GAZİ ÜNİVERSİTESİ

1926

INTERVAL AND PARAMETRIC LINEAR PROGRAMMING AND FUZZY DATA ENVELOPMENT ANALYSIS - NEURAL APPROACH MODELS WITH THE APPLICATION IN TURKISH BANKS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
GAZİ UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF Ph.D. OF SCIENCE
IN
STATISTICS

DECEMBER 2018

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INTERVAL AND PARAMETRIC LINEAR PROGRAMMING AND FUZZY DATA ENVELOPMENT ANALYSIS - NEURAL APPROACH MODELS WITH THE APPLICATION IN TURKISH BANKS

(Ph.D. Thesis)

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December 2018

ABSTRACT

In this study, we have used a new approaches to obtain ranking efficiency with fuzzy data numbers which are considered. Most fuzzy data evelopment analysis (DEA) models are introduced in the literary words are parametric models structured on alpha cuts. Yet, the selection of decision making units (DMUs) and variables which would be used in DEA is rather important because of its high effect on efficiency values. From the theoretical perspective, the objective of this study is to develop a simple and effective Fuzzy DEA-CCR and DEA-BCC models. The most and maximum possible efficiency scores of each DMU are estimated a few α-cut. This model can be applied to determine many which are associated with qualitative factors. It is checked by applying the proposed method in four numerical examples are compared with the results of eight current models of fuzzy DEA. and it is checked by applying the proposed method to calculate of relative efficiency values of 18 commercial banks working in Turkey, we took data for 10 years during the period 2007-2016 and worked on their simulation of DMU using for training the multilayer artificial neural network feedforward algorithm was aimed. Fuzzy CCR and BCC model was used to calculate efficiency values of the banks.

Science Code : 20517
Key Words : Operations Research, Data mining, Statistical analysis
Page Number : 149
Supervisor : Prof. Dr. Hasan BAL

ARALIK VE PARAMETRİK LİNEER PROGRAMLAMA VE BULANIK VERİ ZARFLAMA ANALİZİ - SİNİR YAKLAŞIM MODELLERİ TÜRKİYE BANKALARINDA UYGULAMA İLE

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Aralık 2018

ÖZET

Bu çalışmada, bulanık sayıları ile etkinlik skorlarını sıralamak için yeni yaklaşımlar önerilmiştir. Çoğu bulanık veri zarflama analizi (VZA) modelleri literatürde alfa kesimleri üzerine yapılandırılan parametrik modellerdir. VZA ' da kullanılacak olan karar verme birimleri (KVB) ve değişkenlerin seçimi, etkinlik değerleri üzerindeki yüksek etkisinden dolayı önemlidir. Teorik açıdan bakıldığında, bu çalışmanın amacı basit ve kullanışlı Bulanık VZA-CCR ve VZA-BCC modellerini geliştirmektir. Her bir KVB' nin mümkün mertebe en yüksek ve azami etkinlik puanları, bazı α-duzeyleri/değerleri için tahmin edilmiştir. Bu model, nitel faktörlerle ilişkili olan bazı sorunları belirlemeye de uygulanabilir. Önerilen metodlar dört sayısal örneğe uygulanıp bulanık VZA' nın önerilen yeni sekiz modelinin sonuçları elde edilmiş ve karşılaştırılmıştır. Türkiye'de kullanılan 18 ticari bankanın göreceli etkinlik değerlerini hesaplamak için önerilen yöntemin uygulanıp uygulanmadığı kontrol edilmiş, 2007-2016 yılları arasında 10 yıl boyunca veri toplanıp üzerinde çalışılmıştır. KVB' nin simülasyonu çok katmanlı yapay sinir ağı bellek algoritma eğitimi için kullanılması amaçlanmıştır. Bulanık CCR ve BCC modelleri ile bankaların etkinlik değerleri hesaplanmıştır.

Bilim Kodu	: 20517
Anahtar Kelimeler	: Yöneylem Araştırması,Veri Madenciliği, İstatistiksel Analiz
Sayfa Adedi	: 149
Danışman	: Prof. Dr. Hasan BAL

ACKNOWLEDGEMENT

Firstly, I would like to express my sincere gratitude to my advisor Prof. Hasan BAL for the continuous support of my Ph.D. study and related research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D. study. Special mention goes to my enthusiastic supervisor, Prof. Hasan BAL. My Ph.D. has been an amazing experience and I thank Prof. Ihsan ALP, not only for his tremendous academic support, but also for giving me so many wonderful opportunities. I also thank the professors and assistant researchers of the statistics department at the University of Gazi, Thank you very much Turkey. Finally, but by no means least, thanks go to mum, dad God's mercy in his grave, my wife and my children for almost unbelievable support. They are the most important people in my world and I dedicate this thesis to them.

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1. INTRODUCTION

Data envelopment analysis (DEA) is a linear programming method (LP), which measures the relative efficiency of the associated with the decision-making units (DMUs) when multiple inputs and outputs are present. To examine the radial technical efficiency of a given DMUp, Charnes et al. (1978) proposed the constant returns to scale (RTS) model (CRS) or Charnes-Cooper-Rhodes (CCR). Assume that there are n DMUs to be evaluated, where every DMUj, j = 1, 2, . . . , n, produces s outputs, y_{rj} (r = 1, 2, . . . , s), using m inputs, x_{ij} (i = 1, 2, . . . ,m). The CCR model (Charnes et al., 1978) is proposed to evaluate the efficiency of a specific DMUp. And to see the other basic models of DEA, the defendant is able see. The advanced DEA models divided DMUs into two efficient and inefficient groups while in practice. So, there are variety of researches classified ranking methods and fuzzy data envelopment analysis (FDEA) are presented in the DEA literature. This method removes the unit under the assessment from a group of DMUs, and measures the distance of DMU from the new efficient frontier.The α-level approach is the most popular fuzzy DEA models. Many articles are published about this method in the literatary DEA search.

From this approach, the key two ideas are to transform the fuzzy CCR model in some parametric programs in order to find the lower and upper limits or interval of the α- level membership functions of the efficiency scores. Then find the fuzzy efficiency scores of the DMUs using fuzzy linear programs which require ranking fuzzy sets. It is created by the membership function for spaces of input and output on the most basis of the interpretation of the allow limits.

Kao and Liu (2000) followed up on the basic idea of transforming a fuzzy DEA model to a family of conventional crisp DEA models and developed a solution procedure to measure the efficiencies of the DMUs with fuzzy observations in the BCC model. Kao and Liu (2003) proposed a pair of two-level mathematical models to calculate the lower bound and upper bound of the fuzzy efficiency score for a specific α -level. Then used the ranking fuzzy numbers method of Chen and Klein (1997a) ve (1997b) to rank the obtained fuzzy efficiencies. Kao and Liu (2003) used the maximum set–minimum set method of Chen (1997) into the fuzzy DEA model proposed by Kao and Liu (2000a) and built pairs of nonlinear programs and ranked the DMUs with fuzzy data. Saati et al. (2002) suggested a fuzzy CCR model as a possibilistic programming problem and transformed it into an interval programming problem using α -level based approach. The resulting interval programming problem could be solved as a crisp LP model for a given a with some variable substitutions, use triangular fuzzy inputs and the triangular fuzzy outputs, and x'_{ij} and y'_{rj} are the decision variables obtained from variable substitutions used to transform the original fuzzy model proposed into a parametric LP model with $\alpha \in [0,1]$ Saati and Memariani (2005) suggested a procedure for determining a common set of weights in fuzzy DEA based on the α -level method proposed by Saati et al. (2002) with triangular fuzzy data. In this method, the upper bounds of the input and output weights were determined by solving some fuzzy LP models and then a common set of weights were obtained by solving another fuzzy LP model. Hatami-Marbini and Saati (2009) developed a fuzzy BCC model which considered fuzziness in the input and output data as well as the u_0 variable. Consequently, they obtained the stability of the fuzzy u_0 as an interval by means of the method proposed by Saati et al. (2002). Hatami-Marbini et al. (2010a) used the method of Saati et al. (2002) and proposed a four-phase fuzzy DEA framework based on the theory of displaced ideal. Liu et al. (2007) developed a modified fuzzy DEA model to handle fuzzy and incomplete information on weight indices in product design evaluation transformed fuzzy information into trapezoidal fuzzy numbers and considered incomplete information on indices weights as constraints. They used an α-level approach to convert their fuzzy DEA model into a family of conventional crisp DEA models. Jahanshahloo et al. (2007b) developed a fuzzy L_1-norm model with trapezoidal fuzzy inputs/outputs that was initially suggested by Jahanshahloo et al. (2004c) for solving the crisp data in DEA. Wang et al. (2009b) proposed a fuzzy DEA–Neural approach with a self-organizing map for classification in their neural network.

They used the upper and lower bounds or enterval of efficiency score at different possibilistic levels in their model. Hosseinzadeh Lotfi et al. (2009a) developed two methods for solving fuzzy CCR model with respect to fuzzy, ordinal and exact data.They used an analogue function to transform the fuzzy data into exact values in the first method.

1.1. Data Envelopment Analysis (DEA) Model

This guide introduces Data Envelopment Analysis (DEA), a performance measurement technique, In a manner that is appropriate for decision makers with little or no background in economics and operational research. While keeping the use of mathematics to a minimum. This guide therefore adopts a strong practical approach in order to allow decision makers to conduct their own efficiency analysis and to easily interpret results. DEA helps decision makers with the following reasons;

- With calculates the degree of efficiency, it indicates whether the company is effective or has the ability to optimize.
- With setting target values for input and output, it calculates the amount of inputs that should be reduced or the output increased to be effective.
- With determining the nature of the standard returns, it indicates whether the company should reduce or increase its size in order to reduce the average cost.
- With defining a set of criteria, identify the operations of other companies that need analysis to improve their own practices.

A common measure of relative efficiency when there are multiple inputs and multiple outputs is a ratio of the weighted sum of outputs to the weighted sum of inputs. DEA uses this concept and generalizes the efficiency measurement for a set of homogenous decision making units (DMUs) from a single-input, single-output ratio to a multiple-input, multiple-output ratio. Specifically, it determines a set of weights such that the efficiency of the target (DMUJ) relative to the other DMUs is maximized.

Suppose that there are n DMUs, each of which has n inputs and v outputs of the same type. All inputs and outputs are assumed to be nonnegative. DEA is formulated as the following fractional mathematical programming model:

$$\frac{u_1 y_1 + u_2 y_2 + \dots + u_n y_n}{v_1 x_1 + v_2 x_2 + \dots + v_m x_m} \tag{1.1}$$

Specifically, efficiency is defined as a ratio of a weighted sum of outputs to a weighted sum of inputs. There are n outputs and m inputs for the decision point. Here, u_n n. the weight of the output, y_n n. the amount of output, v_m m. the weight of the entrance and x_m m. the amount of entry.

The CCR (Charnes-Cooper-Rhodes) method is based on the fixed return assumption on a scale. If the activity of the decision-making unit is objective, the objective should be maximization of this value. In this case, the objective function can be expressed as in (1.2) under the assumption of input-orientation:

$$Max h_j = \frac{\sum_{r=1}^{n} u_r y_r}{\sum_{i=1}^{m} v_i x_i} \text{ s.t.} \tag{1.2}$$

$$\frac{\sum_{r=1}^{n} u_r y_r}{\sum_{i=1}^{m} v_i x_i} \leq 1$$

$$u_r \geq 0$$

$$v_i \geq 0$$

It is a model obtained by modifying the assumptions of the CCR model. This model is basically based on the assumption of variable return on a scale. Developed by (Banker-Charnes-Cooper) using the BCC model, the scale type for all decision units can also be determined. The BCC limit is always below the CCR limit. Therefore, the CCR efficacy score will be less than or equal to the BCC efficacy score.

The only difference from the CCR model of the BCC model is that the sum of the values of the sum of the linear program results to be solved for each decision unit under the assumption of a variable return on a scale basis (the value that provides the information needed to generate the effective input output effect for an ineffective decision point). The model of the BCC method is given in equation (1.3).

$$Min \Theta_k$$

s.t.

$$\Theta_k x_{ik} - \sum_{j=1}^{N} x_{ij}\lambda_{jk} \geq 0 \tag{1.3}$$

$$\sum_{j=1}^{N} \lambda_j = 1$$

Some of the basic characteristics of DEA are summarized as follows;

- DEA is used to measure the efficiency of homogeneous units called decision-making units (DMUs), which consume the same input type and produce the same type of output.
- DEA is an atypical approach and, therefore, there is no limitation on the functional form that links inputs to outputs.
- DEA is a fractional mathematical programming technique. However, it can be converted to a linear programming model and resolved by a continuous LP solver.
- DEA circulates the concept of individual per capita income management and individual access to the state of multiple inputs and multiple outputs by calculating relative efficiency points as a ratio of default outputs to default inputs.
- DEA is an approach that focuses on borders rather than central tendencies. It evaluates the efficiency of each DMU relative to similar DMUs. Thus, it provides an effective interface or circumstance for all effective DMU modules rather than the appropriate level of regression through the data center.
- DEA determines the relative efficiency of a single data unit management unit at a time on all other DMUs by finding the most appropriate weights from that point of view, target DMU.

1.2. Fuzzy DEA Model

The observed values in real-world problems are often imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and non obtainable information. Imprecise or vague data is often expressed with bounded intervals, ordinal (rank order) data or fuzzy numbers. In recent years, many researchers have formulated fuzzy DEA models to deal with situations where some of the input and output data are imprecise or vague.

In the fuzzy ranking approach of efficiency measurement was initially developed by Guo and Tanaka (2001). Furthermore, they proposed a fuzzy BCC model using the fuzzy number ranking method proposed by Yao and Wu (2000) for trapezoidal fuzzy data in DEA. Hosseinzadeh Lotfi et al. (2007a) applied trapezoidal fuzzy data to Jahanshahloo et al. (2004b) DEA method, in which a fuzzy fixed cost was equitably assigned to all DMUs in such a way that the efficiency scores were not changed Hatami-Marbini et al. (2010c) developed a fully fuzzified CCR model to obtain the fuzzy efficiency of the DMUs by utilizing a fully fuzzified LP model, in which all of the input-output data and variables (including their weights) were fuzzy numbers.

In this study, we determine the maximum possible efficiency scores of each DMU at some α-levels. The obtained score of each DMU at level α can be represented by an interval. A number of these crisp intervals are used to represent the scores as a fuzzy interval. Finally, DMUs are ranked according to their fuzzy scores. So far, several papers have been presented on the theoretical development of DEA with fuzzy data (FDEA models).

The defuzzification approach was developed by Lertworasirikul et al. (2003a). In this approach the fuzzy inputs and outputs are defuzzified into crisp values. This approach is simple but ignores the uncertainty in data. The α-level based approach was introduced by Kao and Liu (2003), and Lertworasirikul et al. (2003b). In this approach, the FDEA model is solved by parametric programming using α-cuts. Solving the model at a given α-level produces a corresponding interval efficiency for the target DMU. A number of such intervals can be used to construct the corresponding fuzzy efficiency. This approach provides fuzzy efficiency but requires the ranking of fuzzy efficiency sets. The fuzzy ranking approach was developed by Guo and Tanaka (2008). In this approach, both fuzzy inequalities and equalities in the fuzzy CCR model are defined by ranking methods so that the resulting model is α -level PLP model. This approach provides fuzzy efficiency for an evaluated DMU at a specified α-level. Guo and Tanaka (2008) compare fuzzy efficiencies using only one number at a given α-level. Lertworasirikul et al. (2003b) show that for the special case, in which fuzzy membership functions of fuzzy data are trapezoidal types.

Charnes et al. (1978) proposed the constant returns to scale (RTS) model (CRS or CCR) to evaluate the radial technical efficiency of a given DMUp. Assume that there are n DMUs to be evaluated, where every DMUj, j = 1, 2, . . . , n, produces s outputs, y_{rj} (r = 1, 2, . . . , s), using m inputs, x_{ij} (i = 1, 2, . . . ,m). Banker et al. (1984) extended the earlier work of Charnes et al. (1978) by proposing the model for variable RTS (VRS or BCC). The BCC and CCR models differ only in that the former includes an additional convexity constraint, $\sum_{j=1}^{n} \lambda_j = 1$, The CCR model (Charnes et al., 1978) is proposed to evaluate the efficiency of a specific DMUp. The dual CCR models are given in this model (1.4).

$$\max \ \theta_p = \sum_{r=1}^{s} u_r y_{rp}$$

s.t.

$$\sum_{i=1}^{m} v_i x_{ip} = 1 \tag{1.4}$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j$$
$$u_r, v_i \ge 0 \quad \forall r, i$$

The additional variable, u_0, in the dual BCC model as shown in this model (1.5).

$$\max w_p = \sum_{r=1}^{s} u_r y_{rp} - u_0$$

s.t.

$$\sum_{i=1}^{m} v_i x_{ip} = 1 \tag{1.5}$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \le 1 \ , \forall j$$
$$u_r, v_i \ge 0 \quad \forall r, i$$

Sign conditions identify the situation for returns to scale (RTS) for BCC and CCR models which are supplied by Banker and Thrall (1992);

- The case of increasing returns to scale $u_0^* < 0$ for all optimal solutions to (1.4) if and only if $\sum \lambda_j^* < 0$ for all optimal solutions to (1.5).
- The case of decreasing returns to scale $u_0^* > 0$ for all optimal solutions to (1.4) if and only if $\sum \lambda_j^* > 0$ for all optimal solutions to (1.5).
- The case of constant returns to scale $u_0^* = 0$ for some optimal solutions to (1.4) if and only if $\sum \lambda_j^* = 0$ for some optimal solution to (1.5).

1.3. Outline of The Dissertation

This Dissertation is arranged as follows. Chapter 1 gives introduction and data envelopment analysis DEA model and fuzzy DEA model outline of the dissertation. Chapter 2 literature review on DEA and solving fuzzy DEA approaches and some of definitions and concepts. Chapter 3 presents a fuzzy DEA model. The defuzzification approach, α-leve1 and α-cut based approach from our preliminary work are presented in this chapter. Chapter 4 presents a parametric and interval possibility approach to fuzzy DEA. The latter approach is the extension of the methods discussed in the literature review, but it provides more alternatives to decision makers. A numerical examples is given to demonstrate the implementation of these approaches. In Chapter 5 application and simulation for FBCC and FCCR model which was used to calculate efficiency values of the Turkish banks. And Chapter 6 concludes this dissertation, and discusses some future research directions.

2. LITERATURE REVIEW

2.1. Literature Review on Fuzzy DEA

Data Envelopment Analysis has many applications in financial services, agricultural services, healthcare, education, manufacturing, telecommunications, supply chain management, measuring the efficiency of banks, decision making and many more. For a recent comprehensive list of DEA, see Emrouznejad et al. (2008). This method has been introduced in DEA to measure the efficiency of decision-making units under uncertainty mainly when accurate data are not available. This study focuses on the use of blurry groups in DEA.

Target values in real world problems are often inaccurate, ambiguous or incomplete. Inaccurate or ambiguous data may be the result of undetectable, incomplete, and unverifiable information. Insufficient or vague data is often expressed at specified intervals, ordinal data, or fuzzy numbers. In recent years, many researchers have formulated fuzzy DEA models to deal with situations where some input and output data are inaccurate or ambiguous.

The Fuzzy group theory has been developed to deal with the concept of relativistic truth values ranging from absolute truth to total falsehood. Fuzzy group theory has become the outstanding tool for dealing with inaccuracies or ambiguities that aim at traceability, strength, and low-cost solutions to real-world problems. According to Zadeh (1975), it is very difficult to estimate conventional quantities as a reasonable expression of complex situations and it is necessary to use values of linguistic variables that are words or phrases in a natural or artificial language. The advantages that contributed to the spread of this approach are the potential to work with language variables, low cost and easy to understand. The elaborate algebraic development developed by Zadeh (1965) is the formal body of the theory that allows for the processing of inaccurate and ambiguous estimates in an uncertain environment.

The data in traditional CCR and BCC models assume the form of specific numerical values. However, the observed value of input and output data is sometimes inaccurate or ambiguous.

Sengupta (1992) was the first to introduce a fuzzy mathematical programming approach in which ambiguity was incorporated into the DEA model by determining allow levels in both the target function and constraints. Let's assume that DMUs consume different amounts of m input and produces n outputs suppose that $\tilde{x}_{ij}$ $(i = 1,2,\dots, m)$ and $\tilde{y}_{rj}$ $(r = 1,2,\dots, s)$ represent. respectively, the fuzzy input and fuzzy output of the *j*th DMUj $(j = 1,2,\dots, n)$. The primal and its fuzzy CCR models in input-oriented version are formulated as (2.1).

$$\max \quad \sum_{r=1}^{s} u_r \, \tilde{y}_{rp}$$

s.t.

$$\sum_{i=1}^{m} v_i \, \tilde{x}_{ip} = 1 \tag{2.1}$$
$$\sum_{r=1}^{s} u_r \, \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \, \tilde{x}_{ij} \leq 0 \qquad J \neq P$$
$$u_r \,, v_i \geq 0 \qquad \forall r \,, i$$

A fuzzy BCC model is obtained and this added constraint introduces an additional variable, $\tilde{u}_0$, into the model which these models are formulated shown in (2.2).

$$\max \quad w_p = \frac{\sum_{r=1}^{s} u_r \, \tilde{y}_{rp} - u_0}{\sum_{i=1}^{m} v_i \, \tilde{x}_{ip}}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rp} - u_0}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \leq 1 \,, \forall j \tag{2.2}$$
$$u_r \,, v_i \geq 0 \qquad \forall r \,, i$$

2.2. Literature Review Fuzzy DEA Approaches

Theory applications in DEA are usually categorized into four groups (Lertworasirikul et al. 2003a, 2003b, Lertworasirikul 2002, Karsak 2008, Adlera et al. (2002), Hatami-Marbini et al. 2011): (1) Tolerance approach, (2) The fuzzy ranking approach, (3) Possibility approach, (4) α-level based approch. This last approach will build on the parametric programming approach in Chapter 3. In this section, we provide a literature review of each method followed by a brief review of the most common literature related to each method as well as the coupling of the above element, we introduce a new class.

2.2.1. Review by the tolerance approch

The Tolerance Approach is one of the first blurred DEA models developed by Sengupta (1992) and became more efficient by Kahraman and Tolga (1998).In this approach, the main idea is to integrate uncertainty in the DEA models by determining tolerance levels on the projections of constraints. This approach holds signs of inequality or equality but does not treat uncertain transactions directly.

The precise identification of the tolerance approach proposed by Sengupta (1992) is related to the design of a proposed model for the DEA with a Fuzzy objective function and ambiguous limitations that may or may not be met (Triantis and Girod, 1998).

Although uncertainty exists in most production processes, whether in terms of failure to achieve specific targets or in terms of data inaccuracy, the tolerance approach provides flexibility by loosening the envelopment analysis relationships while input and output coefficients are treated as fragile.

2.2.2. Review by the fuzzy ranking approch

The ambiguous arrangement approach is also another popular method that has attracted much attention in the literature of DEA. In this approach, the basic idea is to find the blurred efficiency points of the envelopment units by using blurry linear programs that require ordered fuzzy sets. The random classification approach for measuring efficiency was initially developed by Guo and Tanaka (2001). They proposed a vague CCR model where the unclear restrictions were converted to clear limits by setting a level of probability and using the comparison rule for unclear numbers.

Assuming the presence of n DMUs under evaluation, the efficiency of DMUj with m symmetrical triangular fuzzy inputs and and s symmetrical triangular fuzzy outputs is denoted by $\tilde{x}_{ij} = (x_{ij}, c_{ij})$ and $\tilde{y}_{rj} = (y_{rj}, d_{rj})$, respectively, where x_{ij} and y_{rj} are the center, and c_{ij}

and d_{rj} are the spread of fuzzy numbers , These were given by Guo and Tanaka (2001). Fuzzy efficiency for each DMU under evaluation with similar symmetric trigonometric inputs $\tilde{x}_{ip}$ and outputs $\tilde{y}_{rp}$ is obtained for each h possibility level as a non-symmetrical triangular fuzzy number $\tilde{x} = (x_0 , \sigma , \beta)$.

Guo and Tanaka (2008) extended their previous work (Guo and Tanaka 2001) and presented the foggy aggregation model for the objective rank of the DMUs group by combining multiple ambiguous values. Guo (2009) also applied a new DEA model in a case study of the problem of the restaurant site in China by incorporating the blurred DEA model proposed by Guo and Tanaka (2001) with the ambiguous assembly model proposed by Guo and Tanaka (2008). Sanei et al. (2009) use a sensitivity analysis model for Cooper et al. (2001) with misty data, and applied the Goo and Tanaka (2001) approach to construct their ambiguous model to determine the stability radius of different α values. Along the lines suggested by Guo and Tanaka (2001), Leon et al. (2003) developed a mysterious BCC model (3). However, in the Guo and Tanaka (2001) method, a mysterious degree of efficiency is obtained for each possible level, whereas in Leon et al. (2003) method, a high degree of efficiency is obtained either for all or all possible levels. Leon et al. (2003) proposed two different models for the Department of Drug Control depending on the method of classification used to explain unclear inequality. The first model uses the ranking method in Ramik and Rimanek (1985) to obtain clear efficiency points from DMUp where all possible values of different variables are considered for all DMUs at all probability levels.

Hatami-Marbini et al. (2010b) extended a mysterious CCR model to evaluate units of DMUs from a better perspective and worst possible relative efficiency by using Leon et al. (2003) approach. Then, in order to classify all DMUs, the coefficient of convergence coefficient was obtained by combining the two different elves. Jahanshahloo et al. (2004a), an unclear classification method for the solution of the SBM model was proposed in DEA when inputs and outputs are triangular fuzzy numbers.

Saati and Memariani (2006) dealt with some of the shortcomings of the mysterious DEA proposed by Jahanshahloo et al. (2004a) and proposed several corrections in their own way. Molavi et al. (2005) presented two blurred drug management models in which the objective function and the cluttered constraints of the mysterious CCR model were transformed into fragile conditions using the LR-fuzzy arrangement of Ramik and Rimanek (1985). Solaimani Damaneh et al. (2006) addressed some computational and theoretical deficiencies in many unclear forms of management of drug control, including Kao and Liu (2000), Leon et al. (2003), Lertworasirikul et al. (2003b), Guo and Tanaka (2001) and Jahanshahloo et al (2004a). In addition, they proposed a fuzzy BCC model using the method of ordering ambiguous numbers proposed by Yao and Wu (2000) for trapezoidal fuzzy data in DEA.

Husseinzadeh Lotfi et al. (2007a) trapezoidal fuzzy data were applied to the management of hunger management Jahanshahloo et al. (2004b), where a flat cost was set out that was not clearly defined for all data management units in a manner that did not change efficiency. They used an unclear classification method to solve the ambiguous pattern in which each obscure constraint turned into three obvious constraints. Husseinzadeh Lutfi et al. (2007b) adopted the linear order function proposed by Maleki (2002) to provide cryptic CCR models with trigonometric data. Jahanshahloo et al. (2007a) suggested a way to handle the Malmquist-based DRM index for all DMUs with the input and output of the triangular fuzzy.

They applied the linear order function proposed by Maleki (2002) to transform the blurry linear programming model into a set of clear traditional DEA models. Pal and Pal (2007) uses a mysterious DEA approach and α-parametric inequalities in the diffusion of quality function. They used fuzzy CCR model based on the method proposed by Lai and Huang (1992).

Hussainzadeh and Mansouri (2008) considered the expanded method of tax discrimination (DEA) proposed by Sueyoshi (2001) as fuzzy data and changed its fuzzy model into a crisp model using the linear ranking function proposed by the Maleki (2002). Zhou et al. (2008) developed a fuzzy DEA method to assess the efficiency of real estate investment programs. They applied the fuzzy ranking to solve their model and designed a "relatively effective control tool" that looked at controlling the diversity of the method. Noora and Karami (2008) adopted three fuzzy data to develop a fuzzy non-radial DEA model and applied a classification function proposed by Maleki et al. (2000) to transform fuzzy linear programming into crisp DEA models. Jahanshahloo et al. (2008) showed the linear ranking function proposed by Mahdavi-Amiri and Nasseri (2006) to change the fuzzy cost efficiency model to a traditional linear programming problem. Solaimani-Damaneh (2008) used a fuzzy signed distance and fuzzy bound concepts to formulate a fuzzy addition model in DEA with fuzzy input and output data. Solaimani-Damaneh (2009) presented a theorem about the model of fuzzy DEA proposed by Solaimani-Damaneh (2008) in order to demonstrate the existence of the upper limit based on the distance of the objective function of the model.

Hatami-Marbini et al. (2009) suggested a fuzzy DEA model to assess the efficiency scores in a fuzzy environment. They used the proposed ranking method in Asady and Zendehnam (2007) and obtained specific efficiency grades for the overall order of DMUs. Compare their method with the mysterious DEA methods suggested by Soleimani et al. (2006) and Leon et al. (2003). They have also applied their model to 16 branches in the capital, Iran. Jahanshahloo et. al. (2009b) further introduced an Different approach for solving fuzzy $l_1 - norm$ the method in DEA with fuzzy data is based on a comparison of the ambiguous numbers proposed by Tran and Duckstein (2002) to change the fuzzy linear programming of the crisp model. Husseinazdeh et al. (2009c) circulated DEA multi-activities on network DEA to fuzzy inputs and outputs which were formed by triangular membership functions.

2.2.3. Review by the possibility approach

The basic principles of probability theory are rooted in Zadeh (1978) fuzzy set theory. He suggests that a fuzzy variable is related to a possible distribution in the same way that a random variable is associated with a probability distribution. In fuzzy LP models, confusing transactions can be seen as confusing fuzzy variables and the constraint can be considered fuzzy. Thus, the potential of fuzzy events (ie, fuzzy constraints) can be determined using probability theory. Dubois and Prade (1988) provide an overview of probability theory.

Lertworasirikul (2002) and Lertworasirikul et al. (2002a, 2002b), two approaches were proposed to solve the problem of ranking in fuzzy DEA models called the "probability methodology " and the "credibility methodology". They have introduced a possible approach from an optimistic and pessimistic point of view by considering uncertainties about fuzzy objectives and ambiguities with potential measures. In the credibility approach, the unclear DEA model was converted into a DEA programming model and the fuzzy variables were replaced by "expected balances" obtained using the reliability criteria. The mathematical details of the credibility model can be found in Lertworasirikul et al. (2003b).

Lertworasirikul et al. (2003a, 2003c) suggested a possible way to solve a mysterious CCR model where blurry constraints were treated as fuzzy events. They converted the DEA misty model to a possibility LP problem by using potential actions for confusing events. In the particular case, assuming that the fuzzy data is a trapezoidal distorted number, the potential DEA model becomes an LP model. The proposed CCR model suggested Lertworasirikul et al. (2003a) where they implemented the concept of chance-constrained programming (CCP) and the possibility of fuzzy events.

Lertworasirikul et al. (2003b) Develop more approaches to credibility and credibility to resolve fuzzy BCC models. They applied the concept of chance-constrained programming (CCP) and the possibility of fuzzy events (fuzzy restrictions) to the primal and dual of fuzzy BCC models in order to obtain possibility BCC models. This approach also reveals the relationship between the prototypes and the binary of a mysterious BCC. Ramezanzadeh et al. (2005) suggested a CCR model using a limited-time programming approach using an alpha-level method and a random probabilistic random coordination measure using the classical mean-variance method in Cooper et al. (1996). Jiang and Yang (2007) decided on a fuzzy model of programming credibility data, and introduced a procedure for converting fuzzy programming to confirm programming. They used a potential approach suggested by Lertworasirikul et al. (2003a) in their model to solve their fuzzy DEA problem. Similarly, Wu et al. (2006) used the formulation of Lertworasirikul et al. (2003a) in the undesirable DEA model to deal with quantitative and linguistic variables in analyzing the efficiency of bank branches across regions in Canada.

Wen et al. (2010) The CCR model was extended to a fuzzy DEA model based on Liu et al. (2007) reliability measure. They designed a hybrid algorithm coupled with fuzzy simulations and a genetic algorithm for the rank of all DMUs with fuzzy inputs and outputs.

2.2.4. Review by the α-level based approch

In this approach, the main idea is to transform the fuzzy CCR and fuzzy BCC models into a pair or more of the parametric programs in order to find the lower and upper bounds α level of the membership functions of the efficiency grades.

This approach involves three stages: first, the inaccurate inputs and outputs were determined by the decision maker in terms of risk-free and impossible boundaries. Second, two fuzzy CCR models, BCC were formulated in terms of their risk-free and impossible limits as well as their membership function for different values of a α. Third, they illustrate the implementation of the fuzzy CCR, BCC models in the context of finding the lower and upper bounds of the achieves an efficiency scores for system. Girod (1996) used the approach proposed by Carlsson and Korhonen (1986) for the formulation of blurred BCC and the addition of free disposal hull FDH models which were national measures of efficiency, they illustrated the implementation of their fuzzy BCC model in the context of Pre-printing and packaging line that lists business brochures into news papers. Their paper detailed their roadmap for implementation by Girod and Triantis (1999). Triantis (2003) extended his previous work on DEA (Triantis and Girod, 1998) to non-discrete, non-disruptive DEA measures of technical competence to support an integrated performance measurement system. He also compared his technique to the radiological efficiency of the same production line described in detail (Girod, 1996) and Girod and Triantis (1999). The α-level approach provides an inefficient level of efficiency but requires the ordering of fuzzy efficiency groups as suggested by Meada et al. (1998).

Kau and Liu (2000a) followed the basic idea of transforming a fuzzy DEA model into a family of conventional crisp DEA models and developed a solution to measure DMUs with fuzzy observations in the BCC model. Their method almost found membership functions for the fuzzy efficiency measure by applying the alpha-level approach and the Zadeh (1978) approach (Zimmermann1996).They transformed the fuzzy DEA model into a pair of parametric mathematical programs and used the dice sorting method and Chen et al. (1997). Solving this model at the given level of α-level produced the interval efficiency of the DMU under consideration.

