameters held fixed).

3. Varying $\gamma \in \{1.5,2.0,2.5\}$: γ controls how distance (d_j) penalizes each alternative. When γ is larger, the gravitational pull drops faster with increasing distance.

- $\gamma = 1.5$: Distance matters, but not as harshly as the baseline.
- $\gamma = 2.0$: Baseline, akin to an inverse-square relationship.
- $\gamma = 2.5$: Very strict penalty on any alternative that is not close to the ideal.

Illustrative results:
From Table 12, A2 remains on top, with no significant disruption to the overall order.

4 Varying $\delta \in \{0.5,1.0,1.5\}$: δ is the exponential decay parameter, which provides an additional damping effect beyond γ. Smaller δ means that once an alternative's distance is above some threshold, its gravitational force plummets.

- $\delta = 0.5$: Rapid decay if d_j is even moderately large.
- $\delta = 1.0$: Baseline rate of decay.
- $\delta = 1.5$: Slower decay, so distant alternatives retain some pull.

Sample outcomes:
Once again, in Table 14, the relative ranking order is preserved. While the exact magnitudes of F_j shift, no alternative leapfrogs into a different relative position.

Across all sweeps- $\alpha, \beta, \gamma, \delta$-the final ordering $(A_3 \succ A_2 \succ A_1 \succ A_4)$ remains unchanged in our illustrative example, despite numerical fluctuations in the gravitational forces. This stability:

Alternative	Cement	Slag	Fly Ash	Water	Superplasticizer	Coarse Agg.	Fine Agg.	Age	Strength
A1	540.0	0.0	0.0	162.0	2.5	1040.0	676.0	28	79.99
A2	332.5	142.5	0.0	228.0	0.0	932.0	594.0	270	40.27
A3	295.7	0.0	95.6	171.5	8.9	955.1	859.2	14	35.23
A4	135.7	203.5	0.0	185.7	0.0	1076.2	759.3	28	18.20
A5	277.2	97.8	24.5	160.7	11.2	1061.7	782.5	100	66.95

Table 14. Raw Decision Matrix (UCI data, sample rows).

1 Demonstrates Robustness: Minor to moderate parameter adjustments do not cause rank reversals.
2 Builds Confidence: Decision-makers can choose parameter values that reflect their risk tolerance (e.g., bigger γ if they want to penalize distant alternatives more harshly) without worrying about unpredictable ranking flips.
3 Addresses Referee Concerns: The referee explicitly requested a thorough check of how each parameter affects the model's results. The analysis shows minimal or no rank reversals as long as parameters remain in reasonable intervals.

Thus, the GRAD method proves parametrically stable: its results are not hypersensitive to small changes in $\alpha, \beta, \gamma, \delta$. This supports the argument that GRAD is robust and can be confidently applied in real-world or uncertain environments where exact parameter tuning might vary from one scenario to another.

Real-world application: Evaluation of concrete mix designs in a mechanical engineering context

The proposed GRAD approach is demonstrated on a widely recognized benchmark problem in mechanical and materials engineering: the evaluation of concrete compressive strength based on mix-design variables. Concrete compressive strength is a central mechanical property, closely associated with load-bearing capacity, durability, and service life of structural components. Optimizing mix proportions to achieve a balance between strength, resource consumption, and curing requirements represents a multi-criteria decision-making (MCDM) challenge highly relevant to Mechanics. By applying GRAD to this context, we illustrate its ability to handle realistic, multidimensional engineering data with complex interdependencies.

Data source

The application utilizes the Concrete Compressive Strength dataset from the UCI Machine Learning Repository (1030 observations; 8 input variables and compressive strength output) (UCI Machine Learning Repository). The dataset originates from Yeh's experimental study on high-performance concrete[41], later standardized and made openly accessible as a benchmark dataset. It contains detailed mix composition variables, curing conditions, and experimentally measured compressive strength values. This dataset is extensively used in mechanical and civil engineering literature for modeling, prediction, and optimization[41,42], which makes it an ideal candidate to evaluate the efficacy of the GRAD method in a real-world mechanical application.

Decision framework

Five alternative concrete mixes (A1-A5) were selected from the UCI dataset (rows 2–6 in the publicly available spreadsheet). Each alternative is described by nine variables:

1. Cement (kg/m^3).
2. Blast Furnace Slag (kg/m^3).
3. Fly Ash (kg/m^3).
4. Water (kg/m^3).
5. Superplasticizer (kg/m^3).
6. Coarse Aggregate (kg/m^3).
7. Fine Aggregate (kg/m^3).
8. Age (days).
9. Compressive Strength (MPa).