2.3. Definitions and Concepts

2.3.1. Fuzzy number

The fuzzy numbers identified by Klir and Yuan (1995) are a special case of fuzzy sets $\tilde{x}$ is a convex normalized fuzzy set $\tilde{x}$ of the real line R such that;

- There exist exactly one $x_0 \in$ R with $\mu_{\tilde{x}}(x_0)$ = 1 (*unimodal*).
- $\mu_{\tilde{x}}(x)$ is piecewise continuous.

We review some basic definitions of fuzzy sets (Dubois and Prade 1988, Kaufmann and Gupta 1991, Zimmermann 1996).

Definition 1: A fuzzy number $\tilde{x}$ in parametric form is a pair $(\tilde{x}^i, \tilde{x}^d)$ of functions $\tilde{x}^i(r), \tilde{x}^d(r), 0 \leq r \leq 1$, which satisfy the following requirements;

- $\tilde{x}^i$ is a bounded monotonic increasing left continuous function,
- $\tilde{x}^d$ is a bounded monotonic decreasing left continuous function,
- $\tilde{x}^i \leq \tilde{x}^d$, $0 \leq r \leq 1$

and its parametric form is:

$$\tilde{x}^i(r) = (x_0 - \sigma + \sigma r), \quad \tilde{x}^d(r) = (y_0 + \beta - \beta r)$$

Provided that, $x_0 = y_0$ then $\tilde{x}$ is a triangular fuzzy number, and we write;

$$\tilde{x} = (x_0, \sigma, \beta).$$

Definition 2: The support of fuzzy number $\tilde{x}$ is defined as follows;

$$\text{supp}(\tilde{x}) = \overline{\{x \mid \mu_{\tilde{x}}(x) > 0\}},$$

where $\overline{\{x \mid \mu_{\tilde{x}}(x) > 0\}}$ is closure of set $\{x \mid \mu_{\tilde{x}}(x) > 0\}$

Definition 3:The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented in (Zimmermann 1991, Ying-Ming et al 2005) as follows. For arbitrary $\tilde{x} = (\tilde{x}^i, \tilde{x}^d)$, $\tilde{y} = (\tilde{y}^i, \tilde{y}^d)$ we define addition $(\tilde{x} + \tilde{y})$ and multiplication by scalar k > 0 as.

$$(\tilde{x}^i + \tilde{y}^i)(r) = \tilde{x}^i(r) + \tilde{y}^i(r), \ (\tilde{x}^d + \tilde{y}^d)(r) = \tilde{x}^d(r) + \tilde{y}^d(r)$$
$$(k\tilde{x})^i(r) = k\tilde{x}^i(r), \ (k\tilde{x})^d(r) = k\tilde{x}^d(r) \quad (2.3)$$
$$(k\tilde{y})^i(r) = k\tilde{y}^i(r), \ (k\tilde{y})^d(r) = k\tilde{y}^d(r)$$

To emphasize the collection of all fuzzy numbers with addition and multiplication as defined by (2.3) is denoted by E, which is a convex cone. The image (opposite) of $\tilde{x}$=$(x_0, y_0, \sigma, \beta)$, can be defined by -$\tilde{x}$=$(-x_0, -y_0, \beta, \sigma)$, see (Hosseinzadeh et al. 2007a, Zimmermann 1991).

Definition 4: α-cut of fuzzy set*:* A α-cut of fuzzy set $\tilde{x}$ is α crisp subset of X which, denoted by;

$$\tilde{x}^{\alpha}=\{x \in X | \mu_{\tilde{x}}(x) \geq \alpha\} = [x^{\alpha L}; x^{\alpha U}] = [X^{min}\{x \in X | \mu_{\tilde{x}}(x) \geq \alpha\}; X^{max}\{x \in X | \mu_{\tilde{x}}(x) \geq \alpha\}] \quad (2.4)$$

The α-cuts of $\tilde{x}$ and $\tilde{y}$ are defined as:

$\tilde{x}^{\alpha}=\{x \in X | \mu_{\tilde{x}}(x) \geq \alpha\} = [x^{\alpha L}; x^{\alpha U}]$, and (2.5)

$$\tilde{y}^{\alpha}=\{x \in X | \mu_{\tilde{y}}(x) \geq \alpha\} = [y^{\alpha L}; y^{\alpha U}] \quad (2.6)$$

We can draw a membership function of triangular fuzzy number with concept of local α-cut like (Figure 2.1).

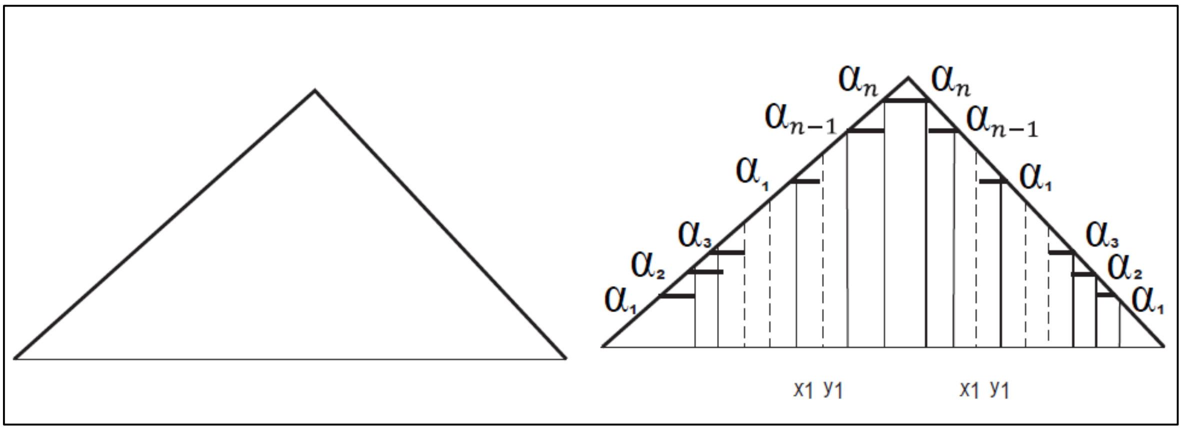

Figure 2.1. Membership function concept of local α-level

Definition 5: The trapezoidal fuzzy number $\tilde{x}$ =$(x_0\ , y_0 , \sigma\ , \beta)$ is reduced to a real number $\tilde{x}\ \ if\ x_0\ = y_0\ = \sigma = \beta$. Conversely, a real number *R* can be written as a trapezoidal fuzzy number $\tilde{x}$ =(x, x, x, x) Similarly, the α – level $\tilde{x}$ =$(x_0\ , y_0 , \sigma\ , \beta)$ can easily be determined as:

$$[\tilde{x}]^{\alpha} = [\alpha x_0 + (1-\alpha)\sigma, \alpha y_0 + (1-\alpha)\beta] \tag{2.7}$$

where $\alpha \in [0, 1]$. If x = $x_0\ = y_0$, then $\tilde{x}$ =$(x_0\ , \sigma\ , \beta)$ is called a triangular fuzzy number.

Definition 6: x real fuzzy number denoted by $\tilde{x}$ =$(x_0\ , y_0 , \sigma\ , \beta, w)$ is described as any fuzzy subset of the real line *R* with a membership function $\mu_{\tilde{x}}$, which satisfies the following properties;

$\mu_{\tilde{x}}$ is a semicontinuous mapping from *R* to the closed interval [0, *w*], 0 ≤ *w* ≤1,
$\mu_{\tilde{x}}(x)$ = 0, for all $x \in [-\infty, \sigma\ ]$,
$\mu_{\tilde{x}}$ is increasing on $[\sigma\ , x_0\ ]$,
$\mu_{\tilde{x}}(x) = w$ for all $x \in [x_0\ , y_0]$, where *w* is a constant and 0 < *w* ≤ 1,
$\mu_{\tilde{x}}$ is decreasing on $[y_0, \beta\ ]$, and
$\mu_{\tilde{x}}(x)$ = 0, for all $x \in [\beta, \infty]$,
where $x_0\ , y_0 , \sigma$ and β are real numbers.

Unless elsewhere specified, it is assumed that x̃ is convex and bounded; i.e., $-\infty < \sigma$, $\beta < \infty$.
If w = 1, x̃ is a normal fuzzy number, and if 0 < w < 1, x̃ is a nonnormal fuzzy number.

Definition 7: Suppose that we have two positive trapezoidal fuzzy numbers $\tilde{A} = (x_a, y_a, \sigma_a, \beta_a)$ and $\tilde{B} = (x_b, y_b, \sigma_b, \beta_b)$, then the arithmetic operations of these two trapezoidal fuzzy numbers are defined as follows Goetschel and Voxman (1986);

$\tilde{A}(+)\tilde{B} = (x_a + x_b\,, y_a + y_b\,, \sigma_a + \sigma_b\,, \beta_a + \beta_b\,),$

$\tilde{A}(-)\tilde{B} = (x_a - x_b\,, y_a - y_b\,, \sigma_a - \sigma_b\,, \beta_a - \beta_b\,),$

$\tilde{A}(\times)\tilde{B} = (x_a x_b\,, y_a y_b\,, \sigma_a \sigma_{bA}\,, \beta_a \beta_b\,),$

$\mathrm{K}\tilde{A} = (kx_a, ky_a, k\sigma_a, k\beta_a) \quad \forall\, k \in,$

$\mathrm{K}\tilde{B} = (kx_b, ky_b, k\sigma_b, k\beta_b) \quad \forall\, k \in R^{+},$

$(\tilde{A})^{-1} = (\frac{1}{y_a}, \frac{1}{x_a}, \frac{1}{\beta_a}, \frac{1}{\sigma_a}),$

$(\tilde{B})^{-1} = (\frac{1}{y_b}, \frac{1}{x_b}, \frac{1}{\beta_b}, \frac{1}{\sigma_b})$,and

$$\tilde{A}(\div)\tilde{B} = \tilde{A}(\times)\tilde{B}^{-1} = (\frac{x_a}{y_b}, \frac{y_a}{x_b}, \frac{\sigma_a}{\beta_b}, \frac{\beta_a}{\sigma_b})$$

Definition 8: An alternative way of fuzzy arithmetic can be defined based on interval arithmetic of α-level intervals. The interval arithmetic If $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with α-level intervals $\tilde{A}^{\alpha} = [\tilde{A}^{\alpha L}, \tilde{A}^{\alpha U}]$ and $\tilde{B}^{\alpha} = [\tilde{B}^{\alpha L}, \tilde{B}^{\alpha U}]$ then the definition (7) can be achieved as follows;

$\tilde{A}^{\alpha}(+)\tilde{B}^{\alpha} = [\tilde{A}^{\alpha L} + \tilde{B}^{\alpha L}, \tilde{A}^{\alpha U} + \tilde{B}^{\alpha U}]$

$\tilde{A}^{\alpha}(-)\tilde{B}^{\alpha} = [\tilde{A}^{\alpha L} - \tilde{B}^{\alpha L}, \tilde{A}^{\alpha U} - \tilde{B}^{\alpha U}]$

$$\tilde{A}^{\alpha}(\times)\tilde{B}^{\alpha} = \begin{bmatrix} min\{\tilde{A}^{\alpha L}\,\tilde{B}^{\alpha L}, \tilde{A}^{\alpha L}\tilde{B}^{\alpha U}, \tilde{A}^{\alpha U}\tilde{B}^{\alpha L}, \tilde{A}^{\alpha U}\tilde{B}^{\alpha U}\}, \\ max\{\tilde{A}^{\alpha L}\,\tilde{B}^{\alpha L}, \tilde{A}^{\alpha L}\tilde{B}^{\alpha U}, \tilde{A}^{\alpha U}\tilde{B}^{\alpha L}, \tilde{A}^{\alpha U}\tilde{B}^{\alpha U}\} \end{bmatrix}$$

$(\tilde{A}^{\alpha})^{-1} = \left[\frac{1}{\tilde{A}^{\alpha L}}, \frac{1}{\tilde{A}^{\alpha U}}\right]$, $(\tilde{B}^{\alpha})^{-1} = \left[\frac{1}{\tilde{B}^{\alpha L}}, \frac{1}{\tilde{B}^{\alpha U}}\right]$

$$\tilde{A}^{\alpha}(\div)\tilde{B}^{\alpha} = \tilde{A}(\times)\tilde{B}^{-1} = \tilde{A}^{\alpha}(\times)\frac{1}{\tilde{B}^{\alpha}}$$

Definition 9: The minimum t-norm is usually applied in fuzzy linear programming to assess a linear combination of fuzzy quantities. Therefore, a given set of trapezoidal fuzzy numbers $\tilde{x_j} = (x_{0j}\,, y_{0j}\,, \sigma_j\,, \beta_j)$ j= 1,2,…….,n and $\lambda_j \geq 0$, $\sum_{j=1}^{n} \lambda_j \tilde{x_j}$ are defined as follows:

$$\sum_{j=1}^{n} \lambda_j \tilde{x_j} = \left(\sum_{j=1}^{n} \lambda_j x_{0j}\,, \sum_{j=1}^{n} \lambda_j y_{0j}\,, \sum_{j=1}^{n} \lambda_j \sigma_j\,, \sum_{j=1}^{n} \lambda_j \beta_j\,,\right) \qquad (2.8)$$

Where $\sum_{j=1}^{n} \lambda_j \tilde{x}_j$ denotes the combination $\lambda_1 \tilde{x}_1 \oplus \lambda_2 \tilde{x}_2 \oplus \dots \oplus \lambda_n \tilde{x}_n$.

A trapezoidal fuzzy number is widely used for solving practical problems. Hence, for the sake of simplicity and without loss of generality, we assume that all fuzzy numbers used throughout in this section are trapezoidal fuzzy numbers.

2.3.2. Fuzzy set theory

One way to describe the vagueness and lack of precision of data is fuzzy sets theory which was introduced by (Zadeh 1965). Here some of the definitions of this theory are presented (Lee et al. 2005, Bellman and Zadeh 1970).

Definition 1: Fuzzy set: If X is a collection of objects denoted by x, then a fuzzy set $\tilde{x}$ in X is a set of ordered pairs;

$\tilde{x} = \{(x;\mu_{\tilde{x}}(x))|x \in X\}$

that, $\mu_{\tilde{x}}(x)$ is called the membership function which for each $x \in X$ associates a number in [0,1], indicating to what degree x is a member of X.

Definition 2: Convex fuzzy set: A fuzzy set $\tilde{x}$ is convex if

$$\mu_{\tilde{x}}(\lambda_{x1}+(1-\lambda)x_2) \geq \min\{ \mu_{\tilde{x}}(x_1), \mu_{\tilde{x}}(x2)\}; x_1, x_2 \in X ; \lambda \in [0;1] \quad (2.9)$$

Definition 3: Normal fuzzy set: A fuzzy set $\tilde{x}$ in X is said to be normal if there exist $x \in X$ such that $\mu_{\tilde{x}}(x) =1$.

Definition 4:Positive fuzzy number: A fuzzy number $\tilde{x}$ is called positive (negative), denoted by $\tilde{x} > 0$ ($\tilde{x} < 0$), if its membership function , $\mu_{\tilde{x}}(x)$ satisfies , $\mu_{\tilde{x}}(x) = 0$, $x < 0$ ($x > 0$).

Definition 5: LR fuzzy number: A fuzzy number $\tilde{x}$ is said to be LR if

$$\mu_{\tilde{x}}(x) = \begin{cases} L\left(\frac{a-x}{\sigma}\right) & x < a, \sigma > 0 \\ \\ R\left(\frac{x-b}{\beta}\right) & x > b, \beta > 0 \end{cases} \tag{2.10}$$

Where σ and β are left and right spreads, respectively, and a function L(.) is the left shape function satisfying;

1. L(x)=L(-x).
2. L(0)=1 and L(1)=0.
3. L(x) is non-decreasing on [0;∞).

Naturally, a right shape function R(.) is similarly defined as L(.).

Definition 6: LR fuzzy interval: A fuzzy set $\tilde{x}$ is said to be an LR fuzzy interval if

$$\mu_{\tilde{x}}(x) = \begin{cases} L\left(\frac{a-x}{\sigma}\right) & x < a, \sigma > 0 \\ 1 & a \leq x \leq b \\ R\left(\frac{x-b}{\beta}\right) & x > b, \beta > 0 \end{cases} \tag{2.11}$$

where [a;b] is the peak or core of $\tilde{x}$ and a and b are left and right spreads, respectively, and the functions L(.) and R(.) are the same as the functions of LR fuzzy number.

Definition 7: Triangular fuzzy number: A LR fuzzy number $\tilde{x}$ is said to be triangular if L(.) and R(.) be linear functions.

Remark: A membership function of triangular fuzzy number like figure 2, $\tilde{x}$ = (L; M; R) (L ≤ M ≤ R) is as follows;

$$\mu_{\tilde{x}}(x) = \begin{cases} \left(\frac{x-L}{M-L}\right) & L \leq x < M \\ 1 & x = M \\ \left(\frac{x-R}{M-R}\right) & M < x \leq R \end{cases} \tag{2.12}$$

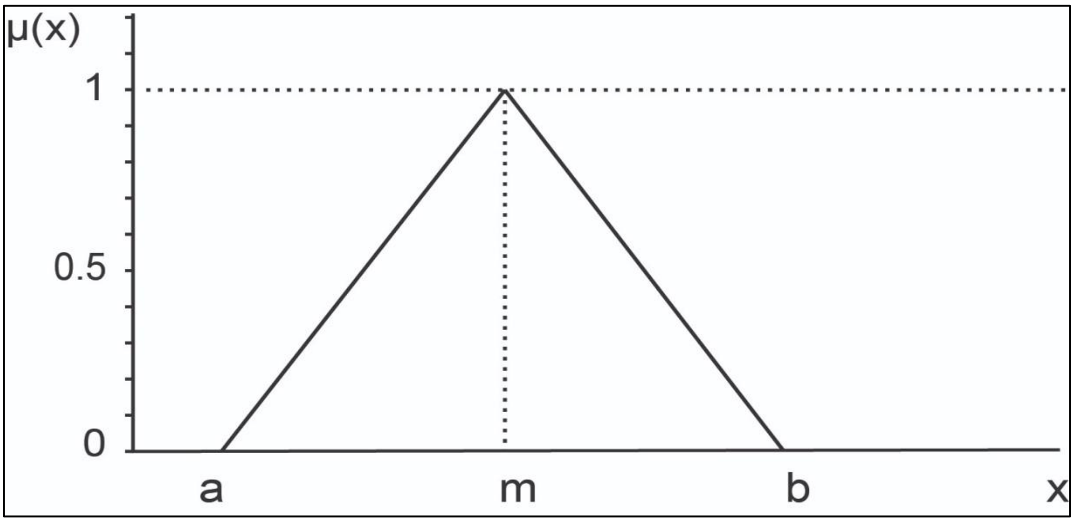

Figure 2.2. Membership function of triangular fuzzy number

Definition 8: Trapezoidal fuzzy number: $\tilde{x} = (x_0, y_0, \sigma, \beta)$ with two defuzzifier x_0, y_0, and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membership function like (Figure 2.3) as;

$$\mu_{\tilde{x}}(x) = \begin{cases} \frac{1}{\sigma}(x - x_0 + \sigma) & x_0 - \sigma \le x \le x_0 \\ 1 & x \in [x_0, y_0] \\ \frac{1}{\beta}(y_0 - x + \beta) & y_0 \le x \le y_0 + \beta \\ 0 & \text{otherwise} \end{cases} \tag{2.13}$$

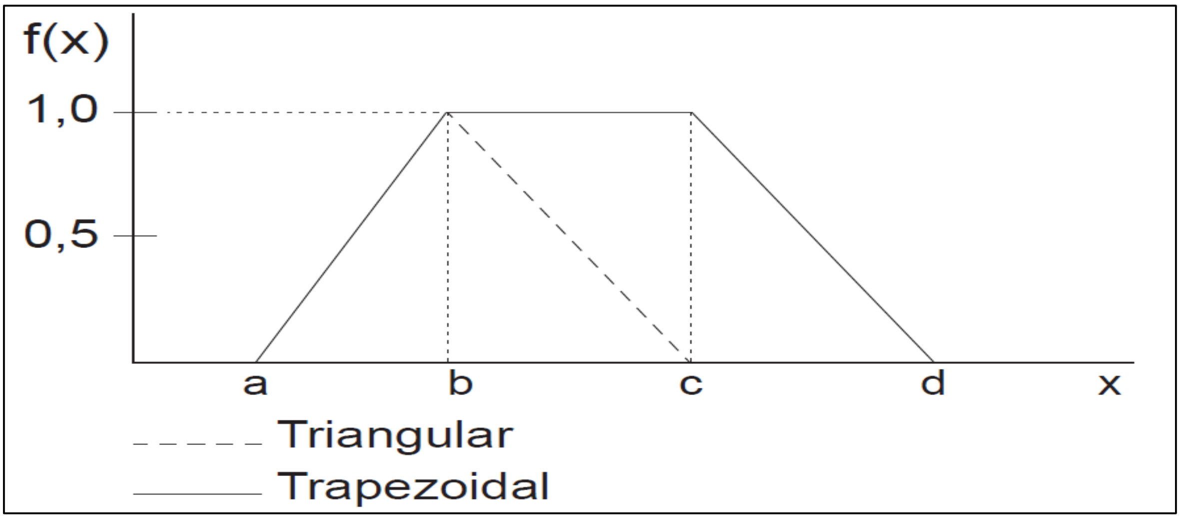

Figure 2.3. Membership function of trapezoidal fuzzy number

2.3.3. Fuzzy decision making

Decision making is defined as a choice between uncertain future alternatives. It is an option between different ways to accomplish the end. Decision-making plays an important role in business, finance, economics as well as in engineering, social, physical and medical sciences. It must be emphasized that all decision-making decisions relate to the future. Where there are no alternatives, no decision can be taken. It is a difficult process due to factors such as incomplete information, inaccurate information and uncertainty. These factors show that decisions are made in a foggy logical environment. Decision-making is characterized by the selection of available alternatives. In this specific process, objectives must be met with constraints in mind.

Multi-criteria decision making has been one of the fastest growing areas of the past decades, depending on changes in business and the economy. Decision maker need a decision help to make a decision among the available alternatives. The speed of trade-off between alternatives is higher. With the help of computers, decision-making methods have been widely accepted in all areas of decision-making. Since multi-standard decision-making has been accepted in the fields of operational research, management science and economics, discipline has established many methodologies. Especially in recent years, where the use of computer significantly increased. Application of MCDM methods has become much easier for decision-making users since the application of most methods is in line with complex mathematics. In the various multiple alternative problems of multiple decisions, the main concern of decision makers assistance is the following;

1. choose the most preferred alternative to the decision maker,
2. choosing the most realistic alternative according to the data of the decision maker,
3. alternatives to ranking in terms of importance to selection problems,
4. alternatives to screening for final decision.

The main steps of multiple criteria decision making are the following;

a. establishing system evaluation criteria that relate system capabilities to goals,
b. developing alternative systems for attaining the goals,
c. evaluating alternatives in terms of criteria,
d. applying a normative multicriteria analysis method,
e. accepting one alternative as optimal,

f. if the final solution is not accepted, gather new information and go into the next iteration of multicriteria optimization.

2.3.4. Fuzzy decision making conceptual model

The fuzzy decision-making process was represented in a four-level decision making (Figure 2.4), in which the first layer consisted of n fuzzy inference systems (FISs) for estimating the probabilities of a given the outputs of this laye. When defining the input variables, we used the decision maker considered to be central to the decisions they made. The second layer is minimal operational viability which opserved the actual inputs, and thus check maximum achivity in the third layer. Then in the fourth layer the decision maker will then have actual inputs (targets), and give feedback to the center of decision-making units.

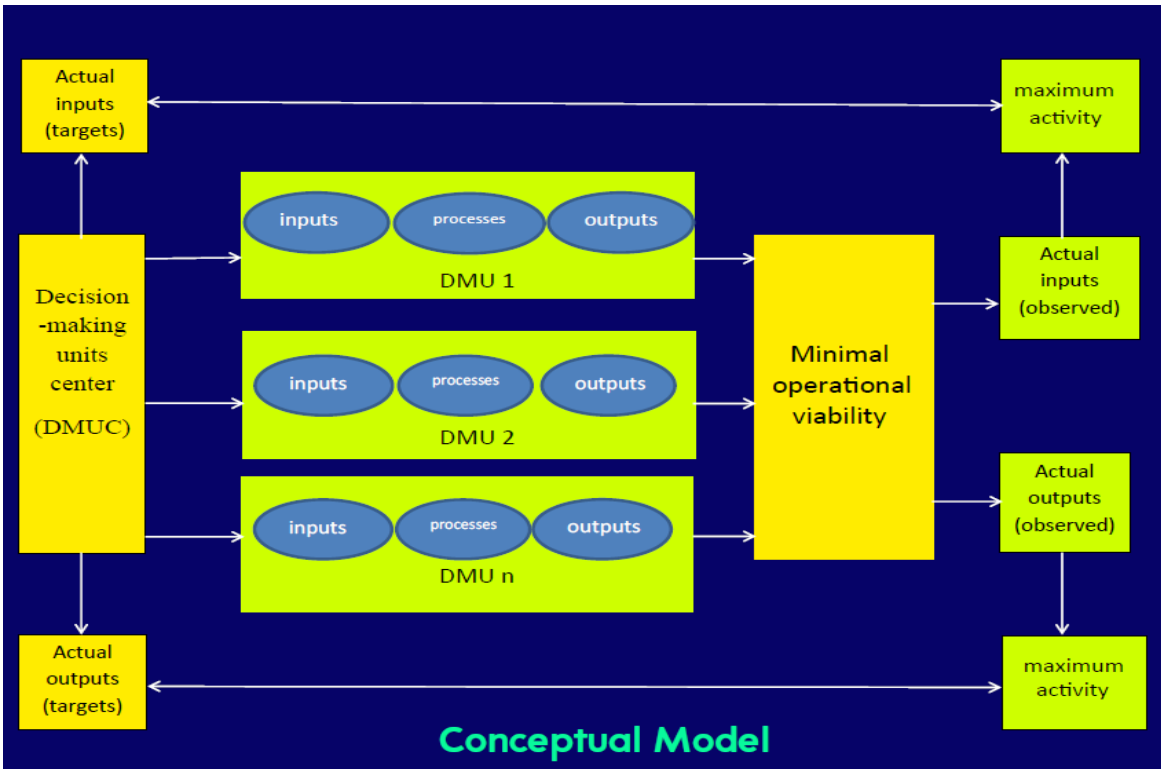

Figure 2.4. Fuzzy decision making conceptual model

3. FUZZY DATA ENVELOPMENT ANALYSIS MODEL

3.1. Fuzzy CCR Model (Input-Oriented Model)

We consider it to be a fuzzy input-oriented CCR model. Suppose there are DMUs with each input and output of the same type. All inputs and outputs are supposed to be non-negative, but there is at least one positive input and product. The CCR model is given with ambiguous and binary variables in equations (3.1) (3.2).

$$FCCR_{P}\ model \quad max_{u,v} \quad v^T \tilde{y}_0$$

$$\begin{aligned} s.t. \quad & u^T \tilde{x}_0 = 1 \\ & -u^T \tilde{x} + v^T \tilde{y} \leq O \\ & u \geq o \\ & v \geq o \end{aligned} \tag{3.1}$$

$$FCCR_D\ \ model \quad min_{\theta_C,\mu} \quad \theta_C$$

$$\begin{aligned} s.t.\ \ & \theta_C \tilde{x}_0 - \tilde{x}\mu \geq 0 \\ & \tilde{y}\mu \geq \tilde{y}_0 \\ & \mu \geq o \end{aligned} \tag{3.2}$$

The model CCR fuzzy alternative to evaluate the efficiency of a group of DMUs with input and output fuzzy derived from α cutting approach proposed by Saati et al. (2002) and Hatami et al. (2011). Let's consider n DMUs, each of which uses a different fuzzy input (m) to generate different fuzzy output (s). Is written on the form model (1.4) the standard model of the CCR model to assess DMUs with typical fuzzy CCR model with fuzzy data can be expressed as the following:

max $\sum_{r=1}^{s} u_r \, \tilde{y}_{rp}$

s.t.

$$\begin{aligned} & \textstyle\sum_{i=1}^{m} v_i \, \tilde{x}_{ip} = 1 \\ & \textstyle\sum_{r=1}^{s} u_r \, \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \, \tilde{x}_{ij} \ \leq 0 \qquad J \neq P \\ & u_r \, , v_i \ \geq 0 \quad \forall r \, , i \end{aligned} \tag{3.3}$$

where $\tilde{y}_{rj} = (x_{rj}, y_{rj}, \sigma_{rj}, \beta_{rj})$ and $\tilde{x}_{ij} = (x_{ij}, y_{ij}, \sigma_{ij}, \beta_{ij})$, are the rth fuzzy output and ith fuzzy input values of the jth DMU, respectively, are characterized as trapezoidal fuzzy numbers. Therefore, model (3.3) can be rewritten as follows:

$$\max \quad \sum_{r=1}^{s} u_r \, (x_{rp}, y_{rp}, \sigma_{rp}, \beta_{rp})$$

s.t.

$$\begin{aligned} &\sum_{i=1}^{m} v_i \, (x_{ip}, y_{ip}, \sigma_{ip}, \beta_{ip}) = 1 \\ &\sum_{r=1}^{s} u_r \, (x_{rj}, y_{rj}, \sigma_{rj}, \beta_{rj}) - \sum_{i=1}^{m} v_i \, (x_{ij}, y_{ij}, \sigma_{ij}, \beta_{ij}) \leq 0 \qquad J \neq P \\ &u_r, v_i \geq 0 \quad \forall r, i \end{aligned} \tag{3.4}$$

The right-hand side of the first constraint in Models (3.4) must be equal to 1 because of the normalization of the efficiency scores of the DMUs.

For all DMUs as well as for the measures of both the lower and apper bound efficiencies (Ying-Ming et al. 2005):

Let

$$E_j = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}}, \text{ j=1,2,..........,n}$$

$$E_j = \frac{\sum_{r=1}^{s} u_r [\tilde{y}_{rj}{}^{L}; \tilde{y}_{rj}{}^{U}]}{\sum_{i=1}^{m} v_i [\tilde{x}_{ij}{}^{L}; \tilde{x}_{ij}{}^{U}]} = \frac{[\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}; \sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{U}]}{[\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L}; \sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}]}$$

$$= \left[\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}}, \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{U}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L}} \right], \text{ j=1,2,..........,n} \tag{3.5}$$

Then

$$\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{U}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L}} \leq 1, \text{ j=1,2,..........,n}$$

$$\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}} > 0, \text{ j=1,2,..........,n}$$

3.2. Fuzzy BCC Model (Output-Oriented Model)

We consider it to be a fuzzy output-oriented BCC model. Suppose there are DMUs with each input and output of the same type. All inputs and outputs are supposed to be non-negative, but there is at least one positive input and product. The BCC model is given with ambiguous and binary variables in equations (3.6) (3.7).

$$FBCC_{\ P}\ model \quad max_{u,v} \quad v^T\tilde{y}_0 - u_0$$

$$\begin{aligned} \text{s.t} \quad & u^T\tilde{x}_0 = 1 \\ & -u^T\tilde{x} + v^T\tilde{y} - e^Tu_0 \leq O \\ & u \geq o \\ & v \geq o \end{aligned} \tag{3.6}$$

$$FBCC_D\ \ model \quad min_{\theta_B,\mu} \qquad \theta_B$$

$$\begin{aligned} s.t. \quad & \theta_B\tilde{x}_0 - \tilde{x}\mu \geq 0 \\ & \tilde{y}\mu \geq \tilde{y}_0 \\ & e^T\mu = 1 \\ & \mu \geq o \end{aligned} \tag{3.7}$$

An alternative fuzzy BCC model for evaluating the efficiency of a set of DMUs with fuzzy inputs and outputs derived from the α-level approach proposed by Saati et al. (2002) and Marbini et al (2012). Let us consider *n* DMUs, each of which uses *m* different fuzzy inputs to generate *s* different fuzzy outputs. The model (1.5) standard form of BCC model for assessing DMUs is written with typical fuzzy BCC model with fuzzy data can be expressed as.

$$\max \quad w_p = \frac{\sum_{r=1}^{s} u_r\, \tilde{y}_{rp} - u_0}{\sum_{i=1}^{m} v_i\, \tilde{x}_{ip}}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r\tilde{y}_{rp} - u_0}{\sum_{i=1}^{m} v_i\tilde{x}_{ij}} \leq 1\ , \forall j \tag{3.8}$$

$$u_r\ , v_i\ \geq 0 \qquad \forall r\ , i$$

Therefore, model (3.8) can be rewritten as follows:

$$\max \quad w_p = \frac{\sum_{r=1}^{s} u_r\,(x_{rp}\ , y_{rp}\,, \sigma_{rp}\ , \beta_{rp}) - u_0}{\sum_{i=1}^{m} v_i\,(x_{ip}\ , y_{ip}\,, \sigma_{ip}\ , \beta_{ip})}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r(x_{rj}\ , y_{rj}\,, \sigma_{rj}\ , \beta_{rj}) - u_0}{\sum_{i=1}^{m} v_i(x_{ij}\ , y_{ij}\,, \sigma_{ij}\ , \beta_{ij})} \leq 1\ , \forall j \tag{3.9}$$

$$u_r\ , v_i\ \geq 0 \qquad \forall r\ , i$$

The right-hand side of the first constraint in model (3.8) must be equal to 1 because of the normalization of the efficiency scores of the DMUs.

For all DMUs as well as for the measures of both the lower and apper bound efficiencies (Ying-Ming et al. 2005).

Let

$$\mathrm{E_j} = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} - u_0^*\ , \text{ j=1,2,..........,n}$$

$$\mathrm{E_j} = \frac{\sum_{r=1}^{s} u_r [\tilde{y}_{rj}{}^{L}; \tilde{y}_{rj}{}^{U}]}{\sum_{i=1}^{m} v_i [\tilde{x}_{ij}{}^{L}; \tilde{x}_{ij}{}^{U}]} - u_0^* = \frac{[\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}; \sum_{r=1}^{s} u_r\, \tilde{y}_{rj}{}^{U}]}{[\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L};\ \sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}]} - u_0^*$$

$$= \left[\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}} - u_0^*, \frac{\sum_{r=1}^{s} u_r\, \tilde{y}_{rj}{}^{U}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L}} - u_0^*\right], \text{ j=1,2,..........,n} \tag{3.10}$$

should also be an interval number, which we denote by $\left[\mathrm{E_j}^{L}, \mathrm{E_j}^{U}\right] \subseteq (0,1]$, j= 1,……,n.

Then

$$\frac{\sum_{r=1}^{s} u_r\, \tilde{y}_{rj}{}^{U}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L}} \leq 1\ , \text{ j=1,2,..........,n}$$

$$\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}} > 0\ , \text{ j=1,2,..........,n}$$

Hence ,the score of DMU0.

4. INTERVAL AND PARAMETRIC POSSIBILITY APPROACH TO FUZZY DEA

4.1. Interval Programming Approch

We develop the fuzzy CCR model in order to measure the crisp efficiency of each DMU under consideration for different α values ($\alpha -$ cuts *)*. for all DMUs as well as for the measures of both the lower and upper bound efficiencies. We can applying the $\alpha -$ cuts of fuzzy DEA, Using the equations (2.4-2.6) in model (3.4), ($\alpha -$ cuts *)*FCCR model the following model would be achieved:

$$\max \ E = \ \sum_{r=1}^{s} u_r \, [\tilde{y}_{rp}{}^{\alpha L}; \tilde{y}_{rp}{}^{\alpha U}]$$

s.t.

$$\sum_{i=1}^{m} v_i \, [\tilde{x}_{ip}{}^{\alpha L}; \tilde{x}_{ip}{}^{\alpha U}] \ = 1$$

$$\sum_{r=1}^{s} u_r [\tilde{y}_{rj}{}^{\alpha L}; \tilde{y}_{rj}{}^{\alpha U}] - \sum_{i=1}^{m} v_i \, [\tilde{x}_{ij}{}^{\alpha L}; \tilde{x}_{ij}{}^{\alpha U}] \ \leq 0 \qquad J \neq P \tag{4.1}$$

$$u_r \, , v_i \ \geq 0 \qquad \forall r \, , i$$

Now interval DEA model is developed for measuring the upper lower and bounds of the best relative efficiency of each DMU with interval input and output data Oday and Hasan (2017).

$$\max \ E^{\alpha U} = \ \sum_{r=1}^{s} u_r \, [\tilde{y}_{rp}{}^{\alpha U}]$$

s.t.

$$\sum_{i=1}^{m} v_i \, [\tilde{x}_{ip}{}^{\alpha L}] \ = 1$$

$$\sum_{r=1}^{s} u_r [\tilde{y}_{rj}{}^{\alpha U}] - \sum_{i=1}^{m} v_i \, [\tilde{x}_{ij}{}^{\alpha L}] \ \leq 0$$

$$\sum_{r=1}^{s} u_r [\tilde{y}_{rj}{}^{\alpha L}] - \sum_{i=1}^{m} v_i \left[\tilde{x}_{ij}{}^{\alpha U}\right] \ \leq 0 \qquad j = 1,2, \dots, n \quad j \neq 0 \tag{4.2}$$

$$u_r \, , v_i \ \geq 0 \qquad \forall r \, , i$$

$$\max \ E^{\alpha L} = \ \sum_{r=1}^{s} u_r \, [\tilde{y}_{rp}{}^{\alpha L}]$$

s.t.

$$\sum_{i=1}^{m} v_i\,[\tilde{x}_{ip}{}^{\alpha U}] = 1$$
$$\sum_{r=1}^{s} u_r[\tilde{y}_{rj}{}^{\alpha L}] - \sum_{i=1}^{m} v_i\,[\tilde{x}_{ij}{}^{\alpha U}] \le 0$$
$$\sum_{r=1}^{s} u_r[\tilde{y}_{rj}{}^{\alpha U}] - \sum_{i=1}^{m} v_i\,\left[\tilde{x}_{ij}{}^{\alpha L}\right] \le 0 \quad j = 1,2,\ldots,n \quad j \ne 0 \tag{4.3}$$
$$u_r\,, v_i \ge 0 \quad \forall r\,, i$$

Theorem 4.1.1. For every α , $E^{\alpha L} \le E^{\alpha U}$

Theorem 4.1.2. If $\alpha_1 \le \alpha_2$ then $[E^{\alpha_2\,L}, E^{\alpha_2\,U}] \subseteq [E^{\alpha_1 L}, E^{\alpha_1 U}]$

where $E^{\alpha U}$stands for the best possible relative efficiency achieved by DMU0 when all the DMUs are in the state of best production activity, while $E^{\alpha L}$ stands for the lower bound of the best possible relative efficiency of DMU_0. They constitute a possible best relative efficiency interval $[E^{\alpha L}, E^{\alpha U}]$. Note that model (4.2) determines the production frontier as a benchmark to measure the upper bound for all the DMUs and model (4.3) uses the production frontier as a benchmark to measure the lower bound efficiency of each DMUs.

are characterized as trapezoidal fuzzy numbers. Therefore, model (4.2) and (4.3) can be rewritten as follows:

$$\max \;\; E^{\alpha U} = \sum_{r=1}^{s} u_r\,(x_{rp}\,, y_{rp}\,, \sigma_{rp}\,, \beta_{rp})^{\alpha U}$$

s.t.

$$\sum_{i=1}^{m} v_i\,(x_{ip}\,, y_{ip}\,, \sigma_{ip}\,, \beta_{ip})^{\alpha L} = 1$$
$$\sum_{r=1}^{s} u_r\,(x_{rj}\,, y_{rj}\,, \sigma_{rj}\,, \beta_{rj})^{\alpha U} - \sum_{i=1}^{m} v_i\,(x_{ij}\,, y_{ij}\,, \sigma_{ij}\,, \beta_{ij})^{\alpha L} \le 0$$
$$\sum_{r=1}^{s} u_r\,(x_{rj}\,, y_{rj}\,, \sigma_{rj}\,, \beta_{rj})^{\alpha L} - \sum_{i=1}^{m} v_i\,(x_{ij}\,, y_{ij}\,, \sigma_{ij}\,, \beta_{ij})^{\alpha U} \le 0$$
$$j = 1,2,\ldots,n \quad j \ne 0 \tag{4.4}$$
$$u_r\,, v_i \ge 0 \quad \forall r\,, i$$

$$\max \;\; E^{\alpha L} = \sum_{r=1}^{s} u_r\,(x_{rp}\,, y_{rp}\,, \sigma_{rp}\,, \beta_{rp})^{\alpha L}$$

s.t.

$$\sum_{i=1}^{m} v_i\,(x_{ip}\;, y_{ip}\,, \sigma_{ip}\;, \beta_{ip})^{\alpha U}\;=1$$
$$\sum_{r=1}^{s} u_r\,(x_{rj}\;, y_{rj}\,, \sigma_{rj}\;, \beta_{rj})^{\alpha L} - \sum_{i=1}^{m} v_i\,(x_{ij}\;, y_{ij}\,, \sigma_{ij}\;, \beta_{ij})^{\alpha U}\;\leq 0 \tag{4.5}$$
$$\sum_{r=1}^{s} u_r\,(x_{rj}\;, y_{rj}\,, \sigma_{rj}\;, \beta_{rj})^{\alpha U} - \sum_{i=1}^{m} v_i(x_{ij}\;, y_{ij}\,, \sigma_{ij}\;, \beta_{ij})^{\alpha L}\;\leq 0$$
$$j = 1,2, \ldots, n \quad j \neq 0$$
$$u_r\,, v_i\;\geq 0 \quad \forall r\,, i$$

In model (4.4) and (4.5), the Trading and rotation variables :

$u_r(x_{rp}\;, y_{rp}\,, \sigma_{rp}\;, \beta_{rp})^{\alpha} = \bar{y}_{rp}$ (r = 1, 2, . . . , s, p = 1, 2, . . . , n)

, $v_i(x_{ip}\;, y_{ip}\,, \sigma_{ip}\;, \beta_{ip})^{\alpha} = \bar{x}_{ip}$ (i = 1, 2, . . . , m, p = 1, 2, . . . , n),

$u_r(x_{rj}\;, y_{rj}\,, \sigma_{rj}\;, \beta_{rj})^{\alpha} = \bar{y}_{rj}$ (r = 1, 2, . . . , s, j = 1, 2, . . . , n) and (4.6)

$v_i(x_{ij}\;, y_{ij}\,, \sigma_{ij}\;, \beta_{ij})^{\alpha} = \bar{x}_{ij}$ (i = 1, 2, …, m, j = 1, 2, …, n)

are introduced to receive the following (α − cuts *)*FCCR proposed model:

$$\max\;\; E^{\alpha U} = \;\sum_{r=1}^{s} \bar{y}_{rp}{}^{U}$$

s.t.

$$\sum_{i=1}^{m} \bar{x}_{ip}{}^{L}\;=1$$
$$\sum_{r=1}^{s} \bar{y}_{rj}{}^{U}\; - \sum_{i=1}^{m} \bar{x}_{ij}{}^{L}\quad \leq 0$$
$$\sum_{r=1}^{s} \bar{y}_{rj}{}^{L} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{U}\;\leq 0 \qquad j = 1,2, \ldots, n \quad j \neq 0 \tag{4.7}$$
$$u_r\,, v_i\;\geq 0 \quad \forall r\,, i$$

$$\max\;\; E^{\alpha L} = \;\sum_{r=1}^{s} \bar{y}_{rp}{}^{L}$$

s.t.

$$\sum_{i=1}^{m} \bar{x}_{ip}{}^{U}\;=1$$
$$\sum_{r=1}^{s} \bar{y}_{rj}{}^{L} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{U}\;\leq 0$$
$$\sum_{r=1}^{s} \bar{y}_{rj}{}^{U} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{L}\;\leq 0 \qquad j = 1,2, \ldots, n \quad j \neq 0 \tag{4.8}$$
$$u_r\,, v_i\;\geq 0 \quad \forall r\,, i$$

Model (4.7) and (4.8) is equivalent to a interval programming model with α ∈ [0, 1].

Therefore, analyzing the efficiency of DMUs with the proposed method, for a set of n different values of α, e.g. αi, i = 1, 2, ..., n results in a set of efficiency α-cuts if $E^{\alpha U}$ *and* $E^{\alpha L}$ = 1 in models (4.7) and (4.8) respectively. Therefore, it will be necessary to obtain an integrated efficiency score for DMUs to rank them. the fuzzy linear programming problem given by (4.7) and (4.8) is equivalent to a crisp parametric linear programming problem. Using the optimal value, we can determine the situation for returns to scale RTS for BCC and CCR when a DMUp is efficient. Similar to the conventional DEA model of Banker and Thrall (1992).

from (3.5) the lower and upper bounds of the efficiency score for a given α can be reduced as:

$$E_j = \left[\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{L}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{U}}, \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}{}^{U}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}{}^{L}}\right]$$

We develop the fuzzy BCC model in order to measure the crisp efficiency of each DMU under consideration for different α values (α – cut). for all DMUs as well as for the measures of both the lower and upper bound efficiencies. And we can applying the α – cuts of fuzzy DEA, Using the equations (2.4-2.6) in model (3.8) we get (α – cuts) FBCC the following model would be achieved:

$$\max \quad E_\alpha = \frac{\sum_{r=1}^{s} u_r \left[\tilde{y}_{rp}{}^{\alpha L}; \tilde{y}_{rp}{}^{\alpha U}\right] - u_0}{\sum_{i=1}^{m} v_i \left[\tilde{x}_{ip}{}^{\alpha L}; \tilde{x}_{ip}{}^{\alpha U}\right]}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r \left[\tilde{y}_{rj}{}^{\alpha L}; \tilde{y}_{rj}{}^{\alpha U}\right] - u_0}{\sum_{i=1}^{m} v_i \left[\tilde{x}_{ij}{}^{\alpha L}; \tilde{x}_{ij}{}^{\alpha U}\right]} \leq 1, \forall j \tag{4.9}$$

$$u_r, v_i \geq 0 \quad \forall r, i$$

Now interval DEA model is developed for measuring the upper and lower bounds of the best relative efficiency of each DMU with interval input andoutput data.

$$\max \; E^{\alpha U} = \sum_{r=1}^{s} u_r \left[\tilde{y}_{rp}{}^{\alpha U}\right] - u_0$$

s.t.

$\sum_{i=1}^{m} v_i \, [\tilde{x}_{ip}{}^{\alpha L}] \ = 1$

$\sum_{r=1}^{s} u_r[\tilde{y}_{rj}{}^{\alpha U}] - \sum_{i=1}^{m} v_i \left[\tilde{x}_{ij}{}^{\alpha L}\right] - \ u_0 \ \le 0$

$\sum_{r=1}^{s} u_r[\tilde{y}_{rj}{}^{\alpha L}] - \sum_{i=1}^{m} v_i \left[\tilde{x}_{ij}{}^{\alpha U}\right] - \ u_0 \ \le 0 \quad j = 1,2, \ldots, n \quad j \ne 0$ (4.10)

$u_r \, , v_i \ \ge 0 \quad \forall r \, , i$

max $E^{\alpha L} = \ \sum_{r=1}^{s} u_r \, [\tilde{y}_{rp}{}^{\alpha L}]$ - u_0

s.t.

$\sum_{i=1}^{m} v_i \, [\tilde{x}_{ip}{}^{\alpha U}] \ = 1$

$\sum_{r=1}^{s} u_r[\tilde{y}_{rj}{}^{\alpha L}] - \sum_{i=1}^{m} v_i \left[\tilde{x}_{ij}{}^{\alpha U}\right] - \ u_0 \ \le 0$

$\sum_{r=1}^{s} u_r[\tilde{y}_{rj}{}^{\alpha U}] - \sum_{i=1}^{m} v_i \left[\tilde{x}_{ij}{}^{\alpha L}\right] - \ u_0 \ \le 0 \quad j = 1,2, \ldots, n \quad j \ne 0$ (4.11)

$u_r \, , v_i \ \ge 0 \quad \forall r \, , i$

In model (4.9) and (4.10), the Rotated variables:

$u_r(x_{rp} \, , y_{rp} \, , \sigma_{rp} \, , \beta_{rp})^{\alpha} = \bar{y}_{rp}$ (r = 1, 2, . . . , s, p = 1, 2, . . . , n),

$v_i(x_{ip} \, , y_{ip} \, , \sigma_{ip} \, , \beta_{ip})^{\alpha} = \bar{x}_{ip}$ (i = 1, 2, . . . , m, p = 1, 2, . . . , n),

$u_r(x_{rj} \, , y_{rj} \, , \sigma_{rj} \, , \beta_{rj})^{\alpha} = \bar{y}_{rj}$ (r = 1, 2, . . . , s, j = 1, 2, . . . , n) and (4.12)

$v_i(x_{ij} \, , y_{ij} \, , \sigma_{ij} \, , \beta_{ij})^{\alpha} = \bar{x}_{ij} \ \ (i \ = \ 1, 2, \ldots, m, j = 1, 2, \ldots, n)$

are introduced to receive the following (α – cuts) FBCC proposed model:

max $E^{\alpha U} = \ \sum_{r=1}^{s} \bar{y}_{rp}{}^{U}$ - u_0

s.t.

$\sum_{i=1}^{m} \bar{x}_{ip}{}^{L} \ = 1$

$\sum_{r=1}^{s} \bar{y}_{rj}{}^{U} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{L} - \ u_0 \ \le 0$

$\sum_{r=1}^{s} \bar{y}_{rj}{}^{L} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{U} - \ u_0 \ \le 0 \quad j = 1,2, \ldots, n \quad j \ne 0$ (4.13)

$u_r \, , v_i \ \ge 0 \quad \forall r \, , i$

$$\max \ E^{\alpha L} = \sum_{r=1}^{s} \bar{y}_{rp}{}^{L} - u_0$$

s.t.

$$\sum_{i=1}^{m} \bar{x}_{ip}{}^{U} = 1$$
$$\sum_{r=1}^{s} \bar{y}_{rj}{}^{L} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{U} - u_0 \le 0$$
$$\sum_{r=1}^{s} \bar{y}_{rj}{}^{U} - \sum_{i=1}^{m} \bar{x}_{ij}{}^{L} - u_0 \le 0 \quad j = 1,2,\ldots,n \quad j \ne 0 \tag{4.14}$$
$$u_r, v_i \ge 0 \quad \forall r, i$$

The fuzzy linear programming problem given by (4.13) and (4.14) is equivalent to a crisp parametric linear programming problem. Using the optimal value, we can determine the situation for returns to scale RTS when a DMUp is efficient. Similar to the conventional DEA model of Banker and Thrall (1992).

from (3.10) the interval lower and upper bounds of the efficiency score for a given α can be reduced as:

$$E_j = \left[\frac{\sum_{r=1}^{s} u_r^* \left[\alpha x_{rp} + (1-\alpha)\sigma_{rp}\right]}{\sum_{i=1}^{m} v_i^* \left[\alpha y_{ip} + (1-\alpha)\beta_{ip}\right]} - u_0^*, \frac{\sum_{r=1}^{s} u_r^* \left[\alpha y_{rp} + (1-\alpha)\beta_{rp}\right]}{\sum_{i=1}^{m} v_i^* \left[\alpha x_{ip} + (1-\alpha)\sigma_{ip}\right]} - u_0^*\right]$$

4.2. Parametric Programming Approch

We develop the fuzzy CCR model in order to measure the crisp efficiency of each DMU under consideration for different α values ($\alpha -$ level *)*. for all DMUs as well as for the measures of both the lower and upper bound efficiencies. And we can applying the $\alpha -$ level of fuzzy DEA, using the equations (2.7) in model (3.4) we get ($\alpha -$level *)* FCCR the following model would be achieved:

$$\max \ \sum_{r=1}^{s} u_r \left[\alpha x_{rp} + (1-\alpha)\sigma_{rp}, \alpha y_{rp} + (1-\alpha)\beta_{rp}\right]$$

s.t.

$$\sum_{i=1}^{m} v_i \left[\alpha x_{ip} + (1-\alpha)\sigma_{ip}, \alpha y_{ip} + (1-\alpha)\beta_{ip}\right] = 1$$
$$\sum_{r=1}^{s} u_r \left[\alpha x_{rj} + (1-\alpha)\sigma_{rj}, \alpha y_{rj} + (1-\alpha)\beta_{rj}\right]$$

$$-\sum_{i=1}^{m} v_i \left[\alpha x_{ij} + (1-\alpha)\sigma_{ij}, \alpha y_{ij} + (1-\alpha)\beta_{ij}\right] \leq 0 \quad J \neq P \qquad (4.15)$$

$$u_r, v_i \geq 0 \quad \forall r, i$$

Where $\tilde{y}_{rj} = (x_{rj}, y_{rj}, \sigma_{rj}, \beta_{rj})$ and $\tilde{x}_{ij} = (x_{ij}, y_{ij}, \sigma_{ij}, \beta_{ij})$, are the rth fuzzy output and ith fuzzy input values of the *j*th DMU, respectively, are characterized as trapezoidal fuzzy numbers.

Model (4.15) is an parametric programming model that can be solved by standard optimization methods. Hence, we transform the interval model (4.15) into a programming model using the following parametric alteration variables:

$$\left[\alpha x_{rp} + (1-\alpha)\sigma_{rp}, \alpha y_{rp} + (1-\alpha)\beta_{rp}\right] = \hat{y}_{rp} \quad \forall r, p$$

$$\left[\alpha x_{ip} + (1-\alpha)\sigma_{ip}, \alpha y_{ip} + (1-\alpha)\beta_{ip}\right] = \hat{x}_{ip} \quad \forall i, p$$

$$\left[\alpha x_{rj} + (1-\alpha)\sigma_{rj}, \alpha y_{rj} + (1-\alpha)\beta_{rj}\right] = \hat{y}_{rj} \quad \forall r, j \qquad (4.16)$$

$$\left[\alpha x_{ij} + (1-\alpha)\sigma_{ij}, \alpha y_{ij} + (1-\alpha)\beta_{ij}\right] = \hat{x}_{ij} \quad \forall i, j$$

The substitutions of the above parametric alteration variables in model (4.15) will result in the following programming model:

$$\max \quad \sum_{r=1}^{s} u_r \hat{y}_{rp}$$

s.t.

$$\sum_{i=1}^{m} v_i \hat{x}_{ip} = 1$$

$$\sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \leq 0 \quad J \neq P \qquad (4.17)$$

$$\alpha x_{rj} + (1-\alpha)\sigma_{rj} \leq \hat{y}_{rj} \leq \alpha y_{rj} + (1-\alpha)\beta_{rj} \quad \forall r, j$$

$$\alpha x_{ij} + (1-\alpha)\sigma_{ij} \leq \hat{x}_{ij} \leq \alpha y_{ij} + (1-\alpha)\beta_{ij} \quad \forall i, j$$

$$u_r, v_i \geq 0 \quad \forall r, i$$

In model (4.15), the alternation variables like (4.18):

$u_r\hat{y}_{rp} = \bar{x}_{rp}$ (r = 1, 2, . . . , s, p = 1, 2, . . . , n),

$v_i\hat{x}_{ip} = \bar{x}_{ip}$ (i = 1, 2, . . . , m, p = 1, 2, . . . , n),

$u_r\hat{y}_{rj} = \bar{x}_{rj}$ (r = 1, 2, . . . , s, j = 1, 2, . . . , n) and (4.18)

$v_i \hat{x}_{ij} = \bar{x}_{ij} \ \ (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$

are introduced to receive the following (α − level)FCCR proposed model:

$$\max \ \ \sum_{r=1}^{s} \bar{y}_{rp}$$

s.t.

$$\begin{aligned}
&\sum_{i=1}^{m} \bar{x}_{ip} = 1 \\
&\sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} \le 0 \qquad J \neq P \qquad\qquad (4.19)\\
&u_r(\alpha x_{rj} + (1-\alpha)\sigma_{rj}) \le \bar{y}_{rj} \le u_r(\alpha y_{rj} + (1-\alpha)\beta_{rj}) \qquad \forall r, j \\
&v_i(\alpha x_{ij} + (1-\alpha)\sigma_{ij}) \le \bar{x}_{ij} \le v_i(\alpha y_{ij} + (1-\alpha)\beta_{ij}) \qquad \forall i, j \\
&u_r, v_i \ge 0 \qquad \forall r, i
\end{aligned}$$

Model (4.19) is equivalent to a parametric programming model with α ∈ [0, 1]. Therefore, Analyzing the efficiency of DMUs with the proposed method, for a set of *n* different values of *α*, e.g. *αi*, *i* = 1, 2, ..., *n* . Therefore, it will be necessary to obtain an integrated efficiency score for DMUs to rank them. the fuzzy linear programming problem given by (4.19) is equivalent to a crisp parametric linear programming problem. Using the optimal value, we can determine the situation for returns to scale RTS for BCC and CCR when a DMUp is efficient. Similar to the conventional DEA model of Banker and Thrall (1992).

from (3.5) the interval of the efficiency score for a given α can be reduced as:

$$E_j = \left[\frac{\sum_{r=1}^{s} u_r^* \left[\alpha x_{rj} + (1-\alpha)\sigma_{rj}\right]}{\sum_{i=1}^{m} v_i^* \left[\alpha y_{ij} + (1-\alpha)\beta_{ij}\right]}, \frac{\sum_{r=1}^{s} u_r^* \left[\alpha x_{rj} + (1-\alpha)\sigma_{rj}\right]}{\sum_{i=1}^{m} v_i^* \left[\alpha x_{ij} + (1-\alpha)\sigma_{ij}\right]}, \frac{\sum_{r=1}^{s} u_r^* \left[\alpha y_{rj} + (1-\alpha)\beta_{rj}\right]}{\sum_{i=1}^{m} v_i^* \left[\alpha y_{ij} + (1-\alpha)\beta_{ij}\right]}, \frac{\sum_{r=1}^{s} u_r^* \left[\alpha y_{rj} + (1-\alpha)\beta_{rj}\right]}{\sum_{i=1}^{m} v_i^* \left[\alpha x_{ij} + (1-\alpha)\sigma_{ij}\right]} \right]$$

We can applying the α − level of fuzzy DEA, Using the equations (2.7) in model (3.8) we get (α – level) FBCC the following model would be achieved:

$$\max \quad w_p = \frac{\sum_{r=1}^{s} u_r \left[\alpha x_{rp} + (1-\alpha)\sigma_{rp}, \alpha y_{rp} + (1-\alpha)\beta_{rp}\right] - u_0}{\sum_{i=1}^{m} v_i \left[\alpha x_{ip} + (1-\alpha)\sigma_{ip}, \alpha y_{ip} + (1-\alpha)\beta_{ip}\right]}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r[\alpha x_{rj} + (1-\alpha)\sigma_{rj}, \alpha y_{rj} + (1-\alpha)\beta_{rj}] - u_0}{\sum_{i=1}^{m} v_i[\alpha x_{ij} + (1-\alpha)\sigma_{ij}, \alpha y_{ij} + (1-\alpha)\beta_{ij}]} \leq 1, \forall j \qquad (4.20)$$

$$u_r, v_i \geq 0 \quad \forall r, i$$

where $\tilde{y}_{rj} = (x_{rj}, y_{rj}, \sigma_{rj}, \beta_{rj})$ and $\tilde{x}_{ij} = (x_{ij}, y_{ij}, \sigma_{ij}, \beta_{ij})$, are the rth fuzzy output and ith fuzzy input values of the *j*th DMU, respectively, are characterized as trapezoidal fuzzy numbers.

Model (4.20) is an parametric programming model that can be solved by standard optimization methods. Hence, we transform the parametric model (4.20) into a programming model using the following parametric alteration variables:

$$\begin{aligned}
&[\alpha x_{rp} + (1-\alpha)\sigma_{rp}, \alpha y_{rp} + (1-\alpha)\beta_{rp}] = \hat{y}_{rp} \quad \forall r, p \\
&[\alpha x_{ip} + (1-\alpha)\sigma_{ip}, \alpha y_{ip} + (1-\alpha)\beta_{ip}] = \hat{x}_{ip} \quad \forall i, p \\
&[\alpha x_{rj} + (1-\alpha)\sigma_{rj}, \alpha y_{rj} + (1-\alpha)\beta_{rj}] = \hat{y}_{rj} \quad \forall r, j \qquad (4.21) \\
&[\alpha x_{ij} + (1-\alpha)\sigma_{ij}, \alpha y_{ij} + (1-\alpha)\beta_{ij}] = \hat{x}_{ij} \quad \forall i, j
\end{aligned}$$

The substitutions of the above parametric alteration variables in model (4.20) will result in the following programming model:

$$\max \quad w_p = \frac{\sum_{r=1}^{s} u_r \hat{y}_{rp} - u_0}{\sum_{i=1}^{m} v_i \hat{x}_{ip}}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r \hat{y}_{rj} - u_0}{\sum_{i=1}^{m} v_i \hat{x}_{ij}} \leq 1, \forall j \qquad (4.22)$$

$$u_r, v_i \geq 0 \quad \forall r, i$$

In model (23), the alternation variables as :

$u_r \hat{y}_{rp} = \bar{y}_{rp}$ (r = 1, 2, . . . , s, p = 1, 2, . . . , n),

$v_i \hat{x}_{ip} = \bar{x}_{ip}$ (i = 1, 2, . . . , m, p = 1, 2, . . . , n),

$u_r \hat{y}_{rj} = \bar{y}_{rj}$ (r = 1, 2, . . . , s, j = 1, 2, . . . , n) and (4.23)

$v_i \hat{x}_{ij} = \bar{x}_{ij}$ $(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$

are introduced to receive the following (α − level)FBCC proposed model:

$$\max \sum_{r=1}^{s} \bar{y}_{rp} - u_0$$

s.t.

$$\begin{aligned}
&\textstyle\sum_{i=1}^{m} \bar{x}_{ip} = 1 \\
&\textstyle\sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} - u_0 \leq 0 \qquad J \neq P \\
&u_r(\alpha x_{rj} + (1-\alpha)\sigma_{rj}) \leq \bar{y}_{rj} \leq u_r(\alpha y_{rj} + (1-\alpha)\beta_{rj}) \qquad \forall r, j \\
&v_i(\alpha x_{ij} + (1-\alpha)\sigma_{ij}) \leq \bar{x}_{ij} \leq v_i(\alpha y_{ij} + (1-\alpha)\beta_{ij}) \qquad \forall i, j \\
&u_r, v_i \geq 0 \quad \forall r, i
\end{aligned} \tag{4.24}$$

Model (4.24) is equivalent to a parametric programming model with α ∈ [0, 1]. Therefore, Analyzing the efficiency of DMUs with the proposed method, for a set of n different values of α, e.g. αi, i = 1, 2, ..., n . Therefore, it will be necessary to obtain an integrated efficiency score for DMUs to rank them. the fuzzy linear programming problem given by (4.24) is equivalent to a crisp parametric linear programming problem. Using the optimal value, we can determine the situation for returns to scale RTS for BCC and CCR when a DMUp is efficient. Similar to the conventional DEA model of Banker and Thrall (1992). from (3.10) the intervel of the efficiency score for a given α can be reduced as:

$$E_j = \begin{bmatrix} \dfrac{\sum_{r=1}^{s} u_r^* [\alpha x_{rj} + (1-\alpha)\sigma_{rj}]}{\sum_{i=1}^{m} v_i^* [\alpha y_{ij} + (1-\alpha)\beta_{ij}]} - u_0^*, \dfrac{\sum_{r=1}^{s} u_r^* [\alpha x_{rj} + (1-\alpha)\sigma_{rj}]}{\sum_{i=1}^{m} v_i^* [\alpha x_{ij} + (1-\alpha)\sigma_{ij}]} - u_0^* \\ , \dfrac{\sum_{r=1}^{s} u_r^* [\alpha y_{rj} + (1-\alpha)\beta_{rj}]}{\sum_{i=1}^{m} v_i^* [\alpha y_{ij} + (1-\alpha)\beta_{ij}]} - u_0^*, \dfrac{\sum_{r=1}^{s} u_r^* [\alpha y_{rj} + (1-\alpha)\beta_{rj}]}{\sum_{i=1}^{m} v_i^* [\alpha x_{ij} + (1-\alpha)\sigma_{ij}]} - u_0^* \end{bmatrix}$$

4.3. Numerical Examples

In this section, we use four numerical examples Guo and Tanaka (2001), Kao and Liu (2000), Saati and Memariani (2006), and the same example Guo and Tanaka (2001) are solved by the four proposed methods and the results are compared with previously presented methods.

Example 1. Consider 4 DMUs with inputs and outputs which are presented in Table 4.1, this problem results are compared with two previously presented Kao and Liu (2000) and Razavi, et. al. (2013).

Table 4.1. Inputs and outputs of 4 DMUs

DMU	Input	α –cut	Output	α –cut
A	(11,12,14)	[11+ α,14-2 α]	10	[10,10]
B	30	[30,30]	(12,13,14,16)	[12+ α,16-2 α]
C	40	[40,40]	11	[11,11]
D	(45,47,52,55)	[45+2 α,55-3 α]	(12,15,19,22)	[12+3 α,22-3 α]

Considering the DMU D, by FCCR models (4.7,4.8) for upper and lower objective models are solved:

$$\max \ E^{\alpha U}(D) = \max \ (22 - 3\alpha)\, u_1$$

s.t.

$$(45 + 2\alpha)v_1 \ = 1$$
$$10u_1 - (11 + \alpha)v_1 \ \le 0$$
$$(16 - 2\alpha)u_1 - 30v_1 \ \le 0$$
$$11u_1 - 40\ v_1 \le 0$$
$$(22 - 3\alpha)u_1 - (45 + 2\alpha)v_1 \ \le 0$$
$$10u_1 - (14 - 2\alpha)v_1 \ \le 0$$
$$(12 + \alpha)u_1 - 30v_1 \ \le 0$$
$$11u_1 - 40\ v_1 \le 0$$
$$(12 + 3\alpha)u_1 - (55 - 3\alpha)v_1 \le 0$$
$$u_1\, , v_1 \ \ge 0 \qquad \forall r\, , i$$

$$\max \ E^{\alpha L}(D) = \max \ (12 + 3\alpha)\, u_1$$

s.t.

$$(55-3\alpha)v_1 = 1$$
$$10u_1-(14-2\alpha)v_1 \le 0$$
$$(12+\alpha)u_1-30v_1 \le 0$$
$$11u_1-40\,v_1 \le 0$$
$$(12+3\alpha)u_1-(55-3\alpha)v_1 \le 0$$
$$10u_1-(11+\alpha)v_1 \le 0$$
$$(16-2\alpha)u_1-30v_1 \le 0$$
$$11u_1-40\,v_1 \le 0$$
$$(22-3\alpha)u_1-(45+2\alpha)v_1 \le 0$$
$$u_1\,,v_1 \ge 0$$

The results for these models are solved for different $\alpha -$ value in Table 4.2 and solved by MATLAB R2017a codes Appendix 17.