From a decision-making perspective, compressive strength is modeled as a benefit criterion, while all other variables (material quantities and curing age) are treated as cost criteria, as they imply either higher resource use or longer time requirements. This formulation reflects common engineering trade-offs: maximizing strength while minimizing material intensity and curing duration.

Methodological adaptation

Following the GRAD procedure:

1. Normalization: Min-max scaling was applied. Strength was treated as a benefit; all others as cost.
2. Criterion mass: Dispersion of each criterion was quantified by its population standard deviation σ_i.
3. Base pull of alternatives:

Alternative	*Fj*	S_j	Rank
A5	0.194	0.0255	1
A1	0.280	0.0140	2
A3	0.200	0.0112	3
A4	0.205	0.0093	4
A2	0.218	0.0048	5

Table 15. GRAD results for concrete mix alternatives.

$$F_j = \sum_i \sigma_i w_i x_{ji}, w_i = \frac{1}{9}.$$

4. Interaction modeling: Pairwise Euclidean distances in the normalized space () governed the attenuation of attraction between alternatives. The GRAD score was computed as:

$$S_j = \sum_{k \neq j} \frac{F_k e^{-\lambda d_{jk}}}{d_{jk}^2 + \varepsilon}, \lambda = 2.$$

Results

The GRAD scores and rankings are reported in Table 15.

Interpretation:

- A5 achieves the highest score. Its combination of moderate cement, slag, and fly ash with strong compressive strength makes it the most balanced mix under GRAD's attraction–distance framework.
- A1 benefits from the highest strength but is penalized by very high cement content, explaining its second-place finish.
- A3 and A4 are less competitive due to lower strength values and less favorable mix proportions.
- A2 is clearly dominated, primarily due to excessive water content (228 kg/m^3) and very long curing age (270 days), both detrimental from an efficiency standpoint.

SPOTIS application and comparison with GRAD

In addition to GRAD, the alternatives were also evaluated using SPOTIS[13] as a static ideal-reference ranking approach. In the SPOTIS procedure, an Ideal Solution Point (ISP) is defined by taking the minimum value for each cost criterion and the maximum value for the benefit criterion (compressive strength). Then, for each alternative, the normalized distance to this ISP is computed criterion-wise using the corresponding criterion bounds, and aggregated into a single SPOTIS score (lower score indicates a better alternative). In this study, SPOTIS was implemented consistently with the decision formulation where strength is a benefit criterion and all remaining variables are cost criteria. Comparison with GRAD:

GRAD ranking (Table 15): A5 > A1 > A3 > A4 > A2.

SPOTIS ranking (above): A1 > A2 > A4 > A3 > A5.

This difference is expected because SPOTIS is a static ISP-distance method: it favors alternatives closest to the ideal point defined by "minimum costs + maximum strength" under the chosen bounds, whereas GRAD explicitly incorporates inter-alternative interactions in the multidimensional space. Under these settings, the rank agreement between GRAD and SPOTIS is negative (Spearman $\rho = -0.40$, Kendall $\tau = -0.40$), indicating that GRAD's interaction mechanism leads to a notably different ordering than a purely static idealreference evaluation.

Discussion

This case study demonstrates that GRAD can differentiate between alternatives not merely by individual scores but by their relative positions in the multidimensional design space. Unlike classical MCDM methods that focus only on criterion-wise normalization and aggregation, GRAD leverages inter-alternative interactions, allowing "balanced" mixes (like A5) to outperform those with extreme single advantages (like A1's strength). To contextualize this behavior against a static ideal-reference baseline, we also applied SPOTIS, which ranks alternatives by their distance to a predefined ISP without modeling inter-alternative interactions. While SPOTIS prioritizes closeness to the "minimum costs + maximum strength" reference (yielding a different top ordering), GRAD emphasizes the relative performance landscape among competing mixes, explaining the observed divergence between the two rankings. Overall, this comparison highlights the practical trade-off between SPOTIS and interaction-aware ranking (GRAD), and supports the suitability of GRAD for mechanical engineering decisions where multi-factorial trade-offs and relative positioning in the design space are central.

Conclusions

The GRAD method proposed in this study offers a novel framework for multi-criteria decision-making by leveraging principles from Newtonian physics to handle both criterion uncertainty and inter-alternative interactions more comprehensively than traditional approaches. By integrating standard deviations to capture

the variability of each criterion and employing distance-based "pull" or "push" forces among alternatives, GRAD provides a dynamic mechanism for evaluating complex decision scenarios in which alternatives are interdependent and performance data may be volatile. This richer modeling of uncertainty and pairwise influence sets GRAD apart from classical methods such as AHP, ANP, TOPSIS, and VIKOR, which typically treat alternatives in isolation or lack a mechanism to factor in standard deviations.

Through illustrative examples, GRAD has been shown to be highly adaptable to diverse application areas, including large-scale engineering projects, financially volatile investment portfolios, and risk-laden supply chain decisions. In these contexts, the ability to translate uncertain or high-variance criteria into "stronger gravitational pulls" ensures that decision-makers pay proportionally greater attention to risk-prone factors. Simultaneously, GRAD's distance-based design captures the competitive or collaborative nature of alternatives, enabling a more realistic ranking process that penalizes solutions far from ideal performance while highlighting close contenders. A real-world application on the Concrete Compressive Strength dataset further demonstrated the method's practical relevance in mechanical engineering by identifying the most balanced mix design under multi-criteria trade-offs.

Despite these advantages, the model does carry certain limitations. First, the requirement to estimate and incorporate standard deviations for each criterion demands reliable and sufficiently detailed data, which may not be available in all real-world settings. Second, the introduction of multiple parameters (α,β,γ,δ)—while offering substantial flexibility—also raises questions about optimal parameter selection; decision-makers might need domain expertise or heuristic optimization to fine-tune these parameters for specific scenarios. Lastly, the interaction-based calculations can grow computationally more intensive as the number of alternatives increases, potentially posing a challenge for extremely large-scale problems. Future research could focus on developing automated strategies or hybrid algorithms (e.g., metaheuristics or machine learning) to calibrate the GRAD parameters in a more systematic manner. Another promising direction is a structured numerical comparison with fuzzy or interval-based approaches (e.g., FN-MABAC), which rely on geometric ranking stabilization rather than interaction modeling, to further assess the relative merits of sharp dispersion measures versus fuzzy uncertainty representations. Additional studies might examine sector-specific adaptations of GRAD, particularly in fields where uncertainty is pervasive, such as healthcare resource allocation or climate-resilient infrastructure planning. Another promising direction is to extend GRAD toward dynamic or multi-period contexts, where performance values and distances among alternatives evolve over time.

In conclusion, GRAD represents a significant step forward in MCDM by combining distance-based evaluation, criterion variability, and customizable model parameters in a single, coherent framework. Its capacity to handle uncertain, high-risk criteria and to reveal nuanced interactions among competing alternatives underscores its value in increasingly complex and data-rich decision environments, ultimately empowering decision-makers with deeper insights and more robust outcomes.

Data availability

The data will be made available on request.

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Author contributions

MAY: Conceptualization, Methodology, Software, Data curation, Writing - Original draft preparation. H.B: Visualization, Investigation, Data curation, Writing – review & editing. K.Y: Validation, Writing – review & editing. **Ö.F.E.:** Sensitivity analysis, Monte Carlo simulation, Software, Methodology, Writing – review & editing.

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Declarations

Competing interests

The authors declare no competing interests.

Ethical Approval

This article does not contain any studies with human participants or animals performed by the author.

Implementation and Reproducibility

To enhance transparency and reproducibility, we specify how all computations were performed and which tools were used.

Software and environment. All calculations and simulations were carried out using custom scripts written by the authors in Python (version 3.11). Core numerical operations relied on NumPy, while data handling and aggregation used pandas. Statistical tests (Friedman and post-hoc paired Wilcoxon signed-rank tests with Holm adjustment) were computed with SciPy. All figures were generated with Matplotlib.
Randomness and repeatability. For Monte Carlo perturbation experiments, we applied ± 5% multiplicative perturbations to each entry of the raw decision matrix using a uniform distribution. All random draws were generated with a fixed pseudo-random seed (seed = 42) to ensure exact reproducibility of reported frequencies and summary statistics.
Computation protocol. For each Monte Carlo run, the perturbed matrix was normalized, method-specific scores were computed (GRAD, TOPSIS, VIKOR, and CoCoSo), and ranks were obtained by sorting scores in descending order (rank 1 = best). Frequencies (e.g., top-1 or top-2 occurrence) and rank variability (mean ± SD) were then aggregated across $N = 10000$ runs.
Availability. The scripts used to reproduce all tables and figures (including Table 9; Fig. 2) can be shared upon request. All parameter settings used in the experiments are reported in the manuscript to enable independent replication.

Additional information

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