Table 4.2. The α –cuts of the efficiency at four α –values on the proposed method and two previously presented

	$A[E^{\alpha L},E^{\alpha U}]$			$B[E^{\alpha L},E^{\alpha U}]$		
A	Proposed BCC	Kao and Liu	Razavi et.al.	Proposed BCC	Kao and Liu	Razavi and et.al.
0.0	[0.98,1.0]	[1.0,1.0]	0.9497	[0.72,1.0]	[0.71,1.0]	[0.71,0.95]
0.5	[0.98,1.0]	[1.0,1.0]	0.9725	[0.80,1.0]	[0.79,1.0]	[0.79,0.95]
0.75	[0.98,1.0]	[1.0,1.0]	0.9873	[0.82,1.0]	[0.83,1.0]	[0.83,0.95]
1.0	[0.98,1.0]	[1.0,1.0]	1.0	[0.90,1.0]	[0.88,1.0]	[0.88,0.95]
	$C[E^{\alpha L},E^{\alpha U}]$			$D[E^{\alpha L},E^{\alpha U}]$		
A	Proposed BCC	Kao and Liu	Razavi and et.al.	Proposed BCC	Kao and Liu	Razavi and et.al.
0.0	[0.58,1.0]	[0.54,0.91]	0.5436	[0.72,1.0]	[0.74,1.0]	[0.54,1.0]
0.5	[0.61,1.0]	[0.58,0.85]	0.5890	[0.79,1.0]	[0.89,1.0]	[0.65,1.0]
0.75	[0.64,1.0]	[0.60,0.82]	0.6086	[0.91,1.0]	[0.96,1.0]	[0.70,1.0]
1.0	[0.70,1.0]	[0.63,0.79]	0.6395	[0.96,1.0]	[1.0,1.0]	[0.78,1.0]

Table 4.3. The results for Model (4.7,4.8) and the ranking of each DMU for different α levels interval score

	$\alpha = 0$	Rank	$\alpha = 0.5$	Rank	$\alpha = 0.75$	Rank	$\alpha = 1.0$	Rank
A	[0.98,1.0]	1	[0.98,1.0]	1	[0.98,1.0]	1	[0.98,1.0]	1
B	[0.72,1.0]	2	[0.80,1.0]	2	[0.82,1.0]	3	[0.90,1.0]	3
C	[0.58,1.0]	4	[0.61,1.0]	4	[0.64,1.0]	4	[0.70,1.0]	4
D	[0.72,1.0]	3	[0.79,1.0]	3	[0.91,1.0]	2	[0.96,1.0]	2

Example2. Consider 10 DMUs with 2 inputs and 2 outputs which are presented in Table (4.4), this problem results are compared with two previously presented Saati et. al. (2006) and Razavi et. al. (2013).

Table 4.4. 2 Inputs and 2 outputs of 10 DMUs

DMUs	Input 1	Input 2	Output 1	Output 2
D1	(6.0,7.0,8.0)	(29.0,30.0,32.0)	(35.5,38.0,41.0)	(409.0,411.0,416.0)
D2	(5.5,6.0,6.5)	(33.0,35.0,63.0)	(39.0,40.0,43.0)	(478.0,480.0,484.0)
D3	(7.5,9.0,10.5)	(43.0,45.0,48.0)	(32.0,35.0,38.0)	(297.0,299.0,301.0)
D4	(7.0,8.0,10.0)	(37.5,39.0,42.0)	(28.0,31.0,31.0)	(347.0,352.0,360.0)
D5	(9.0,11.0,12.0)	(43.0,44.0,45.0)	(33.0,35.0,38.0)	(406.0,411.0,415.0)
D6	(10.0,10.0,14.0)	(53.0,55.0,57.0)	(36.0,38.0,40.0)	(282.0,286.0,289.0)
D7	(10.0,12.0,14.0)	(107.0,110.0,113.0)	(34.5,36.0,38.0)	(396.0,400.0,405.0)
D8	(9.0,13.0,16.0)	(95.0,100.0,101.0)	(37.0,41.0,46.0)	(387.0,393.0,402.0)
D9	(12.0,14.0,15.0)	(120.0,125.0,131.0)	(24.0,27.0,28.0)	(400.0,404.0,406.0)
D10	(5.0,8.0,10.0)	(35.0,38.0,39.0)	(48.0,50.0,51.0)	(470.0,470.0,470.0)

Considering the DMU Di, by FCCR model (4.19) for upper and lower objective models are solved:

$$\max \ E^{\alpha}(D1) = \max \ (\bar{y}_{11} + \bar{y}_{21})$$

s.t.

$$\bar{x}_{11} + \bar{x}_{21} = 1$$

$(\bar{y}_{11}+\bar{y}_{21})-(\bar{x}_{11}\ +\bar{x}_{21})\le 0$

$(\bar{y}_{12}+\bar{y}_{22})-(\bar{x}_{12}\ +\bar{x}_{22})\le 0$

$(\bar{y}_{13}+\bar{y}_{23})-(\bar{x}_{13}\ +\bar{x}_{23})\le 0$

$(\bar{y}_{14}+\bar{y}_{24})-(\bar{x}_{14}\ +\bar{x}_{24})\le 0$

$(\bar{y}_{15}+\bar{y}_{25})-(\bar{x}_{15}\ +\bar{x}_{25})\le 0$

$(\bar{y}_{16}+\bar{y}_{26})-(\bar{x}_{16}\ +\bar{x}_{26})\le 0$

$(\bar{y}_{17}+\bar{y}_{27})-(\bar{x}_{17}\ +\bar{x}_{27})\le 0$

$(\bar{y}_{18}+\bar{y}_{28})-(\bar{x}_{18}\ +\bar{x}_{28})\le 0$

$(\bar{y}_{19}+\bar{y}_{29})-(\bar{x}_{19}\ +\bar{x}_{29})\le 0$

$(\bar{y}_{110}+\bar{y}_{210})-(\bar{x}_{110}\ +\bar{x}_{210})\le 0$

$u_1(\alpha(35.5)\ \ +(1-\alpha)38.0)\ \le \bar{y}_{11}\ \le u_1(\alpha(35.5)\ \ +(1-\alpha)41.0)$

$u_1(\alpha(39.0)\ \ +(1-\alpha)40.0)\ \le \bar{y}_{12}\ \le u_1(\alpha(39.0)\ \ +(1-\alpha)43.0)$

$u_1(\alpha(32.0)\ \ +(1-\alpha)35.0)\ \le \bar{y}_{13}\ \le u_1(\alpha(32.5)\ \ +(1-\alpha)38.0)$

$u_1(\alpha(28.0)\ \ +(1-\alpha)31.0)\ \le \bar{y}_{14}\ \le u_1(\alpha(28.0)\ \ +(1-\alpha)31.0)$

$u_1(\alpha(33.0)\ \ +(1-\alpha)35.0)\ \le \bar{y}_{15}\ \le u_1(\alpha(33.0)\ \ +(1-\alpha)38.0)$

$u_1(\alpha(36.0)\ \ +(1-\alpha)38.0)\ \le \bar{y}_{16}\ \le u_1(\alpha(36.0)\ \ +(1-\alpha)40.0)$

$u_1(\alpha(34.0)\ \ +(1-\alpha)36.0)\ \le \bar{y}_{17}\ \le u_1(\alpha(34.0)\ \ +(1-\alpha)38.0)$

$u_1(\alpha(37.0)\ \ +(1-\alpha)41.0)\ \le \bar{y}_{18}\ \le u_1(\alpha(37.0)\ \ +(1-\alpha)46.0)$

$u_1(\alpha(24.0)\ \ +(1-\alpha)27.0)\ \le \bar{y}_{19}\ \le u_1(\alpha(24.0)\ \ +(1-\alpha)28.0)$

$u_1(\alpha(48.0)\ \ +(1-\alpha)50.0)\ \le \bar{y}_{110}\ \le u_1(\alpha(48.0)\ \ +(1-\alpha)51.0)$

$u_2(\alpha(409.0)\ \ +(1-\alpha)411.0)\ \le \bar{y}_{21}\ \le u_2(\alpha(409.0)\ \ +(1-\alpha)416.0)$

$u_2(\alpha(478.0)\ \ +(1-\alpha)40.0)\ \le \bar{y}_{22}\ \le u_2(\alpha(478.0)\ \ +(1-\alpha)484.0)$

$u_2(\alpha(297.0)\ \ +(1-\alpha)299.0)\ \le \bar{y}_{23}\ \le u_2(\alpha(297.5)\ \ +(1-\alpha)301.0)$

$u_2(\alpha(347.0) \ + (1-\alpha)352.0) \ \leq \bar{y}_{24} \ \leq u_2(\alpha(347.0) \ + (1-\alpha)360.0)$

$u_2(\alpha(406.0) \ + (1-\alpha)411.0) \ \leq \bar{y}_{25} \ \leq u_2(\alpha(406.0) \ + (1-\alpha)415.5)$

$u_2(\alpha(282.0) \ + (1-\alpha)286.0) \ \leq \bar{y}_{26} \ \leq u_2(\alpha(282.0) \ + (1-\alpha)289.0)$

$u_2(\alpha(396.0) \ + (1-\alpha)400.0) \ \leq \bar{y}_{27} \ \leq u_2(\alpha(396.0) \ + (1-\alpha)405.0)$

$u_2(\alpha(387.0) \ + (1-\alpha)393.0) \ \leq \bar{y}_{28} \ \leq u_2(\alpha(387.0) \ + (1-\alpha)402.0)$

$u_2(\alpha(400.0) \ + (1-\alpha)404.0) \ \leq \bar{y}_{29} \ \leq u_2(\alpha(400.0) \ + (1-\alpha)406.0)$

$u_2(\alpha(470.0) \ + (1-\alpha)470.0) \ \leq \bar{y}_{210} \ \leq u_2(\alpha(470.0) \ + (1-\alpha)470.0)$

$v_1(\alpha(6.0) \ + (1-\alpha)7.0\,) \leq \bar{x}_{11} \ \leq v_1(\alpha(6.0) \ + \ (1-\alpha)8.0)$

$v_1(\alpha(5.5) \ + (1-\alpha)6.0) \ \leq \bar{x}_{12} \ \leq v_1(\alpha(5.5) \ + \ (1-\alpha)6.5)$

$v_1(\alpha(7.5) \ + (1-\alpha)9.0) \ \leq \bar{x}_{13} \ \leq v_1(\alpha(7.5) \ + \ (1-\alpha)10.5)$

$v_1(\alpha(7.0) \ + (1-\alpha)8.0) \ \leq \bar{x}_{14} \ \leq v_1(\alpha(7.0) \ + \ (1-\alpha)10.0)$

$v_1(\alpha(9.0) \ + (1-\alpha)11.0) \ \leq \bar{x}_{15} \ \leq v_1(\alpha(9.0) \ + \ (1-\alpha)12.0)$

$v_1(\alpha(10.0) \ + (1-\alpha)10.0) \ \leq \bar{x}_{16} \ \leq v_1(\alpha(10.0) \ + \ (1-\alpha)14.0)$

$v_1(\alpha(10.0) \ + (1-\alpha)12.0) \ \leq \bar{x}_{17} \ \leq v_1(\alpha(10.0) \ + \ (1-\alpha)14.0)$

$v_1(\alpha(9.0) \ + (1-\alpha)13.0) \ \leq \bar{x}_{18} \ \leq v_1(\alpha(9.0) \ + \ (1-\alpha)16.0)$

$v_1(\alpha(12.0) \ + (1-\alpha)14.0) \ \leq \bar{x}_{19} \ \leq v_1(\alpha(12.0) \ + \ (1-\alpha)15.0)$

$v_1(\alpha(5.0) \ + (1-\alpha)8.0) \ \leq \bar{x}_{110} \ \leq v_1(\alpha(5.0) \ + \ (1-\alpha)10.0)$

$v_2(\alpha(29.0) \ + (1-\alpha)30.0) \ \leq \bar{x}_{21} \ \leq v_2(\alpha(29.0) \ + \ (1-\alpha)32.0)$

$v_2(\alpha(33.0) \ + (1-\alpha)35.0) \ \leq \bar{x}_{22} \ \leq v_2(\alpha(33.0) \ + \ (1-\alpha)36.0)$

$v_2(\alpha(43.0) \ + (1-\alpha)45.0) \ \leq \bar{x}_{23} \ \leq v_2(\alpha(43.0) \ + \ (1-\alpha)48.0)$

$v_2(\alpha(37.0) \ + (1-\alpha)39.0) \ \leq \bar{x}_{24} \ \leq v_2(\alpha(37.0) \ + \ (1-\alpha)42.0)$

$v_2(\alpha(43.0) \ + (1-\alpha)44.0) \ \leq \bar{x}_{25} \ \leq v_2(\alpha(43.0) \ + \ (1-\alpha)45.0)$

$v_2(\alpha(53.0) \ + (1-\alpha)55.0) \ \leq \bar{x}_{26} \ \leq v_2(\alpha(53.0) \ + \ (1-\alpha)57.0)$

$v_2(\alpha(107.0) + (1-\alpha)110.0) \leq \bar{x}_{27} \leq v_2(\alpha(107.0) + (1-\alpha)113.0)$

$v_2(\alpha(95.0) + (1-\alpha)100.0) \leq \bar{x}_{28} \leq v_2(\alpha(95.0) + (1-\alpha)101.0)$

$v_2(\alpha(120.0) + (1-\alpha)125.0) \leq \bar{x}_{29} \leq v_2(\alpha(120.0) + (1-\alpha)131.0)$

$v_2(\alpha(35.0) + (1-\alpha)38.0) \leq \bar{x}_{210} \leq v_2(\alpha(35.0) + (1-\alpha)39.0)$

$u_1, v_1, u_2, v_2 \geq 0$

$\max\ E^{\alpha}(D2) = \max\ (\bar{y}_{12} + \bar{y}_{22})$

s.t.

$\bar{x}_{12} + \bar{x}_{22} = 1$

$(\bar{y}_{11} + \bar{y}_{21}) - (\bar{x}_{11} + \bar{x}_{21}) \leq 0$

$(\bar{y}_{12} + \bar{y}_{22}) - (\bar{x}_{12} + \bar{x}_{22}) \leq 0$

$(\bar{y}_{13} + \bar{y}_{23}) - (\bar{x}_{13} + \bar{x}_{23}) \leq 0$

$(\bar{y}_{14} + \bar{y}_{24}) - (\bar{x}_{14} + \bar{x}_{24}) \leq 0$

$(\bar{y}_{15} + \bar{y}_{25}) - (\bar{x}_{15} + \bar{x}_{25}) \leq 0$

$(\bar{y}_{16} + \bar{y}_{26}) - (\bar{x}_{16} + \bar{x}_{26}) \leq 0$

$(\bar{y}_{17} + \bar{y}_{27}) - (\bar{x}_{17} + \bar{x}_{27}) \leq 0$

$(\bar{y}_{18} + \bar{y}_{28}) - (\bar{x}_{18} + \bar{x}_{28}) \leq 0$

$(\bar{y}_{19} + \bar{y}_{29}) - (\bar{x}_{19} + \bar{x}_{29}) \leq 0$

$(\bar{y}_{110} + \bar{y}_{210}) - (\bar{x}_{110} + \bar{x}_{210}) \leq 0$

$u_1(\alpha(35.5) + (1-\alpha)38.0) \leq \bar{y}_{11} \leq u_1(\alpha(35.5) + (1-\alpha)41.0)$

$u_1(\alpha(39.0) + (1-\alpha)40.0) \leq \bar{y}_{12} \leq u_1(\alpha(39.0) + (1-\alpha)43.0)$

$u_1(\alpha(32.0) + (1-\alpha)35.0) \leq \bar{y}_{13} \leq u_1(\alpha(32.5) + (1-\alpha)38.0)$

$u_1(\alpha(28.0) + (1-\alpha)31.0) \leq \bar{y}_{14} \leq u_1(\alpha(28.0) + (1-\alpha)31.0)$

$$u_1(\alpha(33.0) \ + (1-\alpha)35.0) \leq \bar{y}_{15} \leq u_1(\alpha(33.0) \ + (1-\alpha)38.0)$$

$$u_1(\alpha(36.0) \ + (1-\alpha)38.0) \leq \bar{y}_{16} \leq u_1(\alpha(36.0) \ + (1-\alpha)40.0)$$

$$u_1(\alpha(34.0) \ + (1-\alpha)36.0) \leq \bar{y}_{17} \leq u_1(\alpha(34.0) \ + (1-\alpha)38.0)$$

$$u_1(\alpha(37.0) \ + (1-\alpha)41.0) \leq \bar{y}_{18} \leq u_1(\alpha(37.0) \ + (1-\alpha)46.0)$$

$$u_1(\alpha(24.0) \ + (1-\alpha)27.0) \leq \bar{y}_{19} \leq u_1(\alpha(24.0) \ + (1-\alpha)28.0)$$

$$u_1(\alpha(48.0) \ + (1-\alpha)50.0) \leq \bar{y}_{110} \leq u_1(\alpha(48.0) \ + (1-\alpha)51.0)$$

$$u_2(\alpha(409.0) \ + (1-\alpha)411.0) \leq \bar{y}_{21} \leq u_2(\alpha(409.0) \ + (1-\alpha)416.0)$$

$$u_2(\alpha(478.0) \ + (1-\alpha)40.0) \leq \bar{y}_{22} \leq u_2(\alpha(478.0) \ + (1-\alpha)484.0)$$

$$u_2(\alpha(297.0) \ + (1-\alpha)299.0) \leq \bar{y}_{23} \leq u_2(\alpha(297.5) \ + (1-\alpha)301.0)$$

$$u_2(\alpha(347.0) \ + (1-\alpha)352.0) \leq \bar{y}_{24} \leq u_2(\alpha(347.0) \ + (1-\alpha)360.0)$$

$$u_2(\alpha(406.0) \ + (1-\alpha)411.0) \leq \bar{y}_{25} \leq u_2(\alpha(406.0) \ + (1-\alpha)415.5)$$

$$u_2(\alpha(282.0) \ + (1-\alpha)286.0) \leq \bar{y}_{26} \leq u_2(\alpha(282.0) \ + (1-\alpha)289.0)$$

$$u_2(\alpha(396.0) \ + (1-\alpha)400.0) \leq \bar{y}_{27} \leq u_2(\alpha(396.0) \ + (1-\alpha)405.0)$$

$$u_2(\alpha(387.0) \ + (1-\alpha)393.0) \leq \bar{y}_{28} \leq u_2(\alpha(387.0) \ + (1-\alpha)402.0)$$

$$u_2(\alpha(400.0) \ + (1-\alpha)404.0) \leq \bar{y}_{29} \leq u_2(\alpha(400.0) \ + (1-\alpha)406.0)$$

$$u_2(\alpha(470.0) \ + (1-\alpha)470.0) \leq \bar{y}_{210} \leq u_2(\alpha(470.0) \ + (1-\alpha)470.0)$$

$$v_1(\alpha(6.0) \ + (1-\alpha)7.0) \leq \bar{x}_{21} \leq v_1(\alpha(6.0) \ + \ (1-\alpha)8.0)$$

$$v_1(\alpha(5.5) \ + (1-\alpha)6.0) \leq \bar{x}_{22} \leq v_1(\alpha(5.5) \ + \ (1-\alpha)6.5)$$

$$v_1(\alpha(7.5) \ + (1-\alpha)9.0) \leq \bar{x}_{23} \leq v_1(\alpha(7.5) \ + \ (1-\alpha)10.5)$$

$$v_1(\alpha(7.0) \ + (1-\alpha)8.0) \leq \bar{x}_{24} \leq v_1(\alpha(7.0) \ + \ (1-\alpha)10.0)$$

$$v_1(\alpha(9.0) \ + (1-\alpha)11.0) \leq \bar{x}_{25} \leq v_1(\alpha(9.0) \ + \ (1-\alpha)12.0)$$

$$v_1(\alpha(10.0) \ + (1-\alpha)10.0) \leq \bar{x}_{26} \leq v_1(\alpha(10.0) \ + \ (1-\alpha)14.0)$$

$$v_1(\alpha(10.0) \ + (1-\alpha)12.0) \leq \bar{x}_{27} \leq v_1(\alpha(10.0) \ + \ (1-\alpha)14.0)$$

$$v_1(\alpha(9.0) \quad + (1-\alpha)13.0) \ \leq \bar{x}_{28} \ \leq v_1(\alpha(9.0) \ + \ (1-\alpha)16.0)$$

$$v_1(\alpha(12.0) \quad + (1-\alpha)14.0) \ \leq \bar{x}_{29} \ \leq v_1(\alpha(12.0) \ + \ (1-\alpha)15.0)$$

$$v_1(\alpha(5.0) \quad + (1-\alpha)8.0) \ \leq \bar{x}_{210} \ \leq v_1(\alpha(5.0) \ + \ (1-\alpha)10.0)$$

$$v_2(\alpha(29.0) \quad + (1-\alpha)30.0) \ \leq \bar{x}_{21} \ \leq v_2(\alpha(29.0) \ + \ (1-\alpha)32.0)$$

$$v_2(\alpha(33.0) \quad + (1-\alpha)35.0) \ \leq \bar{x}_{22} \ \leq v_2(\alpha(33.0) \ + \ (1-\alpha)36.0)$$

$$v_2(\alpha(43.0) \quad + (1-\alpha)45.0) \ \leq \bar{x}_{23} \ \leq v_2(\alpha(43.0) \ + \ (1-\alpha)48.0)$$

$$v_2(\alpha(37.0) \quad + (1-\alpha)39.0) \ \leq \bar{x}_{24} \ \leq v_2(\alpha(37.0) \ + \ (1-\alpha)42.0)$$

$$v_2(\alpha(43.0) \quad + (1-\alpha)44.0) \ \leq \bar{x}_{25} \ \leq v_2(\alpha(43.0) \ + \ (1-\alpha)45.0)$$

$$v_2(\alpha(53.0) \quad + (1-\alpha)55.0) \ \leq \bar{x}_{26} \ \leq v_2(\alpha(53.0) \ + \ (1-\alpha)57.0)$$

$$v_2(\alpha(107.0) \quad + (1-\alpha)110.0) \ \leq \bar{x}_{27} \ \leq v_2(\alpha(107.0) \ + \ (1-\alpha)113.0)$$

$$v_2(\alpha(95.0) \quad + (1-\alpha)100.0) \ \leq \bar{x}_{28} \ \leq v_2(\alpha(95.0) \ + \ (1-\alpha)101.0)$$

$$v_2(\alpha(120.0) \quad + (1-\alpha)125.0) \ \leq \bar{x}_{29} \ \leq v_2(\alpha(120.0) \ + \ (1-\alpha)131.0)$$

$$v_2(\alpha(35.0) \quad + (1-\alpha)38.0) \ \leq \bar{x}_{210} \ \leq v_2(\alpha(35.0) \ + \ (1-\alpha)39.0)$$

$$u_1 , v_1 , u_2 , v_2 \ \geq 0$$

The results for these model are solved for different $\alpha -$ value in Table (4.5) and solved by MATLAB R2017a codes Appendix 18.

Table 4.5. The α – cuts of the efficiency at four α – values on the proposed method and two previously presented

DMU	A					
	0.0			0.5		
	Proposed CCR	Saati et. al.	Razavi et.al.	Proposed CCR	Saati et. al.	Razavi et.al.
D1	[0.91,1.0]	1.00	[0.91,0.94]	[0.95,1.0]	1.00	[0.96,0.98]
D2	[0.94,1.0]	1.00	[0.94,0.95]	[0.95,1.0]	1.00	[0.97,0.98]
D3	[0.81,0.88]	0.84	[0.48,0.57]	[0.69,0.74]	0.71	[0.54,0.58]
D4	[0.72,0.82]	0.76	[0.60,0.62]	[0.67,0.73]	0.70	[0.63,0.64]
D5	[0.76,0.80]	0.78	[0.62,0.64]	[0.70,0.73]	0.71	[0.66,0.66]

Table 4.5. (cont) The α – cuts of the efficiency at four α – values on the proposed method and two previously presented

D6	[0.63,0.72]	0.69	[0.44,0.49]	[0.61,0.66]	0.63	[0.49,0.51]
D7	[0.59,0.68]	0.63	[0.35,0.35]	[0.45,0.53]	0.51	[0.41,0.42]
D8	[0.81,0.89]	0.85	[0.32,0.33]	[0.53,0.60]	0.59	[0.41,0.40]
D9	[0.51,0.54]	0.46	[0.31,0.31]	[0.37,0.42]	0.40	[0.35,0.35]
D10	[0.96,1.0]	1.00	[0.89,0.93]	[0.94,1.0]	1.00	[0.95,0.97]
DMU	0.75			1.0		
	Proposed CCR	Saati et. al.	Razavi et.al.	Proposed CCR	Saati et. al.	Razavi et.al.
D1	[0.97,1.0]	1.0	[0.98,0.99]	[1.0,1.0]	1.00	[1.00,1.00]
D2	[0.97,1.0]	1.0	[0.98,0.99]	[1.0,1.0]	1.00	[1.00,1.00]
D3	[0.62,0.69]	0.66	[0.56,0.58]	[0.58,0.64]	0.61	[0.60,0.61]
D4	[0.63,0.72]	0.68	[0.64,0.65]	[0.61,0.67]	0.66	[0.65,0.65]
D5	[0.68,0.72]	0.69	[0.67,0.67]	[0.67,0.75]	0.68	[0.68,0.68]
D6	[0.60,0.62]	0.60	[0.53,0.54]	[0.53,0.61]	0.58	[0.58,0.58]
D7	[0.50,0.54]	0.48	[0.43,0.44]	[0.41,0.48]	0.45	[0.45,0.45]
D8	[0.51,0.55]	0.53	[0.45,0.46]	[0.45,0.47]	0.47	[0.47,0.47]
D9	[0.39,0.42]	0.38	[0.35,0.35]	[0.35,0.39]	0.36	[0.36,0.36]
D10	[0.96,1.0]	1.00	[0.97,0.98]	[1.0,1.0]	1.00	[1.00,1.00]

Table 4.6. The results for Model (4.19) and the ranking of each DMU for different α values

DMU	Interval score							
	$\alpha = 0$	Rank	$\alpha = 0.5$	Rank	$\alpha = 0.75$	Rank	$\alpha = 1.0$	Rank
D1	[0.97,1.0]	1	[0.95,1.0]	1	[0.97,1.0]	1	[1.0,1.0]	1
D2	[0.94,1.0]	2	[0.95,1.0]	2	[0.97,1.0]	2	[1.0,1.0]	2
D3	[0.81,0.88]	5	[0.69,0.74]	5	[0.62,0.69]	5	[0.58,0.64]	6
D4	[0.72,0.82]	7	[0.67,0.73]	6	[0.63,0.72]	6	[0.61,0.67]	5
D5	[0.76,0.80]	6	[0.70,0.73]	4	[0.68,0.72]	4	[0.67,0.75]	4
D6	[0.63,0.72]	8	[0.61,0.66]	7	[0.60,0.62]	7	[0.53,0.61]	7
D7	[0.59,0.68]	9	[0.45,0.53]	9	[0.50,0.54]	9	[0.41,0.48]	9
D8	[0.81,0.89]	4	[0.53,0.60]	8	[0.51,0.55]	8	[0.45,0.47]	8
D9	[0.51,0.54]	10	[0.37,0.42]	10	[0.39,0.42]	10	[0.35,0.39]	10
D10	[0.93,1.0]	3	[0.94,1.0]	3	[0.96,1.0]	3	[1.0,1.0]	3

Example 3. was adopted to solve the example (1) and compare it with the same two previously presented Kao and Liu (2000) and Razavi, et. al. (2013). Considering the DMU D, by FBCC models (4.13,4.14) for upper and lower objective models are solved:

$$\max\ E^{\alpha U}(D) = \max\ (22 - 3\alpha)\, u_1 - u_0$$

s.t.

$$(45 + 2\alpha)v_1 = 1$$

$$10u_1 - (11 + \alpha)v_1 - u_0 \leq 0$$

$$(16 - 2\alpha)u_1 - 30v_1 - u_0 \leq 0$$

$$11u_1 - 40\, v_1 - u_0 \leq 0$$

$$(22 - 3\alpha)u_1 - (45 + 2\alpha)v_1 - u_0 \leq 0$$

$$10u_1 - (14 - 2\alpha)v_1 - u_0 \leq 0$$

$$(12 + \alpha)u_1 - 30v_1 - u_0 \leq 0$$

$$11u_1 - 40\, v_1 - u_0 \leq 0$$

$$(12 + 3\alpha)u_1 - (55 - 3\alpha)v_1 - u_0 \leq 0$$

$$u_1, v_1 \geq 0\ , u_0 : unrestrieted$$

$$\max\ E^{\alpha L}(D) = \max\ (12 + 3\alpha)\, u_1 - u_0$$

s.t.

$$(55 - 3\alpha)v_1 = 1$$

$$10u_1 - (14 - 2\alpha)v_1 - u_0 \leq 0$$

$$(12 + \alpha)u_1 - 30v_1 - u_0 \leq 0$$

$$11u_1 - 40\, v_1 - u_0 \leq 0$$

$$(12 + 3\alpha)u_1 - (55 - 3\alpha)v_1 - u_0 \leq 0$$

$10u_1 - (11+\alpha)v_1 \ - u_0 \ \leq 0$

$(16-2\alpha)u_1 - 30v_1 - u_0 \ \leq 0$

$11u_1 - 40\ v_1 - u_0 \leq 0$

$(22-3\alpha)u_1 - (45+2\alpha)v_1 - u_0 \ \leq 0$

$u_1\ , v_1 \ \geq 0 \ \ , u_0$: unrestrieted

The results for these models are solved for different $\alpha -$ value in Table (4.7) and solved by MATLAB R2017a codes Appendix 17.

Table 4.7. The α –cuts of the efficiency at four α –values on the proposed method and two previously presented

A	$A[E^{\alpha L}, E^{\alpha U}]$			$B[E^{\alpha L}, E^{\alpha U}]$		
	Proposed BCC	Kao and Liu	Razavi et.al.	Proposed BCC	Kao and Liu	Razavi and et.al.
0.0	[0.98,1.0]	[1.0,1.0]	0.9497	[0.72,1.0]	[0.71,1.0]	[0.71,0.95]
0.5	[0.98,1.0]	[1.0,1.0]	0.9725	[0.80,1.0]	[0.79,1.0]	[0.79,0.95]
0.75	[0.98,1.0]	[1.0,1.0]	0.9873	[0.82,1.0]	[0.83,1.0]	[0.83,0.95]
1.0	[0.98,1.0]	[1.0,1.0]	1.0	[0.90,1.0]	[0.88,1.0]	[0.88,0.95]
A	$C[E^{\alpha L}, E^{\alpha U}]$			$D[E^{\alpha L}, E^{\alpha U}]$		
	Proposed BCC	Kao and Liu	Razavi and et.al.	Proposed BCC	Kao and Liu	Razavi and et.al.
0.0	[0.58,1.0]	[0.54,0.91]	0.5436	[0.72,1.0]	[0.74,1.0]	[0.54,1.0]
0.5	[0.61,1.0]	[0.58,0.85]	0.5890	[0.79,1.0]	[0.89,1.0]	[0.65,1.0]
0.75	[0.64,1.0]	[0.60,0.82]	0.6086	[0.91,1.0]	[0.96,1.0]	[0.70,1.0]
1.0	[0.70,1.0]	[0.63,0.79]	0.6395	[0.96,1.0]	[1.0,1.0]	[0.78,1.0]

Table 4.8. The results for Model (4.13,4.14) and the ranking of each DMU for different α values

DMU	Interval score							
	$\alpha = 0$	Rank	$\alpha = 0.5$	Rank	$\alpha = 0.75$	Rank	$\alpha = 1.0$	Rank
A	[0.98,1.0]	1	[0.98,1.0]	1	[0.98,1.0]	1	[0.98,1.0]	1
B	[0.72,1.0]	2	[0.80,1.0]	2	[0.82,1.0]	3	[0.90,1.0]	3
C	[0.58,1.0]	4	[0.61,1.0]	4	[0.64,1.0]	4	[0.70,1.0]	4
D	[0.72,1.0]	3	[0.79,1.0]	3	[0.91,1.0]	2	[0.96,1.0]	2

Example 4. Consider 5 DMUs with 2 inputs and 2 outputs which are presented in Table (4.9), this problem results are compared with six previously presented Guo and Tanaka (2001), Lertworasirikul et al. (2003a, 2003b) , Saati et al. (2002), Majid et. al.(2010) and Hatami et.al. (2012).

Table 4.9. 2 Inputs and 2 outputs of 5 DMUs

DMU	Input 1	Input 2	Output 1	Output 2
A	(3.5, 4, 4.5)	(1.9, 2.1, 2.3)	(2.4,2.6,2.8)	(3.8, 4.1, 4.4)
B	(2.9, 2.9, 2.9)	(1.4, 1.5, 1.6)	(2.2, 2.2, 2.2)	(3.3, 3.5, 3.7)
C	(4.4, 4.9, 5.4)	(2.2, 2.6, 3.2)	(2.7, 3.2, 3.7)	(4.3, 5.1, 5.9)
D	(3.4, 4.1, 4.8)	(2.2, 2.3, 2.4)	(2.5, 2.9, 3.3)	(5.5, 5.7, 5.9)
E	(5.9, 6.5, 7.1)	(3.6, 4.1, 4.6)	(4.4, 5.1, 5.8)	(6.5, 7.4, 8.3)

Considering the DMU D, by FBCC model (4.24) for intervel of the objective models are solved:

$$\max\ E^{\alpha}(A) = \max\ (\bar{y}_{11} + \bar{y}_{21}) - u_0$$

s.t.

$$\bar{x}_{11} + \bar{x}_{21} = 1$$

$$(\bar{y}_{11} + \bar{y}_{21}) - (\bar{x}_{11} + \bar{x}_{21}) - u_0 \leq 0$$

$$(\bar{y}_{12} + \bar{y}_{22}) - (\bar{x}_{12} + \bar{x}_{22}) - u_0 \leq 0$$

$$(\bar{y}_{13} + \bar{y}_{23}) - (\bar{x}_{13} + \bar{x}_{23}) - u_0 \leq 0$$

$$(\bar{y}_{14} + \bar{y}_{24}) - (\bar{x}_{14} + \bar{x}_{24}) - u_0 \leq 0$$

$$(\bar{y}_{15} + \bar{y}_{25}) - (\bar{x}_{15} + \bar{x}_{25}) - u_0 \leq 0$$

$$u_1(\alpha(2.4) + (1-\alpha)2.6) \leq \bar{y}_{11} \leq u_1(\alpha(2.4) + (1-\alpha)2.8)$$

$$u_1(\alpha(2.2) + (1-\alpha)2.2) \leq \bar{y}_{12} \leq u_1(\alpha(2.2) + (1-\alpha)2.2)$$

$$u_1(\alpha(2.7) + (1-\alpha)3.2) \leq \bar{y}_{13} \leq u_1(\alpha(2.7) + (1-\alpha)3.7)$$

$u_1(\alpha(2.5) \ + (1-\alpha)2.9) \leq \bar{y}_{14} \leq u_1(\alpha(2.5) \ + (1-\alpha)3.3)$

$u_1(\alpha(4.4) \ + (1-\alpha)5.1) \leq \bar{y}_{15} \leq u_1(\alpha(4.4) \ + (1-\alpha)5.8)$

$u_2(\alpha(3.8) \ + (1-\alpha)4.1) \leq \bar{y}_{21} \leq u_2(\alpha(3.8) \ + (1-\alpha)4.4)$

$u_2(\alpha(3.3) \ + (1-\alpha)3.5.) \leq \bar{y}_{22} \leq u_2(\alpha(3.3) \ + (1-\alpha)3.7)$

$u_2(\alpha(4.3) \ + (1-\alpha)5.1) \leq \bar{y}_{23} \leq u_2(\alpha(4.3) \ + (1-\alpha)5.9)$

$u_2(\alpha(5.5) \ + (1-\alpha)5.7) \leq \bar{y}_{24} \leq u_2(\alpha(5.5) \ + (1-\alpha)5.9)$

$u_2(\alpha(6.5) \ + (1-\alpha)7.4) \leq \bar{y}_{25} \leq u_2(\alpha(6.5) \ + (1-\alpha)8.3)$

$v_1(\alpha(3.5) \ + (1-\alpha)4.0) \leq \bar{x}_{11} \leq v_1(\alpha(3.5) \ + \ (1-\alpha)4.5)$

$v_1(\alpha(2.9) \ + (1-\alpha)2.9) \leq \bar{x}_{12} \leq v_1(\alpha(2.9) \ + \ (1-\alpha)2.9)$

$v_1(\alpha(4.4) \ + (1-\alpha)4.9) \leq \bar{x}_{13} \leq v_1(\alpha(4.4) \ + \ (1-\alpha)5.4)$

$v_1(\alpha(3.4) \ + (1-\alpha)4.1) \leq \bar{x}_{14} \leq v_1(\alpha(3.4) \ + \ (1-\alpha)4.8)$

$v_1(\alpha(5.9) \ + (1-\alpha)6.5) \leq \bar{x}_{15} \leq v_1(\alpha(5.9) \ + \ (1-\alpha)7.1)$

$v_2(\alpha(1.9) \ + (1-\alpha)2.1) \leq \bar{x}_{21} \leq v_2(\alpha(1.9) \ + \ (1-\alpha)2.3)$

$v_2(\alpha(1.4) \ + (1-\alpha)1.5) \leq \bar{x}_{22} \leq v_2(\alpha(1.4) \ + \ (1-\alpha)1.6)$

$v_2(\alpha(2.2) \ + (1-\alpha)2.6) \leq \bar{x}_{23} \leq v_2(\alpha(2.2) \ + \ (1-\alpha)3.0)$

$v_2(\alpha(2.2) \ + (1-\alpha)2.3) \leq \bar{x}_{24} \leq v_2(\alpha(2.2) \ + \ (1-\alpha)2.4)$

$v_2(\alpha(3.6) \ + (1-\alpha)4.1) \leq \bar{x}_{25} \leq v_2(\alpha(3.6) \ + \ (1-\alpha)4.6)$

$u_1, v_1, u_2, v_2 \geq 0 \quad , u_0$: unrestrieted

$\max \ E^{\alpha}(B) = \max \ (\bar{y}_{12} + \bar{y}_{22}) - u_0$

s.t.

$\bar{x}_{12} + \bar{x}_{22} \ = 1$

$(\bar{y}_{11} + \bar{y}_{21}) - (\bar{x}_{11} + \bar{x}_{21}) - u_0 \leq 0$

$(\bar{y}_{12} + \bar{y}_{22}) - (\bar{x}_{12} + \bar{x}_{22}) - u_0 \leq 0$

$(\bar{y}_{13} + \bar{y}_{23}) - (\bar{x}_{13} + \bar{x}_{23}) - u_0 \leq 0$

$(\bar{y}_{14} + \bar{y}_{24}) - (\bar{x}_{14} + \bar{x}_{24}) - u_0 \leq 0$

$(\bar{y}_{15} + \bar{y}_{25}) - (\bar{x}_{15} + \bar{x}_{25}) - u_0 \leq 0$

$u_1(\alpha(2.4) + (1-\alpha)2.6) \leq \bar{y}_{11} \leq u_1(\alpha(2.4) + (1-\alpha)2.8)$

$u_1(\alpha(2.2) + (1-\alpha)2.2) \leq \bar{y}_{12} \leq u_1(\alpha(2.2) + (1-\alpha)2.2)$

$u_1(\alpha(2.7) + (1-\alpha)3.2) \leq \bar{y}_{13} \leq u_1(\alpha(2.7) + (1-\alpha)3.7)$

$u_1(\alpha(2.5) + (1-\alpha)2.9) \leq \bar{y}_{14} \leq u_1(\alpha(2.5) + (1-\alpha)3.3)$

$u_1(\alpha(4.4) + (1-\alpha)5.1) \leq \bar{y}_{15} \leq u_1(\alpha(4.4) + (1-\alpha)5.8)$

$u_2(\alpha(3.8) + (1-\alpha)4.1) \leq \bar{y}_{21} \leq u_2(\alpha(3.8) + (1-\alpha)4.4)$

$u_2(\alpha(3.3) + (1-\alpha)3.5.) \leq \bar{y}_{22} \leq u_2(\alpha(3.3) + (1-\alpha)3.7)$

$u_2(\alpha(4.3) + (1-\alpha)5.1) \leq \bar{y}_{23} \leq u_2(\alpha(4.3) + (1-\alpha)5.9)$

$u_2(\alpha(5.5) + (1-\alpha)5.7) \leq \bar{y}_{24} \leq u_2(\alpha(5.5) + (1-\alpha)5.9)$

$u_2(\alpha(6.5) + (1-\alpha)7.4) \leq \bar{y}_{25} \leq u_2(\alpha(6.5) + (1-\alpha)8.3)$

$v_1(\alpha(3.5) + (1-\alpha)4.0) \leq \bar{x}_{11} \leq v_1(\alpha(3.5) + (1-\alpha)4.5)$

$v_1(\alpha(2.9) + (1-\alpha)2.9) \leq \bar{x}_{12} \leq v_1(\alpha(2.9) + (1-\alpha)2.9)$

$v_1(\alpha(4.4) + (1-\alpha)4.9) \leq \bar{x}_{13} \leq v_1(\alpha(4.4) + (1-\alpha)5.4)$

$v_1(\alpha(3.4) + (1-\alpha)4.1) \leq \bar{x}_{14} \leq v_1(\alpha(3.4) + (1-\alpha)4.8)$

$v_1(\alpha(5.9) + (1-\alpha)6.5) \leq \bar{x}_{15} \leq v_1(\alpha(5.9) + (1-\alpha)7.1)$

$v_2(\alpha(1.9) + (1-\alpha)2.1) \leq \bar{x}_{21} \leq v_2(\alpha(1.9) + (1-\alpha)2.3)$

$v_2(\alpha(1.4) + (1-\alpha)1.5) \leq \bar{x}_{22} \leq v_2(\alpha(1.4) + (1-\alpha)1.6)$

$v_2(\alpha(2.2) + (1-\alpha)2.6) \leq \bar{x}_{23} \leq v_2(\alpha(2.2) + (1-\alpha)3.0)$

$v_2(\alpha(2.2) + (1-\alpha)2.3) \leq \bar{x}_{24} \leq v_2(\alpha(2.2) + (1-\alpha)2.4)$

$v_2(\alpha(3.6) \quad + (1-\alpha)4.1) \leq \bar{x}_{25} \leq v_2(\alpha(3.6) \; + \; (1-\alpha)4.6)$

$u_1, v_1, u_2, v_2 \geq 0 \quad , u_0$: unrestrieted

The results for these model are solved for different $\alpha -$ value in Table (4.10) and solved by MATLAB R2017a codes Appendix 18.

Table 4.10. The efficiency at four α –values on the proposed method and six previously presented

Method	α	DMUs				
		A	B	C	D	E
Guo and Tanaka (2001) CCR model	0.00	(0.66,0.81,0.99)	(0.88,0.98,1.09)	(0.60,0.82,1.12)	(0.71, 0.3,1.25)	(0.61,0.79,1.02)
	0.50	(0.75,0.83,0.92)	(0.94,0.97,1.00)	(0.12,0.83,0.14)	(0.85, 0.97,1.12)	(0.72,0.82,0.93)
	0.75	(0.77,0.81,0.99)	(0.80,0.98,1.09)	(0.22,0.82,0.30)	(0.71,0.93,1.25)	(0.61,0.79,1.02)
	1.00	(0.85,0.85,0.94)	(1.00,1.00,1.00)	(0.86,0.86,0.86)	(1.00,1.00,1.00)	(1.00,1.00,1.00)
Lertworasirikul et.al. (2003a) CCR model	0.00	1.107	1.238	1.276	1.520	1.296
	0.50	0.963	1.112	1.035	1.258	1.159
	0.75	0.904	1.055	0.932	1.131	1.095
	1.00	1.000	0.855	1.000	0.861	1.000
Saati et al. (2002) CCR model	0.00	1.000	1.000	1.000	1.000	1.000
	0.50	0.954	1.000	1.000	1.000	1.000
	0.75	0.901	1.000	0.929	1.000	1.000
	1.00	0.855	1.000	0.862	1.000	1.000
Lertworasirikul et al. (2003b) BCC model	0.00	1.299	1.247	1.699	1.692	∞
	0.50	1.062	1.119	1.243	1.300	∞
	0.75	0.969	1.059	1.074	1.142	∞
	1.00	0.889	1.000	0.935	1.000	1.000
Majid et.al. (2011) CCR model	0.00	0.915	1.000	0.948	1.000	0.991
	0.50	0.909	1.000	0.945	1.000	0.991
	0.75	0.903	1.000	0.941	1.000	0.840
	1.00	1.000	1.000	1.000	1.000	1.000
Hatami et.al (2012)BCC model	0.00	1.000	1.000	1.000	1.000	1.000
	0.50	1.000	1.000	1.000	1.000	1.000
	0.75	0.982	1.000	1.000	1.000	1.000
	1.00	0.918	1.000	0.960	1.000	1.000
Proposed	0.00	(0.71,0.85,0.99)	1.000	(0.60,0.80,1.00)	(0.70, 0.85,1.00)	1.000
	0.50	(0.76,0.86,0.96)	1.000	(0.62, 0.83,0.94)	(0.85, 0.97,1.12)	1.000
	0.75	(0.76,0.87,0.98)	1.000	(0.62, 0.85,0.98)	(0.71, 0.93,1.25)	1.000
	1.00	(0.83,0.89,0.95)	1.000	(0.86,0.86,0.86)	(1.00,1.00,1.00)	1.000

Table 4.11. The results for Model (4.24) and the ranking of each DMU for different α Values

DMU	Interval Score							
	$\alpha = 0$	Rank	$\alpha = 0.5$	Rank	$\alpha = 0.75$	Rank	$\alpha = 1.00$	Rank
A	(0.71,0.85, 099)	3	(0.76,0.86, 0.96)	4	(0.76,0.87,0.98)	3	(0.83,0.89, 0.95)	5
B	1	1	1	1	1	1	1	1
C	(0.60,0.80, 1.00)	5	(0.62,0.83, 0.94)	5	(0.62,0.85,0.98)	5	(0.86,0.86, 0.86)	4
D	(0.70,0.85, 1.00)	4	(0.85,0.97, 1.12)	3	(0.71,0.93,1.25)	4	(1.00,1.00, 1.00)	3
E	1	2	1	2	1	2	1	2

We use these models (4.7,4.8,4.19) proposed in this study, in order to find the efficiencies of the DMUs presented in Tables as 4.1 and 4.4 On top of that, the optimal values for the successful DMUs are reported in Table 4.2 and 4.5 to look for the efficiency. For example 1 the ranking DMU A is uccessful when α is equal to 0, 0. 5, 0.75 and 1.0. and for example 2 the ranking DMU D1,D2 and D10 are successful for all α values.

We use these models (4.13,4.14,4.24) proposed in this study, in order to find the efficiencies of the DMUs presented in Tables as 4.1 and 4.9 On top of that, the optimal values of u_0 for the successful DMUs are reported in Table 4.7 and 4.10 to look for the efficiency. For example 3 the ranking DMU A is successful when α is equal to 0, 0. 5, 0.75 and 1.0. and for example 4 the ranking DMU B and E are successful for all α values.

5. APPLICATION AND SIMULATION

5.1. Applications of the Fuzzy CCR and BCC Models

In this chapter, we used an alternative fuzzy CCR and fuzzy BCC models for evaluating the efficiency of a set of DMUs with fuzzy inputs and outputs derived from the α-cut approach that we proposed. Let us consider n DMUs, each of which uses m different fuzzy inputs to generate s different fuzzy outputs. To calculate of relative efficiency values of commercial banks working in Turkey in 2011 by using DEA was aimed (Budak and Erpolat 2013). FCCR and FBCC model was used to calculate efficiency values of the banks.

The select of DMUs and variables which would be used in DEA is rather important because of its high effect on efficiency values. For the assumption to be homogeneous of units that were measured efficiency, investment bank, development bank, participation bank and branches of the foreign banks in Turkey were excluded from the study 21 commercial banks selected for analysis. 2(Number of branch, Number of staff) input and 3 (Total credit, Total income, Net profit) output variables which usually used in similar studies were selected and 2011 end-of-year data of aforementioned variables were gathered from the official website of banks association of Turkey as in the Appendix 1.

To express of certain data as fuzzy data, standard deviation values were used and fuzzy data in Appendix 2 and Appendix 3 to blur it 1 percent of standard deviation with the formulas:

Lower Bound Data (L) = Actual Data - Standard Deviation*0,01
Midpoint (M) = Actual Data
Upper Bound Data (U) = Actual Data + Standard Deviation*0,01

And we get the α bounds of input variables and output variables were used and fuzzy data in Appendix 4 and Appendix 5.

We apply the models that we proposed as follows:

5.1.1. Application model FCCR (α-cuts)

For Ziraat Bank

$$\max\ E^{\alpha U}(1) = \max [\ (\alpha 71173.26 + (1-\ \alpha)71488.75)\ u_1 + (\alpha 6209.94 + (1-\alpha)6238.01)u_2 \quad + \quad (\alpha 2100.67 + (1-\ \alpha)2111.2)u_3]$$

s.t.

$$(\alpha 1458 + (1-\alpha)1453.65)v_1 + (\alpha 24374 + (1-\ \alpha)24296.14)v_2\ = 1$$

Constraints U output-L input

$$[(\alpha 71173.26 + (1-\ \alpha)71488.75)u_1 + (\alpha 6209.94 + (1-\ \alpha)6238.01)u_2 \quad + (\alpha 2100.67 + (1-\ \alpha)2111.2)u_3] - [(\alpha 1458 + (1-\alpha)1453.65)v_1 + (\alpha 24374 + (1-\alpha)24296.14)v_2]\ \leq 0$$

$$[(\alpha 91620.64 + (1-\ \alpha)91936.12)u_1 + (\alpha 8163.27 + (1-\ \alpha)8191.34)u_2 \quad + (\alpha 2667.49 + (1-\ \alpha)2678.01)u_3] - [(\alpha 1201 + (1-\ \alpha)1196.65)v_1 + (\alpha 24887 + (1-\ \alpha)24809.14)v_2]\ \leq 0$$

$$[(\alpha 70213.22 + (1-\ \alpha)70528.71)u_1 + (\alpha 6083.46 + (1-\ \alpha)6111.52)u_2 \quad + (\alpha 2394.53 + (1-\ \alpha)2405.05)u_3] - [(\alpha 927 + (1-\alpha)922.65)v_1 + (\alpha 15339 + (1-\alpha)15261.14)v_1]\ \leq 0$$

$$[(\alpha 83532.93 + (1-\ \alpha)83848.42)u_1 + (\alpha 7936.87 + (1-\ \alpha)7964.94)u_2 \quad + (\alpha 3070.58 + (1-\ \alpha)3081.1)u_3] - [(\alpha 914 + (1-\alpha)909.65)v_1 + (\alpha 16773 + (1-\alpha)16695.14)v_2]\ \leq 0$$

$$[(\alpha 67044.87 + (1-\ \alpha)67360.36)u_1 + (\alpha 5821.77 + (1-\ \alpha)5849.84)u_2 \quad + (\alpha 1857.49 + (1-\ \alpha)1868.01)u_3] - [(\alpha 907 + (1-\alpha)902.65)v_1 + (\alpha 14859 + (1-\alpha)14781.14)v_2]\ \leq 0$$

$[(\alpha 55949.45+(1-\alpha)56264.93)u_1+(\alpha 5051.93+(1-\alpha)5080)u_2 + (\alpha 2045.13+(1-\alpha)2055.66)u_3]-[(\alpha 771+(1-\alpha)766.65)v_1+(\alpha 13643+(1-\alpha)13565.14)v_2] \leq 0$

$[(\alpha 57200.61+(1-\alpha)57516.1)u_1+(\alpha 4422.16+(1-\alpha)4450.22)u_2 + (\alpha 1226.79+(1-\alpha)1237.31)u_3]-[(\alpha 680+(1-\alpha)675.65)v_1+(\alpha 12222+(1-\alpha)12144.14)v_2] \leq 0$

$[(\alpha 22196.37+(1-\alpha)22511.86)u_1+(\alpha 2291.8+(1-\alpha)2319.87)u_2 + (\alpha 873.97+(1-\alpha)884.5)u_3]-[(\alpha 588+(1-\alpha)583.65)v_1+(\alpha 9772+(1-\alpha)9694.14)v_2] \leq 0$

$[(\alpha 29867.2+(1-\alpha)30182.68)u_1+(\alpha 3000+(1-\alpha)3028.07)u_2 + (\alpha 848.11+(1-\alpha)858.64)u_3]-[(\alpha 522+(1-\alpha)517.65)v_1+(\alpha 10837+(1-\alpha)10759.14)v_2] \leq 0$

$[(\alpha 25443.64+(1-\alpha)25759.13)u_1+(\alpha 1793.32+(1-\alpha)1821.39)u_2 + (\alpha 206.68+(1-\alpha)217.2)u_3]-[(\alpha 507+(1-\alpha)502.65)v_1+(\alpha 9356+(1-\alpha)9278.14)v_2] \leq 0$

$[(\alpha 13662.37+(1-\alpha)13977.86)u_1+(\alpha 1534.52+(1-\alpha)1562.58)u_2 + (\alpha 240.79+(1-\alpha)251.31)u_3]-[(\alpha 330+(1-\alpha)325.65)v_1+(\alpha 6155+(1-\alpha)6077.14)v_2] \leq 0$

$[(\alpha 15264.74+(1-\alpha)15580.22)u_1+(\alpha 1100.21+(1-\alpha)1128.27)u_2 + \alpha 79.01+(1-\alpha)89.54)u_3]-[(\alpha 322+(1-\alpha)317.65)v_1+(\alpha 3530+(1-\alpha)3452.14)v_2] \leq 0$

$[(\alpha 8282.22+(1-\alpha)8597.71)u_1+(\alpha 876.92+(1-\alpha)904.98)u_2 + (\alpha 118.04+(1-\alpha)128.57)u_3]-[(\alpha 272+(1-\alpha)267.65)v_1+(\alpha 3530+(1-\alpha)3452.14)v_2] \leq 0$

$[(\alpha 3715.39+(1-\alpha)4030.88)u_1+(\alpha 335.91+(1-\alpha)363.98)u_2 + (\alpha 85.23+(1-\alpha)95.76)u_3]-[(\alpha 88+(1-\alpha)83.65)v_1+(\alpha 1911+(1-\alpha)1833.14)v_2] \leq 0$

$[(\alpha 4220.59 + (1-\alpha)4536.07)u_1 + (\alpha 288.19 + (1-\alpha)316.26)u_2 + (\alpha 28.26 + (1-\alpha)38.79)u_3] - [(\alpha 63 + (1-\alpha)58.65)v_1 + (\alpha 1185 + (1-\alpha)1107.14)v_2] \leq 0$

$[(\alpha 2291.94 + (1-\alpha)2607.43)u_1 + (\alpha 209.61 + (1-\alpha)237.67)u_2 + (\alpha 37.37 + (1-\alpha)47.89)u_3] - [(\alpha 59 + (1-\alpha)54.65)v_1 + (\alpha 954 + (1-\alpha)876.14)v_2] \leq 0$

$[(\alpha 2461.06 + (1-\alpha)2776.54)u_1 + (\alpha 147.24 + (1-\alpha)175.31)u_2 + (\alpha 22.02 + (1-\alpha)32.54)u_3] - [(\alpha 44 + (1-\alpha)39.65)v_1 + (\alpha 880 + (1-\alpha)802.14)v_2] \leq 0$

$[(\alpha 2652.96 + (1-\alpha)2968.45)u_1 + (\alpha 421.7 + (1-\alpha)449.77)u_2 + (\alpha 5.51 + (1-\alpha)16.03)u_3] - [(\alpha 37 + (1-\alpha)32.65)v_1 + (\alpha 2233 + (1-\alpha)2155.14)v_2] \leq 0$

$[(\alpha 1445.38 + (1-\alpha)1760.86)u_1 + (\alpha 105.83 + (1-\alpha)133.9)u_2 + (\alpha 3.56 + (1-\alpha)14.09)u_3] - [(\alpha 27 + (1-\alpha)22.65)v_1 + (\alpha 496 + (1-\alpha)418.14)v_2] \leq 0$

$[(\alpha 2095.04 + (1-\alpha)2410.52)u_1 + (\alpha 91 + (1-\alpha)119.06)u_2 + (\alpha 5.19 + (1-\alpha)15.71)u_3] - [(\alpha 21 + (1-\alpha)16.65)v_1 + (\alpha 438 + (1-\alpha)360.14)v_2] \leq 0$

$[(\alpha 256.57 + (1-\alpha)572.06)u_1 + (\alpha 33.39 + (1-\alpha)61.46)u_2 + (\alpha 0.69 + (1-\alpha)11.22)u_3] - [(\alpha 20 + (1-\alpha)15.65)v_1 + (\alpha 284 + (1-\alpha)206.14)v_2] \leq 0$

<u>Constraints L output-U input</u>

$[(\alpha 71173.26 + (1-\alpha)70857.77)u_1 + (\alpha 6209.94 + (1-\alpha)6181.87)u_2 + (\alpha 2100.67 + (1-\alpha)2090.15)u_3] - [(\alpha 1458 + (1-\alpha)1462.35)v_1 + (\alpha 24374 + (1-\alpha)24451.86)v_2] \leq 0$

$[(\alpha 91620.64 + (1-\alpha)91305.15)u_1 + (\alpha 8163.27 + (1-\alpha)8135.21)u_2 + (\alpha 2667.49 + (1-\alpha)2656.96)u_3] - [(\alpha 1201 + (1-\alpha)1205.35)v_1 + (\alpha 24887 + (1-\alpha)24964.86)v_2] \leq 0$

$[(\alpha 70213.22 + (1-\alpha)69897.74)u_1 + (\alpha 6083.46 + (1-\alpha)6055.39)u_2 + (\alpha 2394.53 + (1-\alpha)2384)u_3] - [(\alpha 927 + (1-\alpha)931.35)v_1 + (\alpha 15339 + (1-\alpha)15416.86)v_2] \leq 0$

$[(\alpha 83532.93 + (1-\ \alpha)83217.44)u_1 + (\alpha 7936.87 + (1-\ \alpha)7908.81)u_2 \quad + (\alpha 3070.58 + (1-\ \alpha)3060.05)u_3] - [(\alpha 914 + (1-\alpha)918.35)v_1 + (\alpha 16773 + (1-\alpha)16850.86)v_2] \ \leq 0$

$[(\alpha 67044.87 + (1-\ \alpha)66729.38)u_1 + (\alpha 5821.77 + (1-\ \alpha)5793.71)u_2 \quad + (\alpha 1857.49 + (1-\ \alpha)1846.96)u_3] - [(\alpha 907 + (1-\alpha)911.35)v_1 + (\alpha 14859 + (1-\alpha)14936.86)v_2] \ \leq 0$

$[(\alpha 55949.45 + (1-\ \alpha)55633.96)u_1 + (\alpha 5051.93 + (1-\ \alpha)5023.87)u_2 \quad + (\alpha 2045.13 + (1-\ \alpha)2034.61)u_3] - [(\alpha 771 + (1-\alpha)775.35)v_1 + (\alpha 13643 + (1-\alpha)13720.86)v_2] \ \leq 0$

$[(\alpha 57200.61 + (1-\ \alpha)56885.12)u_1 + (\alpha 4422.16 + (1-\ \alpha)4394.09)u_2 \quad + (\alpha 1226.79 + (1-\ \alpha)1216.26)u_3] - [(\alpha 680 + (1-\alpha)684.35)v_1 + (\alpha 12222 + (1-\alpha)12299.86)v_2] \ \leq 0$

$[(\alpha 22196.37 + (1-\ \alpha)21880.89)u_1 + (\alpha 2291.8 + (1-\ \alpha)2263.74)u_2 \quad + (\alpha 873.97 + (1-\ \alpha)863.45)u_3] - [(\alpha 22196.37 + (1-\ \alpha)22511.86)v_1 + (\alpha 9772 + (1-\alpha)9849.86)v_2] \ \leq 0$

$[(\alpha 29867.2 + (1-\ \alpha)29551.71)u_1 + (\alpha 3000 + (1-\ \alpha)2971.93)u_2 \quad + (\alpha 848.11 + (1-\ \alpha)837.59)u_3] - [(\alpha 522 + (1-\alpha)526.35)v_1 + (\alpha 10837 + (1-\ \alpha)10914.86)v_2] \leq 0$

$[(\alpha 25443.64 + (1-\ \alpha)25128.16)u_1 + (\alpha 1793.32 + (1-\ \alpha)1765.25)u_2 \quad + (\alpha 206.68 + (1-\ \alpha)196.15)u_3] - [(\alpha 507 + (1-\alpha)511.35)v_1 + (\alpha 9356 + (1-\alpha)9433.86)v_2] \ \leq 0$

$[(\alpha 13662.37 + (1-\ \alpha)13346.89)u_1 + (\alpha 1534.52 + (1-\ \alpha)1506.45)u_2 \quad + (\alpha 240.79 + (1-\ \alpha)230.26)u_3] - [(\alpha 330 + (1-\alpha)334.35)v_1 + (\alpha 6155 + (1-\alpha)6232.86)v_2] \ \leq 0$

$[(\alpha 15264.74 + (1 - \alpha)14949.25)u_1 + (\alpha 1100.21 + (1 - \alpha)1072.14)u_2 + (\alpha 79.01 + (1 - \alpha)68.49)u_3] - [(\alpha 322 + (1 - \alpha)326.35)v_1 + (\alpha 5232 + (1 - \alpha)5309.86)v_2] \leq 0$

$[(\alpha 8282.22 + (1 - \alpha)7966.74)u_1 + (\alpha 876.92 + (1 - \alpha)848.85)u_2 + (\alpha 118.04 + (1 - \alpha)107.52)u_3] - [(\alpha 272 + (1 - \alpha)276.35)v_1 + (\alpha 3530 + (1 - \alpha)3607.86)v_2] \leq 0$

$[(\alpha 3715.39 + (1 - \alpha)3399.91)u_1 + (\alpha 335.91 + (1 - \alpha)307.85)u_2 + (\alpha 85.23 + (1 - \alpha)74.7)u_3] - [(\alpha 88 + (1 - \alpha)92.35)v_1 + (\alpha 1911 + (1 - \alpha)1988.86)v_2] \leq 0$

$[(\alpha 4220.59 + (1 - \alpha)3905.1)u_1 + (\alpha 288.19 + (1 - \alpha)260.13)u_2 + (\alpha 28.26 + (1 - \alpha)17.74)u_3] - [(\alpha 63 + (1 - \alpha)67.35)v_1 + (\alpha 1185 + (1 - \alpha)1262.86)v_2] \leq 0$

$[(\alpha 2291.94 + (1 - \alpha)1976.46)u_1 + (\alpha 2291.94 + (1 - \alpha)2607.43)u_2 + (\alpha 37.37 + (1 - \alpha)26.84)u_3] - [(\alpha 59 + (1 - \alpha)63.35)v_1 + (\alpha 954 + (1 - \alpha)1031.86)v_2] \leq 0$

$[(\alpha 2461.06 + (1 - \alpha)2145.57)u_1 + (\alpha 147.24 + (1 - \alpha)119.18)u_2 + (\alpha 22.02 + (1 - \alpha)11.49)u_3] - [(\alpha 44 + (1 - \alpha)48.35)v_1 + (\alpha 880 + (1 - \alpha)957.86)v_2] \leq 0$

$[(\alpha 2652.96 + (1 - \alpha)2337.48)u_1 + (\alpha 421.7 + (1 - \alpha)393.64)u_2 + (\alpha 5.51 + (1 - \alpha) - 5.02)u_3] - [(\alpha 37 + (1 - \alpha)41.35)v_1 + (\alpha 2233 + (1 - \alpha)2310.86)v_2] \leq 0$

$[(\alpha 1445.38 + (1 - \alpha)1129.89)u_1 + (\alpha 105.83 + (1 - \alpha)77.76)u_2 + (\alpha 3.56 + (1 - \alpha) - 6.96)u_3] - [(\alpha 27 + (1 - \alpha)31.35)v_1 + (\alpha 496 + (1 - \alpha)573.86)v_2] \leq 0$

$[(\alpha 2095.04 + (1 - \alpha)1779.55)u_1 + (\alpha 91 + (1 - \alpha)62.93)u_2 + (\alpha 5.19 + (1 - \alpha) - 5.34)u_3] - [(\alpha 21 + (1 - \alpha)25.35)v_1 + (\alpha 438 + (1 - \alpha)515.86)v_2] \leq 0$

$[(\alpha 256.57 + (1 - \alpha) - 58.91)u_1 + (\alpha 33.39 + (1 - \alpha)5.32)u_2 + (\alpha 0.69 + (1 - \alpha) - 9.83)u_3] - [(\alpha 20 + (1 - \alpha)24.35)v_1 + (\alpha 284 + (1 - \alpha)361.86)v_2] \leq 0$

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$

$\max \ E^{\alpha L}(1) = \max [(\alpha 71173.26 + (1-\alpha)70857.77)u_1 + (\alpha 6209.94 + (1-\alpha)6181.87)u_2 + (\alpha 2100.67 + (1-\alpha)2090.15)u_3]$

s.t.

$(\alpha 1458 + (1-\alpha)1462.35)v_1 + (\alpha 24374 + (1-\alpha)24451.86)v_2 = 1$

The same constraints U output-L input

The same constraints L output-U input

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$

<u>For İş Bank</u>

$\max \ E^{\alpha U}(2) = \max [(\alpha 91620.64 + (1-\alpha)91936.12)u_1 + (\alpha 8163.27 + (1-\alpha)\ 8191.34)u_2 + (\alpha 2667.49 + (1-\alpha)2678.01)u_3)]$

s.t.

$(\alpha 1201 + (1-\alpha)1196.65)v_1 + (\alpha 24887 + (1-\alpha)24809.14)v_2 = 1$

The same constraints U output-L input

The same constraints L output –U input

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$

$\max \ E^{\alpha L}(2) = \max [(\alpha 91620.64 + (1-\alpha)91305.15)u_1 + (\alpha 8163.27 + (1-\alpha)8135.21)u_2 + \alpha 2667.49 + (1-\alpha)2656.96)u_3]$

s.t.

$(\alpha 1201 + (1-\alpha)1205.35)v_1 + (\alpha 24887 + (1-\alpha)24964.86)v_2 = 1$

The same constraints L output –U input

The same constraints U output - L input

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$

5.1.2. Application model FCCR (α-level)

For Ziraat Bank

$$\max \ E^{\alpha}(1) = \max \ (\bar{y}_{11} + \bar{y}_{21} + \bar{y}_{31})$$

s.t.

$$\bar{x}_{11} + \bar{x}_{21} = 1$$

$$(\bar{y}_{11} + \bar{y}_{21} + \bar{y}_{31}) - (\bar{x}_{11} + \bar{x}_{21}) \leq 0$$

$$(\bar{y}_{12} + \bar{y}_{22} + \bar{y}_{32}) - (\bar{x}_{12} + \bar{x}_{22}) \leq 0$$

$$(\bar{y}_{13} + \bar{y}_{23} + \bar{y}_{33}) - (\bar{x}_{13} + \bar{x}_{23}) \leq 0$$

$$(\bar{y}_{14} + \bar{y}_{24} + \bar{y}_{34}) - (\bar{x}_{14} + \bar{x}_{24}) \leq 0$$

$$(\bar{y}_{15} + \bar{y}_{25} + \bar{y}_{35}) - (\bar{x}_{15} + \bar{x}_{25}) \leq 0$$

$$(\bar{y}_{16} + \bar{y}_{26} + \bar{y}_{36}) - (\bar{x}_{16} + \bar{x}_{26}) \leq 0$$

$$(\bar{y}_{17} + \bar{y}_{27} + \bar{y}_{37}) - (\bar{x}_{17} + \bar{x}_{27}) \leq 0$$

$$(\bar{y}_{18} + \bar{y}_{28} + \bar{y}_{38}) - (\bar{x}_{18} + \bar{x}_{28}) \leq 0$$

$$(\bar{y}_{19} + \bar{y}_{29} + \bar{y}_{39}) - (\bar{x}_{19} + \bar{x}_{29}) \leq 0$$

$$(\bar{y}_{110} + \bar{y}_{210} + \bar{y}_{310}) - (\bar{x}_{110} + \bar{x}_{210}) \leq 0$$

$$(\bar{y}_{111} + \bar{y}_{211} + \bar{y}_{311}) - (\bar{x}_{111} + \bar{x}_{211}) \leq 0$$

$$(\bar{y}_{112} + \bar{y}_{212} + \bar{y}_{312}) - (\bar{x}_{112} + \bar{x}_{212}) \leq 0$$

$$(\bar{y}_{113} + \bar{y}_{213} + \bar{y}_{313}) - (\bar{x}_{113} + \bar{x}_{213}) \leq 0$$

$$(\bar{y}_{114} + \bar{y}_{214} + \bar{y}_{314}) - (\bar{x}_{114} + \bar{x}_{214}) \leq 0$$

$$(\bar{y}_{115} + \bar{y}_{215} + \bar{y}_{315}) - (\bar{x}_{115} + \bar{x}_{215}) \leq 0$$

$(\bar{y}_{116} + \bar{y}_{216} + \bar{y}_{316}) - (\bar{x}_{116} + \bar{x}_{216}) \le 0$

$(\bar{y}_{117} + \bar{y}_{217} + \bar{y}_{317}) - (\bar{x}_{117} + \bar{x}_{217}) \le 0$

$(\bar{y}_{118} + \bar{y}_{218} + \bar{y}_{318}) - (\bar{x}_{118} + \bar{x}_{218}) \le 0$

$(\bar{y}_{119} + \bar{y}_{219} + \bar{y}_{319}) - (\bar{x}_{119} + \bar{x}_{219}) \le 0$

$(\bar{y}_{120} + \bar{y}_{220} + \bar{y}_{320}) - (\bar{x}_{120} + \bar{x}_{220}) \le 0$

$(\bar{y}_{121} + \bar{y}_{221} + \bar{y}_{321}) - (\bar{x}_{121} + \bar{x}_{221}) \le 0$

$u_1(\alpha 71173.26 + (1-\alpha)70857.77) \le \bar{y}_{11} \le u_1(\alpha 71173.26 + (1-\alpha)71488.75)$

$u_1(\alpha 91620.64 + (1-\alpha)91305.15) \le \bar{y}_{12} \le u_1(\alpha 91620.64 + (1-\alpha)91936.12)$

$u_1(\alpha 70213.22 + (1-\alpha)69897.74) \le \bar{y}_{13} \le u_1(\alpha 70213.22 + (1-\alpha)70528.71)$

$u_1(\alpha 83532.93 + (1-\alpha)83217.44) \le \bar{y}_{14} \le u_1(\alpha 83532.93 + (1-\alpha)83848.42)$

$u_1(\alpha 67044.87 + (1-\alpha)66729.38) \le \bar{y}_{15} \le u_1(\alpha 67044.87 + (1-\alpha)67360.36)$

$u_1(\alpha 55949.45 + (1-\alpha)55633.96) \le \bar{y}_{16} \le u_1(\alpha 55949.45 + (1-\alpha)56264.93)$

$u_1(\alpha 57200.61 + (1-\alpha)56885.12) \le \bar{y}_{17} \le u_1(\alpha 57200.61 + (1-\alpha)57516.1)$

$u_1(\alpha 22196.37 + (1-\alpha)21880.89) \le \bar{y}_{18} \le u_1(\alpha 22196.37 + (1-\alpha)22511.86)$

$u_1(\alpha 29867.2 + (1-\alpha)29551.71) \le \bar{y}_{19} \le u_1(\alpha 29867.2 + (1-\alpha)30182.68)$

$u_1(\alpha 25443.64 + (1-\alpha)25128.16) \le \bar{y}_{110} \le u_1(\alpha 25443.64 + (1-\alpha)25759.13)$

$u_1(\alpha 13662.37 + (1-\alpha)13346.89) \le \bar{y}_{111} \le u_1(\alpha 13662.37 + (1-\alpha)13977.86)$

$u_1(\alpha 15264.74 + (1-\alpha)14949.25) \le \bar{y}_{112} \le u_1(\alpha 15264.74 + (1-\alpha)15580.22)$

$u_1(\alpha 8282.22 + (1-\alpha)7966.74) \le \bar{y}_{113} \le u_1(\alpha 8282.22 + (1-\alpha)8597.71)$

$u_1(\alpha 3715.39 + (1-\alpha)3399.91) \le \bar{y}_{114} \le u_1(\alpha 3715.39 + (1-\alpha)4030.88)$

$u_1(\alpha 4220.59 + (1-\alpha)3905.1) \le \bar{y}_{115} \le u_1(\alpha 4220.59 + (1-\alpha)4536.07)$

$u_1(\alpha 2291.94 + (1-\alpha)1976.46) \le \bar{y}_{116} \le u_1(\alpha 2291.94 + (1-\alpha)2607.43)$

$u_1(\alpha 2461.06 + (1-\alpha)2145.57) \le \bar{y}_{117} \le u_1(\alpha 2461.06 + (1-\alpha)2776.54)$

$u_1(\alpha 2652.96 + (1-\ \alpha)2337.48\ \ ) \le \bar{y}_{118}\ \le u_1(\ \alpha 2652.96 + (1-\ \alpha)2968.45\ )$

$u_1(\alpha 1445.38 + (1-\ \alpha)1129.89\ ) \le \bar{y}_{119}\ \le u_1(\alpha 1445.38 + (1-\ \alpha)1760.86\ )$

$u_1(\alpha 2095.04 + (1-\ \alpha)1779.55\ ) \le \bar{y}_{120}\ \le u_1(\alpha 2095.04 + (1-\ \alpha)2410.52\ )$

$u_1(\alpha 256.57 + (1-\ \alpha) - 58.91\ ) \le \bar{y}_{121}\ \le u_1(\alpha 256.57 + (1-\ \alpha)572.06\ )$

$u_2(\alpha 6209.94 + (1-\ \alpha)6181.87\ ) \le \bar{y}_{21}\ \le u_2(\alpha 6209.94 + (1-\ \alpha)6238.01\ )$

$u_2(\alpha 8163.27 + (1-\ \alpha)8135.21\ ) \le \bar{y}_{22}\ \le u_2(\alpha 8163.27 + (1-\ \alpha)8191.34\ )$

$u_2(\alpha 6083.46 + (1-\ \alpha)6055.39\ ) \le \bar{y}_{23}\ \le u_2(\alpha 6083.46 + (1-\ \alpha)6111.52\ )$

$u_2(\alpha 7936.87 + (1-\ \alpha)7908.81\ ) \le \bar{y}_{24}\ \le u_2(\alpha 7936.87 + (1-\ \alpha)7964.94\ )$

$u_2(\alpha 5821.77 + (1-\ \alpha)5793.71\ ) \le \bar{y}_{25}\ \le u_2(\alpha 5821.77 + (1-\ \alpha)5849.84\ )$

$u_2(\alpha 5051.93 + (1-\ \alpha)5023.87\ ) \le \bar{y}_{26}\ \le u_2(\alpha 5051.93 + (1-\ \alpha)5080\ )$

$u_2(\alpha 4422.16 + (1-\ \alpha)4394.09\ ) \le \bar{y}_{27}\ \le u_2(\alpha 4422.16 + (1-\ \alpha)4450.22\ )$

$u_2(\alpha 2291.8 + (1-\ \alpha)2263.74\ ) \le \bar{y}_{28}\ \le u_2(\alpha 2291.8 + (1-\ \alpha)2319.87\ )$

$u_2(\alpha 3000 + (1-\ \alpha)2971.93\ ) \le \bar{y}_{29}\ \le u_2(\alpha 3000 + (1-\ \alpha)3028.07\ )$

$u_2(\alpha 1793.32 + (1-\ \alpha)1765.25\ ) \le \bar{y}_{210}\ \le u_2(\alpha 1793.32 + (1-\ \alpha)1821.39\ )$

$u_2(\alpha 1534.52 + (1-\ \alpha)1506.45\ ) \le \bar{y}_{211}\ \le u_2(\alpha 1534.52 + (1-\ \alpha)1562.58\ )$

$u_2(\alpha 1100.21 + (1-\ \alpha)1072.14\ ) \le \bar{y}_{212}\ \le u_2(\alpha 1100.21 + (1-\ \alpha)1128.27\ )$

$u_2(\alpha 876.92 + (1-\ \alpha)848.85\ ) \le \bar{y}_{213}\ \le u_2(\alpha 876.92 + (1-\ \alpha)904.98\ )$

$u_2(\alpha 335.91 + (1-\ \alpha)307.85\ ) \le \bar{y}_{214}\ \le u_2(\alpha 335.91 + (1-\ \alpha)363.98\ )$

$u_2(\alpha 288.19 + (1-\ \alpha)260.13\ ) \le \bar{y}_{215}\ \le u_2(\alpha 288.19 + (1-\ \alpha)316.26\ )$

$u_2(\alpha 209.61 + (1-\ \alpha)181.54\ ) \le \bar{y}_{216}\ \le u_2(\alpha 209.61 + (1-\ \alpha)237.67\ )$

$u_2(\alpha 147.24 + (1-\ \alpha)119.18\ ) \le \bar{y}_{217}\ \le u_2(\alpha 147.24 + (1-\ \alpha)175.31\ )$

$u_2(\alpha 421.7 + (1-\ \alpha)393.64\ ) \le \bar{y}_{218}\ \le u_2(\alpha 421.7 + (1-\ \alpha)449.77\ )$

$u_2(\alpha 105.83 + (1-\ \alpha)77.76\ ) \le \bar{y}_{219}\ \le u_2(\alpha 105.83 + (1-\ \alpha)133.9\ )$

$$u_2(\alpha 91 + (1-\alpha)62.93) \le \bar{y}_{220} \le u_2(\alpha 91 + (1-\alpha)119.06)$$

$$u_2(\alpha 33.39 + (1-\alpha)5.32) \le \bar{y}_{221} \le u_2(\alpha 33.39 + (1-\alpha)61.46)$$

$$u_3(\alpha 2100.67 + (1-\alpha)2090.15) \le \bar{y}_{31} \le u_3(\alpha 2100.67 + (1-\alpha)2111.2)$$

$$u_3(\alpha 2667.49 + (1-\alpha)2656.96) \le \bar{y}_{32} \le u_3(\alpha 2667.49 + (1-\alpha)2678.01)$$

$$u_3(\alpha 2394.53 + (1-\alpha)2384) \le \bar{y}_{33} \le u_3(\alpha 2394.53 + (1-\alpha)2405.05)$$

$$u_3(\alpha 3070.58 + (1-\alpha)3060.05) \le \bar{y}_{34} \le u_3(\alpha 3070.58 + (1-\alpha)3081.1)$$

$$u_3(\alpha 1857.49 + (1-\alpha)1846.96) \le \bar{y}_{35} \le u_3(\alpha 1857.49 + (1-\alpha)1868.01)$$

$$u_3(\alpha 2045.13 + (1-\alpha)2034.61) \le \bar{y}_{36} \le u_3(\alpha 2045.13 + (1-\alpha)2055.66)$$

$$u_3(\alpha 1226.79 + (1-\alpha)1216.26) \le \bar{y}_{37} \le u_3(\alpha 1226.79 + (1-\alpha)1237.31)$$

$$u_3(\alpha 873.97 + (1-\alpha)863.45) \le \bar{y}_{38} \le u_3(\alpha 873.97 + (1-\alpha)884.5)$$

$$u_3(\alpha 848.11 + (1-\alpha)837.59) \le \bar{y}_{39} \le u_3(\alpha 848.11 + (1-\alpha)858.64)$$

$$u_3(\alpha 206.68 + (1-\alpha)196.15) \le \bar{y}_{310} \le u_3(\alpha 206.68 + (1-\alpha)217.2)$$

$$u_3(\alpha 240.79 + (1-\alpha)230.26) \le \bar{y}_{311} \le u_3(\alpha 240.79 + (1-\alpha)251.31)$$

$$u_3(\alpha 79.01 + (1-\alpha)68.49) \le \bar{y}_{312} \le u_3(\alpha 79.01 + (1-\alpha)89.54)$$

$$u_3(\alpha 118.04 + (1-\alpha)107.52) \le \bar{y}_{313} \le u_3(\alpha 118.04 + (1-\alpha)128.57)$$

$$u_3(\alpha 85.23 + (1-\alpha)74.7) \le \bar{y}_{314} \le u_3(\alpha 85.23 + (1-\alpha)95.76)$$

$$u_3(\alpha 28.26 + (1-\alpha)17.74) \le \bar{y}_{315} \le u_3(\alpha 28.26 + (1-\alpha)38.79)$$

$$u_3(\alpha 37.37 + (1-\alpha)26.84) \le \bar{y}_{316} \le u_3(\alpha 37.37 + (1-\alpha)47.89)$$

$$u_3(\alpha 22.02 + (1-\alpha)11.49) \le \bar{y}_{317} \le u_3(\alpha 22.02 + (1-\alpha)32.54)$$

$$u_3(\alpha 5.51 + (1-\alpha) - 5.02) \le \bar{y}_{318} \le u_3(\alpha 5.51 + (1-\alpha)16.03)$$

$$u_3(\alpha 3.56 + (1-\alpha) - 6.96) \le \bar{y}_{319} \le u_3(\alpha 3.56 + (1-\alpha)14.09)$$

$$u_3(\alpha 5.19 + (1-\alpha) - 5.34) \le \bar{y}_{320} \le u_3(\alpha 5.19 + (1-\alpha)15.71)$$

$$u_3(\alpha 0.69 + (1-\alpha) - 9.83) \le \bar{y}_{321} \le u_3(\alpha 0.69 + (1-\alpha)11.22)$$

$v_1(\alpha 1458 + (1-\alpha)1453.65) \quad \leq \bar{x}_{11} \ \leq v_1(\alpha 1458 + (1-\ \alpha)1462.35)$

$v_1(\alpha 1201 + (1-\ \alpha)1196.65) \quad \leq \bar{x}_{12} \ \leq v_1(\alpha 1201 + (1-\ \alpha)1205.35)$

$v_1(\alpha 927 + (1-\alpha)922.65) \quad \leq \bar{x}_{13} \ \leq v_1(\alpha 927 + (1-\alpha)931.35)$

$v_1(\alpha 914 + (1-\alpha)909.65) \quad \leq \bar{x}_{14} \ \leq v_1(\ \alpha 914 + (1-\alpha)918.35)$

$v_1(\alpha 907 + (1-\alpha)902.65) \quad \leq \bar{x}_{15} \ \leq v_1(\alpha 907 + (1-\alpha)911.35)$

$v_1(\alpha 771 + (1-\alpha)766.65) \quad \leq \bar{x}_{16} \ \leq v_1(\alpha 907 + (1-\alpha)911.35)$

$v_1(\alpha 680 + (1-\alpha)675.65) \quad \leq \bar{x}_{17} \ \leq v_1(\alpha 680 + (1-\alpha)684.35)$

$v_1(\alpha 588 + (1-\alpha)583.65) \quad \leq \bar{x}_{18} \ \leq v_1(\alpha 588 + (1-\alpha)592.35)$

$v_1(\alpha 522 + (1-\alpha)517.65) \quad \leq \bar{x}_{19} \ \leq v_1(\alpha 522 + (1-\alpha)526.35)$

$v_1(\alpha 507 + (1-\alpha)502.65) \quad \leq \bar{x}_{110} \ \leq v_1(\alpha 507 + (1-\alpha)511.35)$

$v_1(\alpha 330 + (1-\alpha)325.65) \quad \leq \bar{x}_{111} \ \leq v_1(\alpha 330 + (1-\alpha)334.35)$

$v_1(\alpha 322 + (1-\alpha)317.65) \quad \leq \bar{x}_{112} \ \leq v_1(\alpha 322 + (1-\alpha)326.35)$

$v_1(\alpha 272 + (1-\alpha)267.65) \quad \leq \bar{x}_{113} \ \leq v_1(\alpha 272 + (1-\alpha)276.35)$

$v_1(\alpha 88 + (1-\alpha)83.65) \quad \leq \bar{x}_{114} \ \leq v_1(\alpha 88 + (1-\alpha)92.35)$

$v_1(\alpha 63 + (1-\alpha)58.65) \quad \leq \bar{x}_{115} \ \leq v_1(\alpha 63 + (1-\alpha)67.35)$

$v_1(\alpha 59 + (1-\alpha)54.65) \quad \leq \bar{x}_{116} \ \leq v_1(\ \alpha 59 + (1-\alpha)63.35)$

$v_1(\alpha 44 + (1-\alpha)39.65) \quad \leq \bar{x}_{117} \ \leq v_1(\alpha 44 + (1-\alpha)48.35)$

$v_1(\alpha 37 + (1-\alpha)32.65) \quad \leq \bar{x}_{118} \ \leq v_1(\alpha 37 + (1-\alpha)41.35)$

$v_1(\alpha 27 + (1-\alpha)22.65) \quad \leq \bar{x}_{119} \ \leq v_1(\alpha 27 + (1-\alpha)31.35)$

$v_1(\alpha 21 + (1-\alpha)16.65) \quad \leq \bar{x}_{120} \ \leq v_1(\alpha 21 + (1-\alpha)25.35)$

$v_1(\alpha 20 + (1-\alpha)15.65) \quad \leq \bar{x}_{121} \ \leq v_1(\alpha 20 + (1-\alpha)24.35)$

$v_2(\alpha 24374 + (1-\ \alpha)24296.14) \ \leq \bar{x}_{21} \ \leq v_2(\alpha 24374 + (1-\ \alpha)24451.86)$

$v_2(\alpha 24887 + (1-\ \alpha)24809.14) \ \leq \bar{x}_{22} \ \leq v_2(\alpha 24887 + (1-\ \alpha)24964.86)$

$v_2(\alpha 15339 + (1 - \alpha)15261.14) \leq \bar{x}_{23} \leq v_2(\alpha 15339 + (1 - \alpha)15416.86)$

$v_2(\alpha 16773 + (1 - \alpha)16695.14) \leq \bar{x}_{24} \leq v_2(\alpha 16773 + (1 - \alpha)16850.86)$

$v_2(\alpha 14859 + (1 - \alpha)14781.14) \leq \bar{x}_{25} \leq v_2(\alpha 14859 + (1 - \alpha)14936.86)$

$v_2(\alpha 13643 + (1 - \alpha)13565.14) \leq \bar{x}_{26} \leq v_2(\alpha 13643 + (1 - \alpha)13720.86)$

$v_2\alpha 12222 + (1 - \alpha)12144.14() \leq \bar{x}_{27} \leq v_2(\alpha 12222 + (1 - \alpha)12299.86)$

$v_2(\alpha 9772 + (1 - \alpha)9694.14) \leq \bar{x}_{28} \leq v_2(\alpha 9772 + (1 - \alpha)9849.86)$

$v_2(\alpha 10837 + (1 - \alpha)10759.14) \leq \bar{x}_{29} \leq v_2(\alpha 10837 + (1 - \alpha)10914.86)$

$v_2(\alpha 9356 + (1 - \alpha)9278.14) \leq \bar{x}_{210} \leq v_2(\alpha 9356 + (1 - \alpha)9433.86)$

$v_2(\alpha 6155 + (1 - \alpha)6077.14) \leq \bar{x}_{211} \leq v_2(\alpha 6155 + (1 - \alpha)6232.86)$

$v_2(\alpha 5232 + (1 - \alpha)5154.14) \leq \bar{x}_{212} \leq v_2(\alpha 5232 + (1 - \alpha)5309.86)$

$v_2(\alpha 3530 + (1 - \alpha)3452.14) \leq \bar{x}_{213} \leq v_2(\alpha 3530 + (1 - \alpha)3607.86)$

$v_2(\alpha 1911 + (1 - \alpha)1833.14) \leq \bar{x}_{214} \leq v_2(\alpha 1911 + (1 - \alpha)1988.86)$

$v_2(\alpha 1185 + (1 - \alpha)1107.14) \leq \bar{x}_{215} \leq v_2(\alpha 1185 + (1 - \alpha)1262.86)$

$v_2(\alpha 954 + (1 - \alpha)876.14) \leq \bar{x}_{216} \leq v_2(\alpha 954 + (1 - \alpha)1031.86)$

$v_2(\alpha 880 + (1 - \alpha)802.14) \leq \bar{x}_{217} \leq v_2(\alpha 880 + (1 - \alpha)957.86)$

$v_2(\alpha 2233 + (1 - \alpha)2155.14) \leq \bar{x}_{218} \leq v_2(\alpha 2233 + (1 - \alpha)2310.86)$

$v_2(\alpha 496 + (1 - \alpha)418.14) \leq \bar{x}_{219} \leq v_2(\alpha 496 + (1 - \alpha)573.86)$

$v_2(\alpha 438 + (1 - \alpha)360.14) \leq \bar{x}_{220} \leq v_2(\alpha 438 + (1 - \alpha)515.86)$

$v_2(\alpha 284 + (1 - \alpha)206.14) \leq \bar{x}_{221} \leq v_2(\alpha 284 + (1 - \alpha)361.86)$

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$

For İş Bank

$\max\ E^{\alpha}(2) = \max\ (\bar{y}_{12} + \bar{y}_{22} + \bar{y}_{32})$

s.t.

$\bar{x}_{12} + \bar{x}_{22} = 1$

The same constraints

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$

5.1.3. Application model FBCC (α-cuts)

For Ziraat Bank

$\max\ E^{\alpha U}(1) = \max\ [\ (\alpha 71173.26 + (1-\alpha)71488.75)\, u_1 + (\alpha 6209.94 + (1-\alpha)6238.01)u_2 + (\alpha 2100.67 + (1-\alpha)2111.2)u_3]$ - u_0

s.t.

$(\alpha 1458 + (1-\alpha)1453.65)v_1 + (\alpha 24374 + (1-\alpha)24296.14)v_2 = 1$

Constraints U output-L input

$[(\alpha 71173.26 + (1-\alpha)71488.75)u_1 + (\alpha 6209.94 + (1-\alpha)6238.01)u_2 + (\alpha 2100.67 + (1-\alpha)2111.2)u_3] - [(\alpha 1458 + (1-\alpha)1453.65)v_1 + (\alpha 24374 + (1-\alpha)24296.14)v_2] - u_0 \leq 0$

$[(\alpha 91620.64 + (1-\alpha)91936.12)u_1 + (\alpha 8163.27 + (1-\alpha)8191.34)u_2 + (\alpha 2667.49 + (1-\alpha)2678.01)u_3] - [(\alpha 1201 + (1-\alpha)1196.65)v_1 + (\alpha 24887 + (1-\alpha)24809.14)v_2] - u_0 \leq 0$

$\big[(\alpha 70213.22 + (1-\alpha)70528.71)u_1 + (\alpha 6083.46 + (1-\alpha)6111.52)u_2 + (\alpha 2394.53 + (1-\alpha)2405.05)u_3] - [(\alpha 927 + (1-\alpha)922.65)v_1 + (\alpha 15339 + (1-\alpha)15261.14)v_2] - u_0 \leq 0\big]$

$[(\alpha 83532.93 + (1-\ \alpha)83848.42)u_1 + (\alpha 7936.87 + (1-\ \alpha)7964.94)u_2 \quad + (\alpha 3070.58 + (1-\ \alpha)3081.1)u_3] - [(\alpha 914 + (1-\alpha)909.65)v_1 + (\alpha 16773 + (1- \alpha)16695.14)v_2]$ - $u_0 \ \leq 0$

$[(\alpha 67044.87 + (1-\ \alpha)67360.36)u_1 + (\alpha 5821.77 + (1-\ \alpha)5849.84)u_2 \quad + (\alpha 1857.49 + (1-\ \alpha)1868.01)u_3] - [(\alpha 907 + (1-\alpha)902.65)v_1 + (\alpha 14859 + (1- \alpha)14781.14)v_2] -\ u_0 \ \leq 0$

$[(\alpha 55949.45 + (1-\ \alpha)56264.93)u_1 + (\alpha 5051.93 + (1-\ \alpha)5080)u_2 \quad + (\alpha 2045.13 + (1-\ \alpha)2055.66)u_3] - [(\alpha 771 + (1-\alpha)766.65)v_1 + (\alpha 13643 + (1- \alpha)13565.14)v_2]$ - $u_0 \leq 0$

$[(\alpha 57200.61 + (1-\ \alpha)57516.1)u_1 + (\alpha 4422.16 + (1-\ \alpha)4450.22)u_2 \quad + (\alpha 1226.79 + (1-\ \alpha)1237.31)u_3] - [(\alpha 680 + (1-\alpha)675.65)v_1 + (\alpha 12222 + (1- \alpha)12144.14)v_2]$ - $u_0 \ \leq 0$

$[(\alpha 22196.37 + (1-\ \alpha)22511.86)u_1 + (\alpha 2291.8 + (1-\ \alpha)2319.87)u_2 \quad + (\alpha 873.97 + (1-\ \alpha)884.5)u_3] - [(\alpha 588 + (1-\alpha)583.65)v_1 + (\alpha 9772 + (1- \alpha)9694.14)v_2]$ - $u_0 \ \leq 0$

$[(\alpha 29867.2 + (1-\ \alpha)30182.68)u_1 + (\alpha 3000 + (1-\ \alpha)3028.07)u_2 \ + (\alpha 848.11 + (1-\ \alpha)858.64)u_3] - [(\alpha 522 + (1-\alpha)517.65)v_1 + (\alpha 10837 + (1-\ \alpha)10759.14)v_2] - u_0 \ \leq 0$

$[(\alpha 25443.64 + (1-\ \alpha)25759.13)u_1 + (\alpha 1793.32 + (1-\ \alpha)1821.39)u_2 \quad + (\alpha 206.68 + (1-\ \alpha)217.2)u_3] - [(\alpha 507 + (1-\alpha)502.65)v_1 + (\alpha 9356 + (1- \alpha)9278.14)v_2]$ - $u_0 \leq 0$

$[(\alpha 13662.37 + (1-\ \alpha)13977.86)u_1 + (\alpha 1534.52 + (1-\ \alpha)1562.58)u_2 \quad + (\alpha 240.79 + (1-\ \alpha)251.31)u_3] - [(\alpha 330 + (1-\alpha)325.65)v_1 + (\alpha 6155 + (1- \alpha)6077.14)v_2] -\ u_0 \ \leq 0$

$[(\alpha 15264.74 + (1-\alpha)15580.22)u_1 + (\alpha 1100.21 + (1-\alpha)1128.27)u_2 + (\alpha 79.01 + (1-\alpha)89.54)u_3] - [(\alpha 322 + (1-\alpha)317.65)v_1 + (\alpha 3530 + (1-\alpha)3452.14)v_2] - u_0 \leq 0$

$[(\alpha 8282.22 + (1-\alpha)8597.71)u_1 + (\alpha 876.92 + (1-\alpha)904.98)u_2 + (\alpha 118.04 + (1-\alpha)128.57)u_3] - [(\alpha 272 + (1-\alpha)267.65)v_1 + (\alpha 3530 + (1-\alpha)3452.14)v_2] - u_0 \leq 0$

$[(\alpha 3715.39 + (1-\alpha)4030.88)u_1 + (\alpha 335.91 + (1-\alpha)363.98)u_2 + (\alpha 85.23 + (1-\alpha)95.76)u_3] - [(\alpha 88 + (1-\alpha)83.65)v_1 + (\alpha 1911 + (1-\alpha)1833.14)v_2] - u_0 \leq 0$

$[(\alpha 4220.59 + (1-\alpha)4536.07)u_1 + (\alpha 288.19 + (1-\alpha)316.26)u_2 + (\alpha 28.26 + (1-\alpha)38.79)u_3] - [(\alpha 63 + (1-\alpha)58.65)v_1 + (\alpha 1185 + (1-\alpha)1107.14)v_2] - u_0 \leq 0$

$[(\alpha 2291.94 + (1-\alpha)2607.43)u_1 + (\alpha 209.61 + (1-\alpha)237.67)u_2 + (\alpha 37.37 + (1-\alpha)47.89)u_3] - [(\alpha 59 + (1-\alpha)54.65)v_1 + (\alpha 954 + (1-\alpha)876.14)v_2] - u_0 \leq 0$

$[(\alpha 2461.06 + (1-\alpha)2776.54)u_1 + (\alpha 147.24 + (1-\alpha)175.31)u_2 + (\alpha 22.02 + (1-\alpha)32.54)u_3] - [(\alpha 44 + (1-\alpha)39.65)v_1 + (\alpha 880 + (1-\alpha)802.14)v_2] - u_0 \leq 0$

$[(\alpha 2652.96 + (1-\alpha)2968.45)u_1 + (\alpha 421.7 + (1-\alpha)449.77)u_2 + (\alpha 5.51 + (1-\alpha)16.03)u_3] - [(\alpha 37 + (1-\alpha)32.65)v_1 + (\alpha 2233 + (1-\alpha)2155.14)v_2] - u_0 \leq 0$

$[(\alpha 1445.38 + (1-\alpha)1760.86)u_1 + (\alpha 105.83 + (1-\alpha)133.9)u_2 + (\alpha 3.56 + (1-\alpha)14.09)u_3] - [(\alpha 27 + (1-\alpha)22.65)v_1 + (\alpha 496 + (1-\alpha)418.14)v_2] - u_0 \leq 0$

$[(\alpha 2095.04 + (1-\alpha)2410.52)u_1 + (\alpha 91 + (1-\alpha)119.06)u_2 + (\alpha 5.19 + (1-\alpha)15.71)u_3] - [(\alpha 21 + (1-\alpha)16.65)v_1 + (\alpha 438 + (1-\alpha)360.14)v_2] - u_0 \leq 0$

$[(\alpha 256.57 + (1 - \alpha)572.06)u_1 + (\alpha 33.39 + (1 - \alpha)61.46)u_2 + (\alpha 0.69 + (1 - \alpha)11.22)u_3] - [(\alpha 20 + (1 - \alpha)15.65)v_1 + (\alpha 284 + (1 - \alpha)206.14)v_2]$ - $u_0 \le 0$

<u>Constraints L output-U input</u>

$[(\alpha 71173.26 + (1 - \alpha)70857.77)u_1 + (\alpha 6209.94 + (1 - \alpha)6181.87)u_2 + (\alpha 2100.67 + (1 - \alpha)2090.15)u_3] - [(\alpha 1458 + (1 - \alpha)1462.35)v_1 + (\alpha 24374 + (1 - \alpha)24451.86)v_2]$- $u_0 \le 0$

$[(\alpha 91620.64 + (1 - \alpha)91305.15)u_1 + (\alpha 8163.27 + (1 - \alpha)8135.21)u_2 + (\alpha 2667.49 + (1 - \alpha)2656.96)u_3] - [(\alpha 1201 + (1 - \alpha)1205.35)v_1 + (\alpha 24887 + (1 - \alpha)24964.86)v_2]$- $u_0 \le 0$

$[(\alpha 70213.22 + (1 - \alpha)69897.74)u_1 + (\alpha 6083.46 + (1 - \alpha)6055.39)u_2 + (\alpha 2394.53 + (1 - \alpha)2384)u_3] - [(\alpha 927 + (1 - \alpha)931.35)v_1 + (\alpha 15339 + (1 - \alpha)15416.86)v_2]$ - $u_0 \le 0$

$[(\alpha 83532.93 + (1 - \alpha)83217.44)u_1 + (\alpha 7936.87 + (1 - \alpha)7908.81)u_2 + (\alpha 3070.58 + (1 - \alpha)3060.05)u_3] - [(\alpha 914 + (1 - \alpha)918.35)v_1 + (\alpha 16773 + (1 - \alpha)16850.86)v_2]$ - $u_0 \le 0$

$[(\alpha 67044.87 + (1 - \alpha)66729.38)u_1 + (\alpha 5821.77 + (1 - \alpha)5793.71)u_2 + (\alpha 1857.49 + (1 - \alpha)1846.96)u_3] - [(\alpha 907 + (1 - \alpha)911.35)v_1 + (\alpha 14859 + (1 - \alpha)14936.86)v_2] - u_0 \le 0$

$[(\alpha 55949.45 + (1 - \alpha)55633.96)u_1 + (\alpha 5051.93 + (1 - \alpha)5023.87)u_2 + (\alpha 2045.13 + (1 - \alpha)2034.61)u_3] - [(\alpha 771 + (1 - \alpha)775.35)v_1 + (\alpha 13643 + (1 - \alpha)13720.86)v_2] - u_0 \le 0$

$[(\alpha 57200.61 + (1 - \alpha)56885.12)u_1 + (\alpha 4422.16 + (1 - \alpha)4394.09)u_2 + (\alpha 1226.79 + (1 - \alpha)1216.26)u_3] - [(\alpha 680 + (1 - \alpha)684.35)v_1 + (\alpha 12222 + (1 - \alpha)12299.86)v_2] - u_0 \le 0$

$[(\alpha 22196.37 + (1-\alpha)21880.89)u_1 + (\alpha 2291.8 + (1-\alpha)2263.74)u_2 + (\alpha 873.97 + (1-\alpha)863.45)u_3] - [(\alpha 22196.37 + (1-\alpha)22511.86)v_1 + (\alpha 9772 + (1-\alpha)9849.86)v_2]$ - $u_0 \leq 0$

$[(\alpha 29867.2 + (1-\alpha)29551.71)u_1 + (\alpha 3000 + (1-\alpha)2971.93)u_2 \quad (\alpha 848.11 + (1-\alpha)837.59)u_3] - [(\alpha 522 + (1-\alpha)526.35)v_1 + (\alpha 10837 + (1-\alpha)10914.86)v_2]$ - $u_0 \leq 0$

$[(\alpha 25443.64 + (1-\alpha)25128.16)u_1 + (\alpha 1793.32 + (1-\alpha)1765.25)u_2 + (\alpha 206.68 + (1-\alpha)196.15)u_3] - [(\alpha 507 + (1-\alpha)511.35)v_1 + (\alpha 9356 + (1-\alpha)9433.86)v_2]$ - $u_0 \leq 0$

$[(\alpha 13662.37 + (1-\alpha)13346.89)u_1 + (\alpha 1534.52 + (1-\alpha)1506.45)u_2 + (\alpha 240.79 + (1-\alpha)230.26)u_3] - [(\alpha 330 + (1-\alpha)334.35)v_1 + (\alpha 6155 + (1-\alpha)6232.86)v_2] - u_0 \leq 0$

$[(\alpha 15264.74 + (1-\alpha)14949.25)u_1 + (\alpha 1100.21 + (1-\alpha)1072.14)u_2 + (\alpha 79.01 + (1-\alpha)68.49)u_3] - [(\alpha 322 + (1-\alpha)326.35)v_1 + (\alpha 5232 + (1-\alpha)5309.86)v_2] - u_0 \leq 0$

$[(\alpha 8282.22 + (1-\alpha)7966.74)u_1 + (\alpha 876.92 + (1-\alpha)848.85)u_2 + (\alpha 118.04 + (1-\alpha)107.52)u_3] - [(\alpha 272 + (1-\alpha)276.35)v_1 + (\alpha 3530 + (1-\alpha)3607.86)v_2] - u_0 \leq 0$

$[(\alpha 3715.39 + (1-\alpha)3399.91)u_1 + (\alpha 335.91 + (1-\alpha)307.85)u_2 + (\alpha 85.23 + (1-\alpha)74.7)u_3] - [(\alpha 88 + (1-\alpha)92.35)v_1 + (\alpha 1911 + (1-\alpha)1988.86)v_2] - u_0 \leq 0$

$[(\alpha 4220.59 + (1-\alpha)3905.1)u_1 + (\alpha 288.19 + (1-\alpha)260.13)u_2 + (\alpha 28.26 + (1-\alpha)17.74)u_3] - [(\alpha 63 + (1-\alpha)67.35)v_1 + (\alpha 1185 + (1-\alpha)1262.86)v_2] - u_0 \leq 0$

$[(\alpha 2291.94 + (1-\ \alpha)1976.46)u_1 + (\alpha 2291.94 + (1-\ \alpha)2607.43)u_2 \quad +$
$(\alpha 37.37 + (1-\alpha)26.84)u_3] - [(\alpha 59 + (1-\alpha)63.35)v_1 + (\alpha 954 + (1-$
$\alpha)1031.86)v_2] -\ u_0\ \leq 0$

$[(\alpha 2461.06 + (1-\ \alpha)2145.57)u_1 + (\alpha 147.24 + (1-\ \alpha)119.18)u_2 \quad + (\alpha 22.02 +$
$(1-\ \alpha)11.49)u_3] - [(\alpha 44 + (1-\alpha)48.35)v_1 + (\alpha 880 + (1-\alpha)957.86)v_2]$ - $u_0\ \leq 0$

$[(\alpha 2652.96 + (1-\ \alpha)2337.48)u_1 + (\alpha 421.7 + (1-\ \alpha)393.64)u_2 \quad + (\alpha 5.51 +$
$(1-\alpha) -\ 5.02)u_3] - [(\alpha 37 + (1-\alpha)41.35)v_1 + (\alpha 2233 + (1-\alpha)2310.86)v_2]$ - u_0
≤ 0

$[(\alpha 1445.38 + (1-\ \alpha)1129.89)u_1 + (\alpha 105.83 + (1-\ \alpha)77.76)u_2 \quad + (\alpha 3.56 +$
$(1-\alpha) -\ 6.96)u_3] - [(\alpha 27 + (1-\alpha)31.35)v_1 + (\alpha 496 + (1-\alpha)573.86)v_2] -\ u_0\ \leq$
0

$[(\alpha 2095.04 + (1-\ \alpha)1779.55)u_1 + (\alpha 91 + (1-\ \alpha)62.93)u_2 \quad + (\alpha 5.19 + (1-\alpha) -$
$5.34)u_3] - [(\alpha 21 + (1-\alpha)25.35)v_1 + (\alpha 438 + (1-\alpha)515.86)v_2] -\ u_0\ \leq 0$

$[(\alpha 256.57 + (1-\ \alpha) - 58.91)u_1 + (\alpha 33.39 + (1-\ \alpha)5.32)u_2 \quad + (\alpha 0.69 + (1-\alpha) -$
$9.83)u_3] - [(\alpha 20 + (1-\alpha)24.35)v_1 + (\alpha 284 + (1-\alpha)361.86)v_2] -\ u_0\ \leq 0$

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$, u_0: unrestrieted

$\max\ E^{\alpha L}(1) = \max [(\alpha 71173.26 + (1-\ \alpha)70857.77)u_1 + (\alpha 6209.94 +$
$(1-\ \alpha)6181.87)u_2 \quad + \quad (\alpha 2100.67 + (1-\ \alpha)2090.15)u_3]$

s.t.

$(\alpha 1458 + (1-\ \alpha)1462.35)v_1 + (\alpha 24374 + (1-\ \alpha)24451.86)v_2\ = 1$

The same constraints U output - L input

The same constraints L output –U input

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i$, u_0 : unrestrieted

For İş Bank

$$\max\ E^{\alpha U}(2) = \max\ [(\alpha 91620.64 + (1-\alpha)91936.12)u_1 + (\alpha 8163.27 + (1-\alpha)8191.34)u_2 + (\alpha 2667.49 + (1-\alpha)2678.01)u_3)] - u_0$$

s.t.

$$(\alpha 1201 + (1-\alpha)1196.65)v_1 + (\alpha 24887 + (1-\alpha)24809.14)v_2 = 1$$

The same constraints U output-L input

The same constraints L output –U input

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i, u_0$: unrestrieted

$$\max\ E^{\alpha L}(2) = \max\ [(\alpha 91620.64 + (1-\alpha)91305.15)u_1 + (\alpha 8163.27 + (1-\alpha)8135.21)u_2 + \alpha 2667.49 + (1-\alpha)2656.96)u_3] - u_0$$

s.t.

$$(\alpha 1201 + (1-\alpha)1205.35)v_1 + (\alpha 24887 + (1-\alpha)24964.86)v_2 = 1$$

The same constraints L output –U input

The same constraints U output-L input

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i, u_0$: unrestrieted

5.1.4. Application model FBCC (α-level)

For Ziraat Bank

$$\max\ E^{\alpha}(1) = \max\ (\bar{y}_{11} + \bar{y}_{21} + \bar{y}_{31}) - u_0$$

s.t.

$$\bar{x}_{11} + \bar{x}_{21} = 1$$

$(\bar{y}_{11} + \bar{y}_{21} + \bar{y}_{31}) - (\bar{x}_{11} + \bar{x}_{21}) - u_0 \leq 0$

$(\bar{y}_{12} + \bar{y}_{22} + \bar{y}_{32}) - (\bar{x}_{12} + \bar{x}_{22}) - u_0 \leq 0$

$(\bar{y}_{13} + \bar{y}_{23} + \bar{y}_{33}) - (\bar{x}_{13} + \bar{x}_{23}) - u_0 \leq 0$

$(\bar{y}_{14} + \bar{y}_{24} + \bar{y}_{34}) - (\bar{x}_{14} + \bar{x}_{24}) - u_0 \leq 0$

$(\bar{y}_{15} + \bar{y}_{25} + \bar{y}_{35}) - (\bar{x}_{15} + \bar{x}_{25}) - u_0 \leq 0$

$(\bar{y}_{16} + \bar{y}_{26} + \bar{y}_{36}) - (\bar{x}_{16} + \bar{x}_{26}) - u_0 \leq 0$

$(\bar{y}_{17} + \bar{y}_{27} + \bar{y}_{37}) - (\bar{x}_{17} + \bar{x}_{27}) - u_0 \leq 0$

$(\bar{y}_{18} + \bar{y}_{28} + \bar{y}_{38}) - (\bar{x}_{18} + \bar{x}_{28}) - u_0 \leq 0$

$(\bar{y}_{19} + \bar{y}_{29} + \bar{y}_{39}) - (\bar{x}_{19} + \bar{x}_{29}) - u_0 \leq 0$

$(\bar{y}_{110} + \bar{y}_{210} + \bar{y}_{310}) - (\bar{x}_{110} + \bar{x}_{210}) - u_0 \leq 0$

$(\bar{y}_{111} + \bar{y}_{211} + \bar{y}_{311}) - (\bar{x}_{111} + \bar{x}_{211}) - u_0 \leq 0$

$(\bar{y}_{112} + \bar{y}_{212} + \bar{y}_{312}) - (\bar{x}_{112} + \bar{x}_{212}) - u_0 \leq 0$

$(\bar{y}_{113} + \bar{y}_{213} + \bar{y}_{313}) - (\bar{x}_{113} + \bar{x}_{213}) - u_0 \leq 0$

$(\bar{y}_{114} + \bar{y}_{214} + \bar{y}_{314}) - (\bar{x}_{114} + \bar{x}_{214}) - u_0 \leq 0$

$(\bar{y}_{115} + \bar{y}_{215} + \bar{y}_{315}) - (\bar{x}_{115} + \bar{x}_{215}) - u_0 \leq 0$

$(\bar{y}_{116} + \bar{y}_{216} + \bar{y}_{316}) - (\bar{x}_{116} + \bar{x}_{216}) - u_0 \leq 0$

$(\bar{y}_{117} + \bar{y}_{217} + \bar{y}_{317}) - (\bar{x}_{117} + \bar{x}_{217}) - u_0 \leq 0$

$(\bar{y}_{118} + \bar{y}_{218} + \bar{y}_{318}) - (\bar{x}_{118} + \bar{x}_{218}) - u_0 \leq 0$

$(\bar{y}_{119} + \bar{y}_{219} + \bar{y}_{319}) - (\bar{x}_{119} + \bar{x}_{219}) - u_0 \leq 0$

$(\bar{y}_{120} + \bar{y}_{220} + \bar{y}_{320}) - (\bar{x}_{120} + \bar{x}_{220}) - u_0 \leq 0$

$(\bar{y}_{121} + \bar{y}_{221} + \bar{y}_{321}) - (\bar{x}_{121} + \bar{x}_{221}) - u_0 \leq 0$

$u_1(\alpha 71173.26 + (1 - \alpha)70857.77) \leq \bar{y}_{11} \leq u_1(\alpha 71173.26 + (1 - \alpha)71488.75)$

$u_1(\alpha 91620.64 + (1 - \alpha)91305.15) \leq \bar{y}_{12} \leq u_1(\alpha 91620.64 + (1 - \alpha)91936.12)$

$u_1(\alpha 70213.22 + (1-\alpha)69897.74) \le \bar{y}_{13} \le u_1(\alpha 70213.22 + (1-\alpha)70528.71)$

$u_1(\alpha 83532.93 + (1-\alpha)83217.44) \le \bar{y}_{14} \le u_1(\alpha 83532.93 + (1-\alpha)83848.42)$

$u_1(\alpha 67044.87 + (1-\alpha)66729.38) \le \bar{y}_{15} \le u_1(\alpha 67044.87 + (1-\alpha)67360.36)$

$u_1(\alpha 55949.45 + (1-\alpha)55633.96) \le \bar{y}_{16} \le u_1(\alpha 55949.45 + (1-\alpha)56264.93)$

$u_1(\alpha 57200.61 + (1-\alpha)56885.12) \le \bar{y}_{17} \le u_1(\alpha 57200.61 + (1-\alpha)57516.1)$

$u_1(\alpha 22196.37 + (1-\alpha)21880.89) \le \bar{y}_{18} \le u_1(\alpha 22196.37 + (1-\alpha)22511.86)$

$u_1(\alpha 29867.2 + (1-\alpha)29551.71) \le \bar{y}_{19} \le u_1(\alpha 29867.2 + (1-\alpha)30182.68)$

$u_1(\alpha 25443.64 + (1-\alpha)25128.16) \le \bar{y}_{110} \le u_1(\alpha 25443.64 + (1-\alpha)25759.13)$

$u_1(\alpha 13662.37 + (1-\alpha)13346.89) \le \bar{y}_{111} \le u_1(\alpha 13662.37 + (1-\alpha)13977.86)$

$u_1(\alpha 15264.74 + (1-\alpha)14949.25) \le \bar{y}_{112} \le u_1(\alpha 15264.74 + (1-\alpha)15580.22)$

$u_1(\alpha 8282.22 + (1-\alpha)7966.74) \le \bar{y}_{113} \le u_1(\alpha 8282.22 + (1-\alpha)8597.71)$

$u_1(\alpha 3715.39 + (1-\alpha)3399.91) \le \bar{y}_{114} \le u_1(\alpha 3715.39 + (1-\alpha)4030.88)$

$u_1(\alpha 4220.59 + (1-\alpha)3905.1) \le \bar{y}_{115} \le u_1(\alpha 4220.59 + (1-\alpha)4536.07)$

$u_1(\alpha 2291.94 + (1-\alpha)1976.46) \le \bar{y}_{116} \le u_1(\alpha 2291.94 + (1-\alpha)2607.43)$

$u_1(\alpha 2461.06 + (1-\alpha)2145.57) \le \bar{y}_{117} \le u_1(\alpha 2461.06 + (1-\alpha)2776.54)$

$u_1(\alpha 2652.96 + (1-\alpha)2337.48) \le \bar{y}_{118} \le u_1(\alpha 2652.96 + (1-\alpha)2968.45)$

$u_1(\alpha 1445.38 + (1-\alpha)1129.89) \le \bar{y}_{119} \le u_1(\alpha 1445.38 + (1-\alpha)1760.86)$

$u_1(\alpha 2095.04 + (1-\alpha)1779.55) \le \bar{y}_{120} \le u_1(\alpha 2095.04 + (1-\alpha)2410.52)$

$u_1(\alpha 256.57 + (1-\alpha) - 58.91) \le \bar{y}_{121} \le u_1(\alpha 256.57 + (1-\alpha)572.06)$

$u_2(\alpha 6209.94 + (1-\alpha)6181.87) \le \bar{y}_{21} \le u_2(\alpha 6209.94 + (1-\alpha)6238.01)$

$u_2(\alpha 8163.27 + (1-\alpha)8135.21) \le \bar{y}_{22} \le u_2(\alpha 8163.27 + (1-\alpha)8191.34)$

$u_2(\alpha 6083.46 + (1-\alpha)6055.39) \le \bar{y}_{23} \le u_2(\alpha 6083.46 + (1-\alpha)6111.52)$

$u_2(\alpha 7936.87 + (1-\alpha)7908.81) \le \bar{y}_{24} \le u_2(\alpha 7936.87 + (1-\alpha)7964.94)$

$u_2(\alpha 5821.77 + (1-\alpha)5793.71) \le \bar{y}_{25} \le u_2(\alpha 5821.77 + (1-\alpha)5849.84)$

$u_2(\alpha 5051.93 + (1-\alpha)5023.87) \le \bar{y}_{26} \le u_2(\alpha 5051.93 + (1-\alpha)5080)$

$u_2(\alpha 4422.16 + (1-\alpha)4394.09) \le \bar{y}_{27} \le u_2(\alpha 4422.16 + (1-\alpha)4450.22)$

$u_2(\alpha 2291.8 + (1-\alpha)2263.74) \le \bar{y}_{28} \le u_2(\alpha 2291.8 + (1-\alpha)2319.87)$

$u_2(\alpha 3000 + (1-\alpha)2971.93) \le \bar{y}_{29} \le u_2(\alpha 3000 + (1-\alpha)3028.07)$

$u_2(\alpha 1793.32 + (1-\alpha)1765.25) \le \bar{y}_{210} \le u_2(\alpha 1793.32 + (1-\alpha)1821.39)$

$u_2(\alpha 1534.52 + (1-\alpha)1506.45) \le \bar{y}_{211} \le u_2(\alpha 1534.52 + (1-\alpha)1562.58)$

$u_2(\alpha 1100.21 + (1-\alpha)1072.14) \le \bar{y}_{212} \le u_2(\alpha 1100.21 + (1-\alpha)1128.27)$

$u_2(\alpha 876.92 + (1-\alpha)848.85) \le \bar{y}_{213} \le u_2(\alpha 876.92 + (1-\alpha)904.98)$

$u_2(\alpha 335.91 + (1-\alpha)307.85) \le \bar{y}_{214} \le u_2(\alpha 335.91 + (1-\alpha)363.98)$

$u_2(\alpha 288.19 + (1-\alpha)260.13) \le \bar{y}_{215} \le u_2(\alpha 288.19 + (1-\alpha)316.26)$

$u_2(\alpha 209.61 + (1-\alpha)181.54) \le \bar{y}_{216} \le u_2(\alpha 209.61 + (1-\alpha)237.67)$

$u_2(\alpha 147.24 + (1-\alpha)119.18) \le \bar{y}_{217} \le u_2(\alpha 147.24 + (1-\alpha)175.31)$

$u_2(\alpha 421.7 + (1-\alpha)393.64) \le \bar{y}_{218} \le u_2(\alpha 421.7 + (1-\alpha)449.77)$

$u_2(\alpha 105.83 + (1-\alpha)77.76) \le \bar{y}_{219} \le u_2(\alpha 105.83 + (1-\alpha)133.9)$

$u_2(\alpha 91 + (1-\alpha)62.93) \le \bar{y}_{220} \le u_2(\alpha 91 + (1-\alpha)119.06)$

$u_2(\alpha 33.39 + (1-\alpha)5.32) \le \bar{y}_{221} \le u_2(\alpha 33.39 + (1-\alpha)61.46)$

$u_3(\alpha 2100.67 + (1-\alpha)2090.15) \le \bar{y}_{31} \le u_3(\alpha 2100.67 + (1-\alpha)2111.2)$

$u_3(\alpha 2667.49 + (1-\alpha)2656.96) \le \bar{y}_{32} \le u_3(\alpha 2667.49 + (1-\alpha)2678.01)$

$u_3(\alpha 2394.53 + (1-\alpha)2384) \le \bar{y}_{33} \le u_3(\alpha 2394.53 + (1-\alpha)2405.05)$

$u_3(\alpha 3070.58 + (1-\alpha)3060.05) \le \bar{y}_{34} \le u_3(\alpha 3070.58 + (1-\alpha)3081.1)$

$u_3(\alpha 1857.49 + (1-\alpha)1846.96) \le \bar{y}_{35} \le u_3(\alpha 1857.49 + (1-\alpha)1868.01)$

$u_3(\alpha 2045.13 + (1-\alpha)2034.61) \le \bar{y}_{36} \le u_3(\alpha 2045.13 + (1-\alpha)2055.66)$

$u_3(\alpha 1226.79 + (1-\alpha)1216.26) \le \bar{y}_{37} \le u_3(\alpha 1226.79 + (1-\alpha)1237.31)$

$u_3(\alpha 873.97 + (1-\alpha)863.45) \le \bar{y}_{38} \le u_3(\alpha 873.97 + (1-\alpha)884.5)$

$u_3(\alpha 848.11 + (1-\alpha)837.59) \le \bar{y}_{39} \le u_3(\alpha 848.11 + (1-\alpha)858.64)$

$u_3(\alpha 206.68 + (1-\alpha)196.15) \le \bar{y}_{310} \le u_3(\alpha 206.68 + (1-\alpha)217.2)$

$u_3(\alpha 240.79 + (1-\alpha)230.26) \le \bar{y}_{311} \le u_3(\alpha 240.79 + (1-\alpha)251.31)$

$u_3(\alpha 79.01 + (1-\alpha)68.49) \le \bar{y}_{312} \le u_3(\alpha 79.01 + (1-\alpha)89.54)$

$u_3(\alpha 118.04 + (1-\alpha)107.52) \le \bar{y}_{313} \le u_3(\alpha 118.04 + (1-\alpha)128.57)$

$u_3(\alpha 85.23 + (1-\alpha)74.7) \le \bar{y}_{314} \le u_3(\alpha 85.23 + (1-\alpha)95.76)$

$u_3(\alpha 28.26 + (1-\alpha)17.74) \le \bar{y}_{315} \le u_3(\alpha 28.26 + (1-\alpha)38.79)$

$u_3(\alpha 37.37 + (1-\alpha)26.84) \le \bar{y}_{316} \le u_3(\alpha 37.37 + (1-\alpha)47.89)$

$u_3(\alpha 22.02 + (1-\alpha)11.49) \le \bar{y}_{317} \le u_3(\alpha 22.02 + (1-\alpha)32.54)$

$u_3(\alpha 5.51 + (1-\alpha) - 5.02) \le \bar{y}_{318} \le u_3(\alpha 5.51 + (1-\alpha)16.03)$

$u_3(\alpha 3.56 + (1-\alpha) - 6.96) \le \bar{y}_{319} \le u_3(\alpha 3.56 + (1-\alpha)14.09)$

$u_3(\alpha 5.19 + (1-\alpha) - 5.34) \le \bar{y}_{320} \le u_3(\alpha 5.19 + (1-\alpha)15.71)$

$u_3(\alpha 0.69 + (1-\alpha) - 9.83) \le \bar{y}_{321} \le u_3(\alpha 0.69 + (1-\alpha)11.22)$

$v_1(\alpha 1458 + (1-\alpha)1453.65) \le \bar{x}_{11} \le v_1(\alpha 1458 + (1-\alpha)1462.35)$

$v_1(\alpha 1201 + (1-\alpha)1196.65) \le \bar{x}_{12} \le v_1(\alpha 1201 + (1-\alpha)1205.35)$

$v_1(\alpha 927 + (1-\alpha)922.65) \le \bar{x}_{13} \le v_1(\alpha 927 + (1-\alpha)931.35)$

$v_1(\alpha 914 + (1-\alpha)909.65) \le \bar{x}_{14} \le v_1(\alpha 914 + (1-\alpha)918.35)$

$v_1(\alpha 907 + (1-\alpha)902.65) \le \bar{x}_{15} \le v_1(\alpha 907 + (1-\alpha)911.35)$

$v_1(\alpha 771 + (1-\alpha)766.65) \le \bar{x}_{16} \le v_1(\alpha 907 + (1-\alpha)911.35)$

$v_1(\alpha 680 + (1-\alpha)675.65) \le \bar{x}_{17} \le v_1(\alpha 680 + (1-\alpha)684.35)$

$v_1(\alpha 588 + (1-\alpha)583.65) \le \bar{x}_{18} \le v_1(\alpha 588 + (1-\alpha)592.35)$

$v_1(\alpha 522 + (1-\alpha)517.65) \quad \leq \bar{x}_{19} \leq v_1(\alpha 522 + (1-\alpha)526.35)$

$v_1(\alpha 507 + (1-\alpha)502.65) \quad \leq \bar{x}_{110} \leq v_1(\alpha 507 + (1-\alpha)511.35)$

$v_1(\alpha 330 + (1-\alpha)325.65) \quad \leq \bar{x}_{111} \leq v_1(\alpha 330 + (1-\alpha)334.35)$

$v_1(\alpha 322 + (1-\alpha)317.65) \quad \leq \bar{x}_{112} \leq v_1(\alpha 322 + (1-\alpha)326.35)$

$v_1(\alpha 272 + (1-\alpha)267.65) \quad \leq \bar{x}_{113} \leq v_1(\alpha 272 + (1-\alpha)276.35)$

$v_1(\alpha 88 + (1-\alpha)83.65) \quad \leq \bar{x}_{114} \leq v_1(\alpha 88 + (1-\alpha)92.35)$

$v_1(\alpha 63 + (1-\alpha)58.65) \quad \leq \bar{x}_{115} \leq v_1(\alpha 63 + (1-\alpha)67.35)$

$v_1(\alpha 59 + (1-\alpha)54.65) \quad \leq \bar{x}_{116} \leq v_1(\alpha 59 + (1-\alpha)63.35)$

$v_1(\alpha 44 + (1-\alpha)39.65) \quad \leq \bar{x}_{117} \leq v_1(\alpha 44 + (1-\alpha)48.35)$

$v_1(\alpha 37 + (1-\alpha)32.65) \quad \leq \bar{x}_{118} \leq v_1(\alpha 37 + (1-\alpha)41.35)$

$v_1(\alpha 27 + (1-\alpha)22.65) \quad \leq \bar{x}_{119} \leq v_1(\alpha 27 + (1-\alpha)31.35)$

$v_1(\alpha 21 + (1-\alpha)16.65) \quad \leq \bar{x}_{120} \leq v_1(\alpha 21 + (1-\alpha)25.35)$

$v_1(\alpha 20 + (1-\alpha)15.65) \quad \leq \bar{x}_{121} \leq v_1(\alpha 20 + (1-\alpha)24.35)$

$v_2(\alpha 24374 + (1-\alpha)24296.14) \leq \bar{x}_{21} \leq v_2(\alpha 24374 + (1-\alpha)24451.86)$

$v_2(\alpha 24887 + (1-\alpha)24809.14) \leq \bar{x}_{22} \leq v_2(\alpha 24887 + (1-\alpha)24964.86)$

$v_2(\alpha 15339 + (1-\alpha)15261.14) \leq \bar{x}_{23} \leq v_2(\alpha 15339 + (1-\alpha)15416.86)$

$v_2(\alpha 16773 + (1-\alpha)16695.14) \leq \bar{x}_{24} \leq v_2(\alpha 16773 + (1-\alpha)16850.86)$

$v_2(\alpha 14859 + (1-\alpha)14781.14) \leq \bar{x}_{25} \leq v_2(\alpha 14859 + (1-\alpha)14936.86)$

$v_2(\alpha 13643 + (1-\alpha)13565.14) \leq \bar{x}_{26} \leq v_2(\alpha 13643 + (1-\alpha)13720.86)$

$v_2\alpha 12222 + (1-\alpha)12144.14() \leq \bar{x}_{27} \leq v_2(\alpha 12222 + (1-\alpha)12299.86)$

$v_2(\alpha 9772 + (1-\alpha)9694.14) \leq \bar{x}_{28} \leq v_2(\alpha 9772 + (1-\alpha)9849.86)$

$v_2(\alpha 10837 + (1-\alpha)10759.14) \leq \bar{x}_{29} \leq v_2(\alpha 10837 + (1-\alpha)10914.86)$

$v_2(\alpha 9356 + (1-\alpha)9278.14) \leq \bar{x}_{210} \leq v_2(\alpha 9356 + (1-\alpha)9433.86)$

$v_2(\alpha 6155 + (1-\alpha)6077.14) \leq \bar{x}_{211} \leq v_2(\alpha 6155 + (1-\alpha)6232.86)$

$v_2(\alpha 5232 + (1-\alpha)5154.14) \leq \bar{x}_{212} \leq v_2(\alpha 5232 + (1-\alpha)5309.86)$

$v_2(\alpha 3530 + (1-\alpha)3452.14) \leq \bar{x}_{213} \leq v_2(\alpha 3530 + (1-\alpha)3607.86)$

$v_2(\alpha 1911 + (1-\alpha)1833.14) \leq \bar{x}_{214} \leq v_2(\alpha 1911 + (1-\alpha)1988.86)$

$v_2(\alpha 1185 + (1-\alpha)1107.14) \leq \bar{x}_{215} \leq v_2(\alpha 1185 + (1-\alpha)1262.86)$

$v_2(\alpha 954 + (1-\alpha)876.14) \leq \bar{x}_{216} \leq v_2(\alpha 954 + (1-\alpha)1031.86)$

$v_2(\alpha 880 + (1-\alpha)802.14) \leq \bar{x}_{217} \leq v_2(\alpha 880 + (1-\alpha)957.86)$

$v_2(\alpha 2233 + (1-\alpha)2155.14) \leq \bar{x}_{218} \leq v_2(\alpha 2233 + (1-\alpha)2310.86)$

$v_2(\alpha 496 + (1-\alpha)418.14) \leq \bar{x}_{219} \leq v_2(\alpha 496 + (1-\alpha)573.86)$

$v_2(\alpha 438 + (1-\alpha)360.14) \leq \bar{x}_{220} \leq v_2(\alpha 438 + (1-\alpha)515.86)$

$v_2(\alpha 284 + (1-\alpha)206.14) \leq \bar{x}_{221} \leq v_2(\alpha 284 + (1-\alpha)361.86)$

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i, u_0$: unrestrieted

<u>For İş Bank</u>

$\max \; E^{\alpha}(2) = \max \; (\bar{y}_{12} + \bar{y}_{22} + \bar{y}_{32}) - u_0$

s.t.

$\bar{x}_{12} + \bar{x}_{22} = 1$

The same constraints

$u_1, u_2, u_3, v_1, v_2 \geq 0 \quad \forall r, i, u_0$: unrestrieted

We obtain the lower and upper bounds of the efficiency score for DMUs with a FCCR model (α-cuts and α-level) using MATLAB R2017a codes in Appendix 19 as in Figure 5.1.

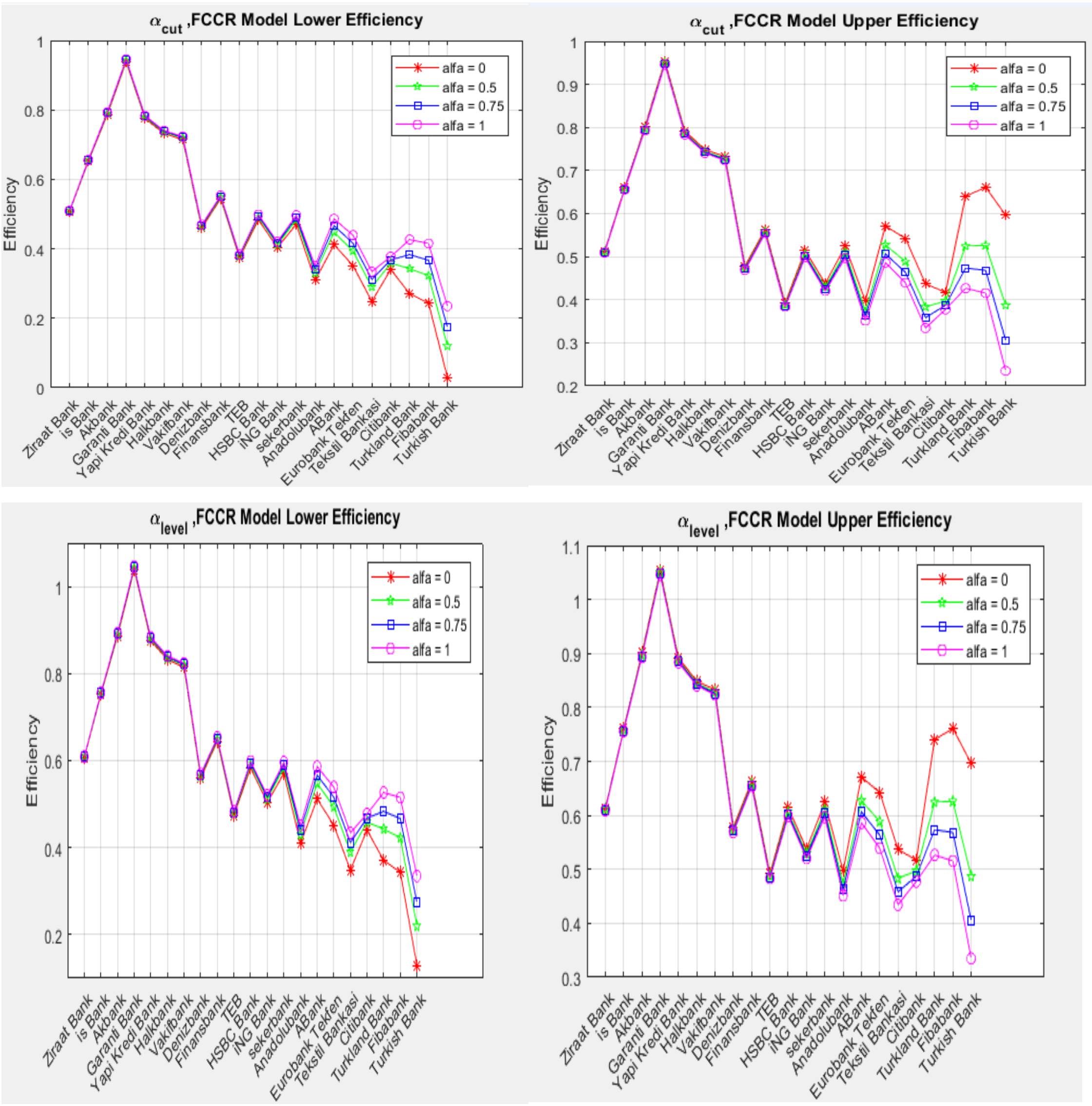

Figure 5.1. Lower and upper bounds of the efficiency score for FCCR model

And we obtain the lower and upper bounds of the efficiency score for DMUs with a FBCC model (α-cuts and α-level) using MATLAB R2017a codes in Appendix 20 as in Figure 5.2.

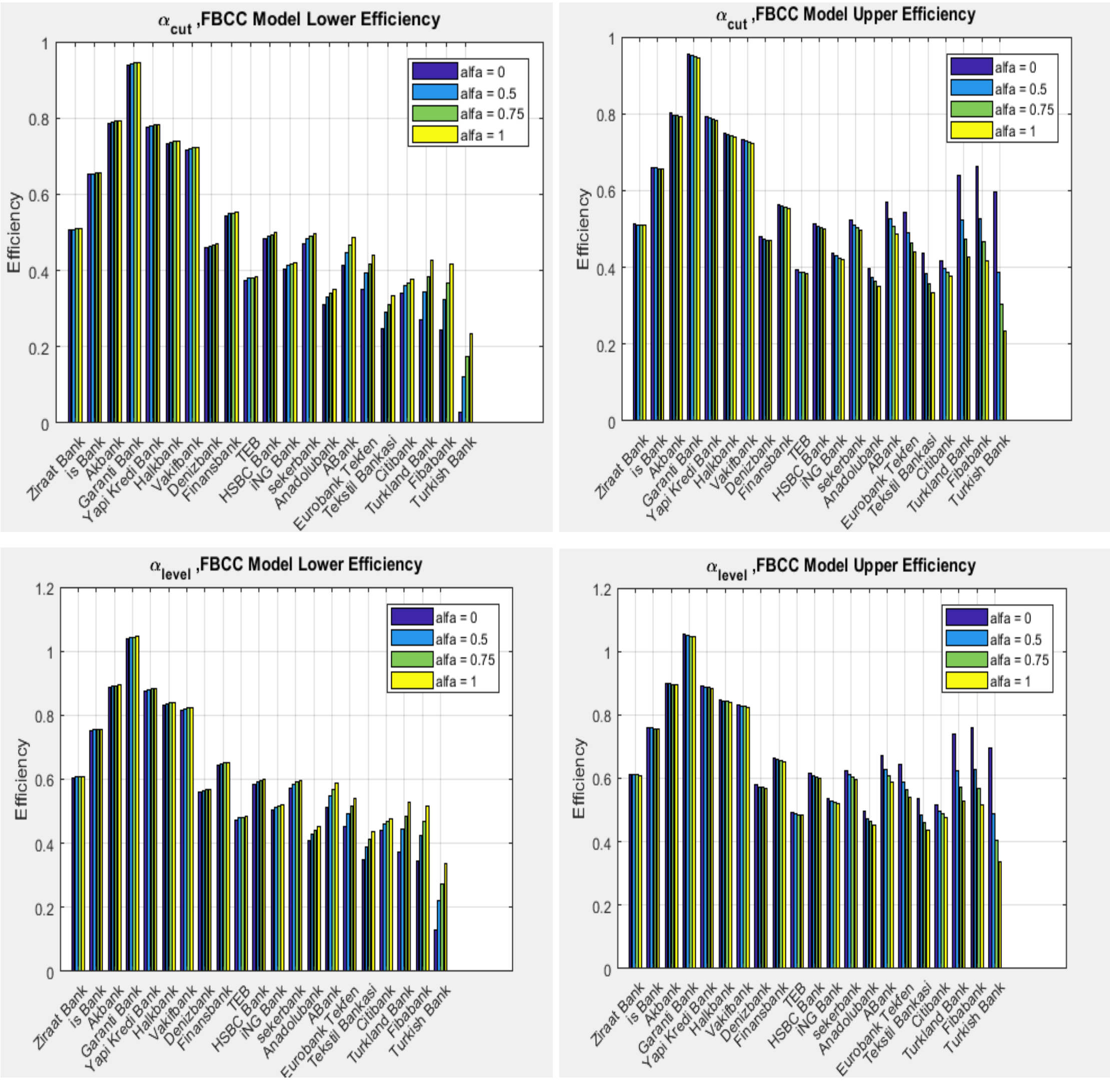

Figure 5.2. Lower and upper bounds of the efficiency score for FBCC model

They constitute a possible best relative efficiency interval $[E^{\alpha L}, E^{\alpha U}]$ note them in the Tables (5.1-5.4) determines the production frontier for all the DMUs to rank them using MATLAB R2017a codes Appendix 19,20.

Table 5.1. FCCR model α-cuts efficiency

DMU		E(L,U)								Rank
		L	U	L	U	L	U	L	U	
		0.0	0.0	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.5056	0.5134	0.5075	0.5115	0.5085	0.5105	0.5095	0.5095	8
2	İş Bank	0.6517	0.6603	0.6538	0.6581	0.6549	0.6571	0.6560	0.6560	6
3	Ak Bank	0.7855	0.8009	0.7893	0.7970	0.7912	0.7951	0.7932	0.7932	2
4	Garanti Bank	0.9386	0.9541	0.9425	0.9502	0.9444	0.9483	0.9463	0.9463	1
5	Yapı Kredi Bank	0.7757	0.7915	0.7796	0.7875	0.7816	0.7855	0.7836	0.7836	3
6	Halk Bank	0.7322	0.7489	0.7364	0.7447	0.7385	0.7426	0.7405	0.7405	4
7	Vakıf Bank	0.7144	0.7328	0.7190	0.7282	0.7213	0.7259	0.7236	0.7236	5
8	Deniz Bank	0.4596	0.4786	0.4643	0.4738	0.4666	0.4714	0.4690	0.4690	12
9	Finans Bank	0.5445	0.5628	0.5490	0.5582	0.5513	0.5559	0.5536	0.5536	7
10	TEB	0.3742	0.3926	0.3787	0.3879	0.3810	0.3856	0.3833	0.3833	14
11	HSBC Bank	0.4833	0.5142	0.4909	0.5063	0.4947	0.5024	0.4986	0.4986	9
12	ING Bank	0.4038	0.4378	0.4121	0.4291	0.4163	0.4248	0.4205	0.4205	13
13	Şeker Bank	0.4705	0.5243	0.4835	0.5104	0.4901	0.5035	0.4968	0.4968	10
14	Anadolu Bank	0.3095	0.3971	0.3301	0.3738	0.3407	0.3625	0.3515	0.3515	17
15	ABank	0.4119	0.5713	0.4479	0.5274	0.4668	0.5065	0.4863	0.4863	11
16	Euro Bank Tekfen	0.3518	0.5425	0.3939	0.4887	0.4162	0.4635	0.4394	0.4394	15
17	Tekstil Bank	0.2488	0.4371	0.2899	0.3834	0.3117	0.3585	0.3346	0.3346	19
18	Citi Bank	0.3406	0.4173	0.3588	0.3971	0.3682	0.3873	0.3776	0.3776	16
19	Turkland Bank	0.2710	0.6404	0.3432	0.5244	0.3833	0.4736	0.4267	0.4267	18
20	Fiba Bank	0.2439	0.6611	0.3227	0.5263	0.3671	0.4683	0.4155	0.4155	20
21	Turkish Bank	0.0294	0.5962	0.1198	0.3870	0.1738	0.3054	0.2351	0.2351	21

Table 5.2. FBCC model α-cuts efficiency

DMU		E(L,U)								Rank
		L	U	L	U	L	U	L	U	
		0.0	0.0	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.5056	0.5134	0.5075	0.5115	0.5085	0.5105	0.5095	0.5095	8
2	İş Bank	0.6517	0.6603	0.6538	0.6581	0.6549	0.6571	0.6560	0.6560	6
3	Ak Bank	0.7855	0.8009	0.7893	0.7970	0.7912	0.7951	0.7932	0.7932	2
4	Garanti Bank	0.9386	0.9541	0.9425	0.9502	0.9444	0.9483	0.9463	0.9463	1
5	Yapı Kredi Bank	0.7757	0.7915	0.7796	0.7875	0.7816	0.7855	0.7836	0.7836	3
6	Halk Bank	0.7322	0.7489	0.7364	0.7447	0.7385	0.7426	0.7405	0.7405	4
7	Vakıf Bank	0.7144	0.7328	0.7190	0.7282	0.7213	0.7259	0.7236	0.7236	5
8	Deniz Bank	0.4596	0.4786	0.4643	0.4738	0.4666	0.4714	0.4690	0.4690	12
9	Finans Bank	0.5445	0.5628	0.5490	0.5582	0.5513	0.5559	0.5536	0.5536	7
10	TEB	0.3742	0.3926	0.3787	0.3879	0.3810	0.3856	0.3833	0.3833	14
11	HSBC Bank	0.4833	0.5142	0.4909	0.5063	0.4947	0.5024	0.4986	0.4986	9
12	ING Bank	0.4038	0.4378	0.4121	0.4291	0.4163	0.4248	0.4205	0.4205	13
13	Şeker Bank	0.4705	0.5243	0.4835	0.5104	0.4901	0.5035	0.4968	0.4968	10
14	Anadolu Bank	0.3095	0.3971	0.3301	0.3738	0.3407	0.3625	0.3515	0.3515	17
15	ABank	0.4119	0.5713	0.4479	0.5274	0.4668	0.5065	0.4863	0.4863	11
16	Euro Bank Tekfen	0.3518	0.5425	0.3939	0.4887	0.4162	0.4635	0.4394	0.4394	15
17	Tekstil Bank	0.2488	0.4371	0.2899	0.3834	0.3117	0.3585	0.3346	0.3346	19
18	Citi Bank	0.3406	0.4173	0.3588	0.3971	0.3682	0.3873	0.3776	0.3776	16
19	Turkland Bank	0.2710	0.6404	0.3432	0.5244	0.3833	0.4736	0.4267	0.4267	18
20	Fiba Bank	0.2439	0.6611	0.3227	0.5263	0.3671	0.4683	0.4155	0.4155	20
21	Turkish Bank	0.0294	0.5962	0.1198	0.3870	0.1738	0.3054	0.2351	0.2351	21

Table 5.3. FCCR model α-level efficiency

DMU		E(L,U)								Rank
		L	U	L	U	L	U	L	U	
		0.0	0.0	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.5656	0.5834	0.5675	0.5815	0.5685	0.5805	0.5695	0.5795	8
2	İş Bank	0.7117	0.7303	0.7138	0.7281	0.7149	0.7271	0.7160	0.7260	6
3	Ak Bank	0.8455	0.8709	0.8493	0.8670	0.8512	0.8651	0.8532	0.8632	2
4	Garanti Bank	0.9986	1.0241	1.0025	1.0202	1.0044	1.0183	1.0063	1.0163	1
5	Yapı Kredi Bank	0.8357	0.8615	0.8396	0.8575	0.8416	0.8555	0.8436	0.8536	3
6	Halk Bank	0.7922	0.8189	0.7964	0.8147	0.7985	0.8126	0.8005	0.8105	4
7	Vakıf Bank	0.7744	0.8028	0.7790	0.7982	0.7813	0.7959	0.7836	0.7936	5
8	Deniz Bank	0.5196	0.5486	0.5243	0.5438	0.5266	0.5414	0.5290	0.5390	12
9	Finans Bank	0.6045	0.6328	0.6090	0.6282	0.6113	0.6259	0.6136	0.6236	7
10	TEB	0.4342	0.4626	0.4387	0.4579	0.4410	0.4556	0.4433	0.4533	14
11	HSBC Bank	0.5433	0.5842	0.5509	0.5763	0.5547	0.5724	0.5586	0.5686	9
12	ING Bank	0.4638	0.5078	0.4721	0.4991	0.4763	0.4948	0.4805	0.4905	13
13	Şeker Bank	0.5305	0.5943	0.5435	0.5804	0.5501	0.5735	0.5568	0.5668	10
14	Anadolu Bank	0.3695	0.4671	0.3901	0.4438	0.4007	0.4325	0.4115	0.4215	17
15	ABank	0.4719	0.6413	0.5079	0.5974	0.5268	0.5765	0.5463	0.5563	11
16	Euro Bank Tekfen	0.4118	0.6125	0.4539	0.5587	0.4762	0.5335	0.4994	0.5094	15
17	Tekstil Bank	0.3088	0.5071	0.3499	0.4534	0.3717	0.4285	0.3946	0.4046	19
18	Citi Bank	0.4006	0.4873	0.4188	0.4671	0.4282	0.4573	0.4376	0.4476	16
19	Turkland Bank	0.3310	0.7104	0.4032	0.5944	0.4433	0.5436	0.4867	0.4967	18
20	Fiba Bank	0.3039	0.7311	0.3827	0.5963	0.4271	0.5383	0.4755	0.4855	20
21	Turkish Bank	0.0894	0.6662	0.1798	0.4570	0.2338	0.3754	0.2951	0.3051	21

Table 5.4. FBCC model α-level efficiency

DMU		E(L,U)								Rank
		L	U	L	U	L	U	L	U	
		0.0	0.0	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.5656	0.5834	0.5675	0.5815	0.5685	0.5805	0.5695	0.5795	8
2	İş Bank	0.7117	0.7303	0.7138	0.7281	0.7149	0.7271	0.7160	0.7260	6
3	Ak Bank	0.8455	0.8709	0.8493	0.8670	0.8512	0.8651	0.8532	0.8632	2
4	Garanti Bank	0.9986	1.0241	1.0025	1.0202	1.0044	1.0183	1.0063	1.0163	1
5	Yapı Kredi Bank	0.8357	0.8615	0.8396	0.8575	0.8416	0.8555	0.8436	0.8536	3
6	Halk Bank	0.7922	0.8189	0.7964	0.8147	0.7985	0.8126	0.8005	0.8105	4
7	Vakıf Bank	0.7744	0.8028	0.7790	0.7982	0.7813	0.7959	0.7836	0.7936	5
8	Deniz Bank	0.5196	0.5486	0.5243	0.5438	0.5266	0.5414	0.5290	0.5390	12
9	Finans Bank	0.6045	0.6328	0.6090	0.6282	0.6113	0.6259	0.6136	0.6236	7
10	TEB	0.4342	0.4626	0.4387	0.4579	0.4410	0.4556	0.4433	0.4533	14
11	HSBC Bank	0.5433	0.5842	0.5509	0.5763	0.5547	0.5724	0.5586	0.5686	9
12	ING Bank	0.4638	0.5078	0.4721	0.4991	0.4763	0.4948	0.4805	0.4905	13
13	Şeker Bank	0.5305	0.5943	0.5435	0.5804	0.5501	0.5735	0.5568	0.5668	10
14	AnadoluBank	0.3695	0.4671	0.3901	0.4438	0.4007	0.4325	0.4115	0.4215	17
15	ABank	0.4719	0.6413	0.5079	0.5974	0.5268	0.5765	0.5463	0.5563	11
16	Euro Bank Tekfen	0.4118	0.6125	0.4539	0.5587	0.4762	0.5335	0.4994	0.5094	15
17	Tekstil Banka	0.3088	0.5071	0.3499	0.4534	0.3717	0.4285	0.3946	0.4046	19
18	Citi Bank	0.4006	0.4873	0.4188	0.4671	0.4282	0.4573	0.4376	0.4476	16
19	Turkland Bank	0.3310	0.7104	0.4032	0.5944	0.4433	0.5436	0.4867	0.4967	18
20	Fiba Bank	0.3039	0.7311	0.3827	0.5963	0.4271	0.5383	0.4755	0.4855	20
21	Turkish Bank	0.0894	0.6662	0.1798	0.4570	0.2338	0.3754	0.2951	0.3051	21

Note that the most efficient bank in the four models is Garanti Bank, It achieved a higher percentage of events 13.1% and achieved a profit increase of 16.9% and 13.4% increase was also recorded in the annual financial report for 2011. And note that when alpha equals one there is no difference between minimum and maximum bounds.

5.2. Simulation

5.2.1. Boundary determination

To calculate of relative efficiency values of 18 commercial banks working within in turkey we took data for 10 years during the period 2007-2016 and worked on them simulation by using DEA was aimed. BCC and CCR model was used to calculate efficiency values of the banks.

The select of DMUs and variables which would be used in DEA is rather important because of its high effect on efficiency values. For the assumption to be homogeneous of units that were measured efficiency, investment bank, development bank, participation bank and branches of the foreign banks in Turkey were excluded from the study 18 commercial banks selected for analysis as in the Appendix 6. 2 input and 3 output variables which usually used in similar studies were selected, We took annual reports of all banks 180 annual reports on 31 december data of aforementioned variables that were gathered from the official website of banks association of Turkey, output units of millions of Turkish lira.

5.2.2. Training data set and testing data set determination

After we do have 75% of the data for training purpose and 25% for testing, the next step is to get new outputs from the input in case of increasing and decreasing to the neural network in Figure (5.3) using MATLAB R2017a codes Appendix 23.

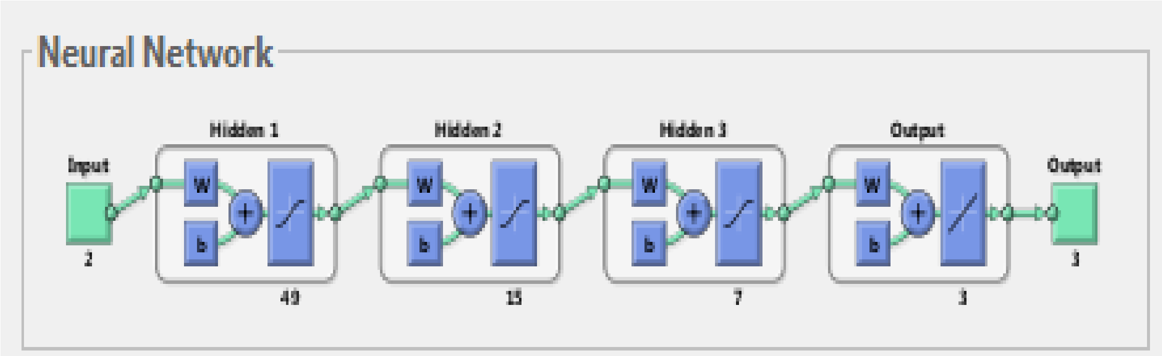

Figure 5.3. Feedforward neural network

5.2.3. Simulation results

In this study, we used multilayer perceptron to train the network with feedforward network algorithm. The number of hidden layer is three and the number of hidden neuron is 49, 15 and 7 respectively. From the training window, we can access three plots: performance, training state, and regression. The performance plot shows the value of the performance function versus the iteration number. Each iteration of the complete training set is called an epoch. In each epoch the network adjusts the weights in the direction that reduces the error. As the iterative process of incremental adjustment continues, the weights gradually converge to the locally optimal set of values. Many epochs are usually required before training is completed. Training automatically stops when generalization stops improving, as indicated by an increase in the mean square error (MSE) of the validation samples. MSE is the average squared difference between outputs and targets. Lower values are better while zero means no error. Regression is used to validate the neural network and justify the relationship between the outputs of the network and the targets. Regression analysis is performed to measure the correlation between outputs and targets. If the training were perfect, the network outputs and the targets would be exactly equal, but the relationship is rarely perfect in inputs.

The relationship is rarely perfect in inputs. For this implementation, the data set is collected from the given input In case of increasing and decreasing feedforward network algorithm is used for training the network. The performance of the proposed network is trained with feedforward network algorithm using MATLAB R2017a in Appendix 23 (Feedforward Neural Network Program Codes to Simulation DMUs of Banks) which is shown in Figure 8. We ran the feedforward network on algorithm 10:28 times and through the figure we can see that the best validation performance happens at epoch 329. From our obtained learning curve we can see that the validation and test curves are very similar so we can conclude that learning with feedforward network algorithm does not indicate any major problems with the training. If the test curve had increased significantly before the validation curve increased, then it is possible that some over fitting might have occurred. But in our case no over fitting is occurred. The regression plot shows the perfect correlation between the outputs and the targets Figure 5.4. The result for training, validation and testing samples is illustrated in Table 5.5. It is observed that the value of R is closest to 1 indicating the accurate relation between outputs and targets. For our network, the training data indicates a good fit between original data and outputs, Figure 5.5 The validation and test results also show R values that greater than 0.9.

The outputs from matlab software are;

- Accuracy = 93% (classification)
- Iteration = 329

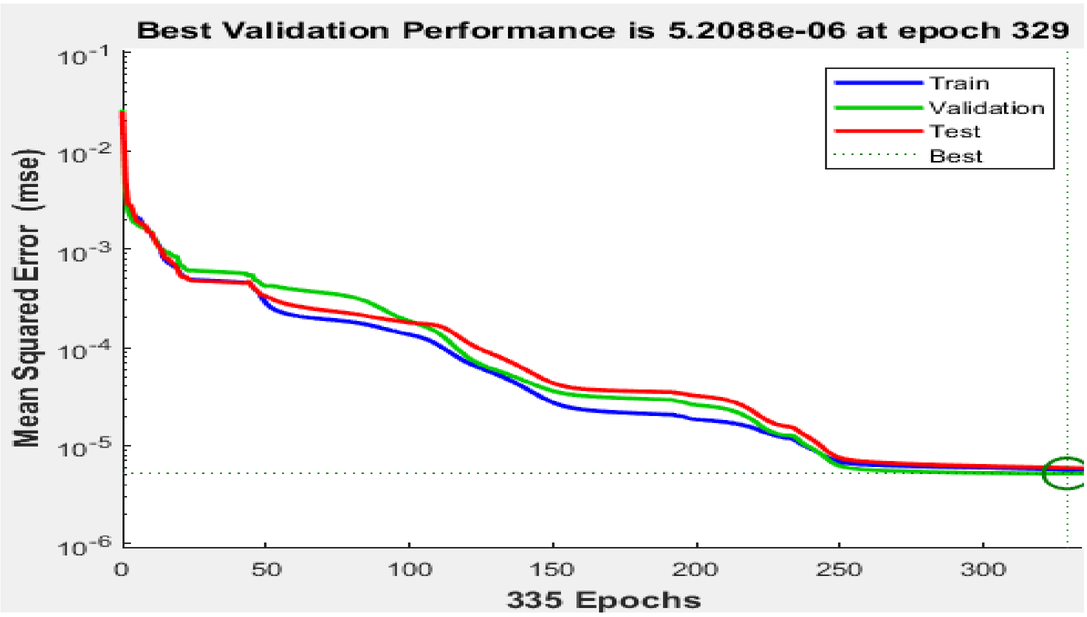

Figure 5.4. The learning curve

Figure 5.5. The simulation outputs fit curve

Table 5.5. The mean square error (MSE) and regression (R) values for the training, validation and testing

Type of data	MSE	R
Training	2.45e-3	0.96312
Testing	2.3e-3	0.95443
Validation	2.8e-3	0.95643

We get a increasing efficiency in Tabels 5.6 and 5.8 by using MATLAB R2017a codes in Appendix 21and 22, and decresing efficiency to rank them in Tabels 5.7 and 5.9 for fuzzy CCR and in Tabels 5.8 and 5.9 by using MATLAB R2017a codes in Appendix 21 and 22 for fuzzy CCR and BCC models respectively.

Table 5.6. Increasing efficiency for FCCR model

DMU		E(L,U)										Rank
		L	U	L	U	L	U	L	U	L	U	
		0.0	0.0	0.25	0.25	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.7880	1.0911	0.7894	1.0897	0.7907	1.0884	0.7921	1.0870	0.7934	1.0857	5
2	İş Bank	0.5902	0.8940	0.5914	0.8928	0.5926	0.8916	0.5938	0.8904	0.5950	0.8892	7
3	Ak Bank	0.7853	1.0969	0.7877	1.0945	0.7901	1.0921	0.7925	1.0896	0.7950	1.0872	4
4	Garanti Bank	0.8170	1.1229	0.8187	1.1211	0.8205	1.1194	0.8222	1.1177	0.8240	1.1159	2
5	Yapı Kredi Bank	0.4658	0.7729	0.4673	0.7714	0.4687	0.7700	0.4702	0.7685	0.4717	0.7671	8
6	Halk Bank	0.4475	0.7555	0.4490	0.7539	0.4506	0.7524	0.4521	0.7508	0.4537	0.7493	9
7	Vakıf Bank	0.9902	1.3000	0.9927	1.2975	0.9951	1.2951	0.9975	1.2926	1	1.2902	1
8	Deniz Bank	0.7017	1.0144	0.7041	1.0119	0.7066	1.0095	0.7090	1.0070	0.7115	1.0046	6
9	Finans Bank	0.7693	1.1634	0.7814	1.1500	0.7938	1.1368	0.8063	1.1238	0.8190	1.1110	3
10	TEB	0.4310	0.7481	0.4336	0.7454	0.4363	0.7427	0.4390	0.7400	0.4417	0.7374	10
11	HSBC Bank	0.0624	0.4063	0.0678	0.4005	0.0732	0.3948	0.0787	0.3891	0.0843	0.3835	17
12	ING Bank	0.0429	0.3688	0.0461	0.3654	0.0494	0.3620	0.0527	0.3587	0.0560	0.3554	18
13	Şeker Bank	0.2523	0.5976	0.2581	0.5914	0.2640	0.5852	0.2699	0.5792	0.2759	0.5732	13
14	Anadolu Bank	0.2243	0.6133	0.2350	0.6010	0.2459	0.5890	0.2570	0.5772	0.2683	0.5657	14
15	A Bank	0.3037	0.7798	0.3235	0.7543	0.3439	0.7297	0.3652	0.7061	0.3872	0.6835	12
16	Turklald Bank	0.2788	0.8287	0.3052	0.7907	0.3329	0.7548	0.3620	0.7210	0.3927	0.6889	11
17	Fiba Bank	0.2025	0.6062	0.2147	0.5917	0.2272	0.5777	0.2400	0.5640	0.2531	0.5506	15
18	Turkish Bank	0.2025	0.6062	0.2147	0.5917	0.2272	0.5777	0.2400	0.5640	0.2531	0.5506	16

Table 5.7. Decreasing efficiency for FCCR model

DMU		E(L,U)										Rank
		L	U	L	U	L	U	L	U	L	U	
		0.0	0.0	0.25	0.25	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.7494	1.0415	0.7481	1.0432	0.7469	1.0450	0.7456	1.0468	0.7444	1.0485	4
2	İş Bank	0.5324	0.7604	0.5313	0.7620	0.5302	0.7635	0.5291	0.7651	0.5280	0.7666	8
3	Akbank	0.8114	1.0556	0.8090	1.0588	0.8066	1.0620	0.8043	1.0653	0.8019	1.0685	3
4	Garanti Bank	0.6347	0.8832	0.6333	0.8853	0.6318	0.8873	0.6303	0.8894	0.6289	0.8915	5
5	YapıKredi Bank	0.6689	1.0926	0.6673	1.0951	0.6657	1.0976	0.6641	1.1000	0.6625	1.1025	2
6	Halk Bank	0.6163	0.8623	0.6146	0.8647	0.6129	0.8671	0.6112	0.8694	0.6095	0.8718	6
7	Vakıf Bank	0.3892	0.4709	0.3877	0.4729	0.3861	0.4750	0.3846	0.4770	0.3831	0.4791	13
8	Deniz Bank	0.2922	0.4315	0.2905	0.4339	0.2888	0.4363	0.2872	0.4387	0.2855	0.4411	15
9	Finans Bank	0.8751	1.3097	0.8600	1.3306	0.8453	1.3519	0.8308	1.3736	0.8166	1.3957	1
10	TEB	0.3255	0.4317	0.3232	0.4349	0.3208	0.4381	0.3185	0.4414	0.3162	0.4446	14
11	HSBC Bank	0.3320	0.1652	0.3244	0.1735	0.3169	0.1819	0.3095	0.1903	0.3022	0.1990	17
12	ING Bank	0.1037	0.0744	0.1003	0.0789	0.0969	0.0834	0.0936	0.0879	0.0902	0.0924	18
13	Şeker Bank	0.3301	0.4442	0.3236	0.4531	0.3171	0.4621	0.3107	0.4712	0.3044	0.4804	12
14	Anadolu Bank	0.4337	0.4500	0.4178	0.4688	0.4023	0.4881	0.3873	0.5079	0.3726	0.5283	11
15	A Bank	0.4888	0.5019	0.4505	0.5424	0.4146	0.5851	0.3810	0.6303	0.3493	0.6782	9
16	Turkland Bank	0.8423	0.5217	0.7461	0.5905	0.6613	0.6658	0.5860	0.7488	0.5186	0.8406	7
17	Fiba Bank	0.3238	0.2861	0.3059	0.3064	0.2887	0.3273	0.2721	0.3489	0.2560	0.3713	16
18	Turkish Bank	0.5238	0.4861	0.5059	0.5064	0.4887	0.5273	0.4721	0.5489	0.4560	0.5713	10

Table 5.8. Increasing efficiency for FBCC model

DMU		E(L,U)										Rank
		L	U	L	U	L	U	L	U	L	U	
		0.0	0.0	0.25	0.25	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.7880	1.0911	0.7894	1.0897	0.7907	1.0884	0.7921	1.0870	0.7934	1.0857	5
2	İş Bank	0.5902	0.8940	0.5914	0.8928	0.5926	0.8916	0.5938	0.8904	0.5950	0.8892	7
3	Ak Bank	0.7853	1.0969	0.7877	1.0945	0.7901	1.0921	0.7925	1.0896	0.7950	1.0872	4
4	Garanti Bank	0.8170	1.1229	0.8187	1.1211	0.8205	1.1194	0.8222	1.1177	0.8240	1.1159	2
5	Yapı Kredi Bank	0.4658	0.7729	0.4673	0.7714	0.4687	0.7700	0.4702	0.7685	0.4717	0.7671	8
6	Halk Bank	0.4475	0.7555	0.4490	0.7539	0.4506	0.7524	0.4521	0.7508	0.4537	0.7493	9
7	Vakıf Bank	0.9902	1.3000	0.9927	1.2975	0.9951	1.2951	0.9975	1.2926	1	1.2902	1
8	Deniz Bank	0.7017	1.0144	0.7041	1.0119	0.7066	1.0095	0.7090	1.0070	0.7115	1.0046	6
9	Finans Bank	0.7693	1.1634	0.7814	1.1500	0.7938	1.1368	0.8063	1.1238	0.8190	1.1110	3
10	TEB	0.4310	0.7481	0.4336	0.7454	0.4363	0.7427	0.4390	0.7400	0.4417	0.7374	10
11	HSBC Bank	0.0624	0.4063	0.0678	0.4005	0.0732	0.3948	0.0787	0.3891	0.0843	0.3835	17
12	ING Bank	0.0429	0.3688	0.0461	0.3654	0.0494	0.3620	0.0527	0.3587	0.0560	0.3554	18
13	Şeker Bank	0.2523	0.5976	0.2581	0.5914	0.2640	0.5852	0.2699	0.5792	0.2759	0.5732	13
14	Anadolu Bank	0.2243	0.6133	0.2350	0.6010	0.2459	0.5890	0.2570	0.5772	0.2683	0.5657	14
15	A Bank	0.3037	0.7798	0.3235	0.7543	0.3439	0.7297	0.3652	0.7061	0.3872	0.6835	12
16	Turkland Bank	0.2788	0.8287	0.3052	0.7907	0.3329	0.7548	0.3620	0.7210	0.3927	0.6889	11
17	Fiba Bank	0.2025	0.6062	0.2147	0.5917	0.2272	0.5777	0.2400	0.5640	0.2531	0.5506	15
18	Turkish Bank	0.2025	0.6062	0.2147	0.5917	0.2272	0.5777	0.2400	0.5640	0.2531	0.5506	16

Table 5.9. Decreasing efficiency for FBCC model

DMU		E(L,U)										Rank
		L	U	L	U	L	U	L	U	L	U	
		0.0	0.0	0.25	0.25	0.5	0.5	0.75	0.75	1.0	1.0	
1	Ziraat Bank	0.7462	1.0093	0.7474	1.0081	0.7487	1.0069	0.7500	1.0057	0.7512	1.0045	4
2	İş Bank	0.5448	0.8193	0.5459	0.8182	0.5470	0.8172	0.5481	0.8161	0.5493	0.8151	8
3	Ak Bank	0.7563	1.0267	0.7586	1.0245	0.7609	1.0223	0.7632	1.0201	0.7656	1.0179	3
4	Garanti Bank	0.6328	0.9046	0.6343	0.9032	0.6357	0.9018	0.6372	0.9004	0.6387	0.8990	5
5	Yapı Kredi Bank	0.7828	1.0475	0.7846	1.0458	0.7864	1.0441	0.7881	1.0424	0.7899	1.0408	2
6	Halk Bank	0.6178	0.8923	0.6195	0.8906	0.6212	0.8890	0.6229	0.8874	0.6246	0.8858	6
7	Vakıf Bank	0.3374	0.6274	0.3388	0.6260	0.3403	0.6246	0.3418	0.6233	0.3432	0.6219	12
8	Deniz Bank	0.3092	0.6029	0.3109	0.6013	0.3126	0.5996	0.3143	0.5980	0.3160	0.5964	14
9	Finans Bank	0.9384	1.3000	0.9534	1.2839	0.9686	1.2682	0.9841	1.2528	1	1.2377	1
10	TEB	0.3093	0.6076	0.3116	0.6054	0.3139	0.6031	0.3162	0.6009	0.3186	0.5987	13
11	HSBC Bank	0.1184	0.4577	0.1243	0.4515	0.1303	0.4455	0.1364	0.4395	0.1425	0.4337	17
12	ING Bank	0.0533	0.3746	0.0565	0.3714	0.0597	0.3683	0.0629	0.3652	0.0662	0.3621	18
13	Şeker Bank	0.3182	0.6483	0.3246	0.6418	0.3310	0.6354	0.3376	0.6290	0.3442	0.6227	11
14	Anadolu Bank	0.3224	0.7136	0.3359	0.6983	0.3497	0.6834	0.3639	0.6690	0.3785	0.6549	10
15	A Bank	0.3596	0.9061	0.3886	0.8648	0.4192	0.8262	0.4516	0.7898	0.4859	0.7557	9
16	Turkland Bank	0.3737	1.1942	0.4230	1.0963	0.4770	1.0100	0.5365	0.9333	0.6022	0.8648	7
17	Fiba Bank	0.2050	0.6150	0.2195	0.5977	0.2345	0.5811	0.2500	0.5650	0.2660	0.5495	15
18	Turkish Bank	0.2050	0.6150	0.2195	0.5977	0.2345	0.5811	0.2500	0.5650	0.2660	0.5495	16

Note that the most efficient bank in the four models are Vakıf Bank, Garanti Bank, Finans Bank, Ak Bank, Ziraat Bank, and Deniz Bank respectively in case of increase the inputs (Number of branch, Number of staff), And the most efficient bank in the four models are

Finans Bank, Yapı Kredi Bank, Ak Bank, and Ziraat Bank in case of decrease the inputs, like the Figure 5.5-5.8.

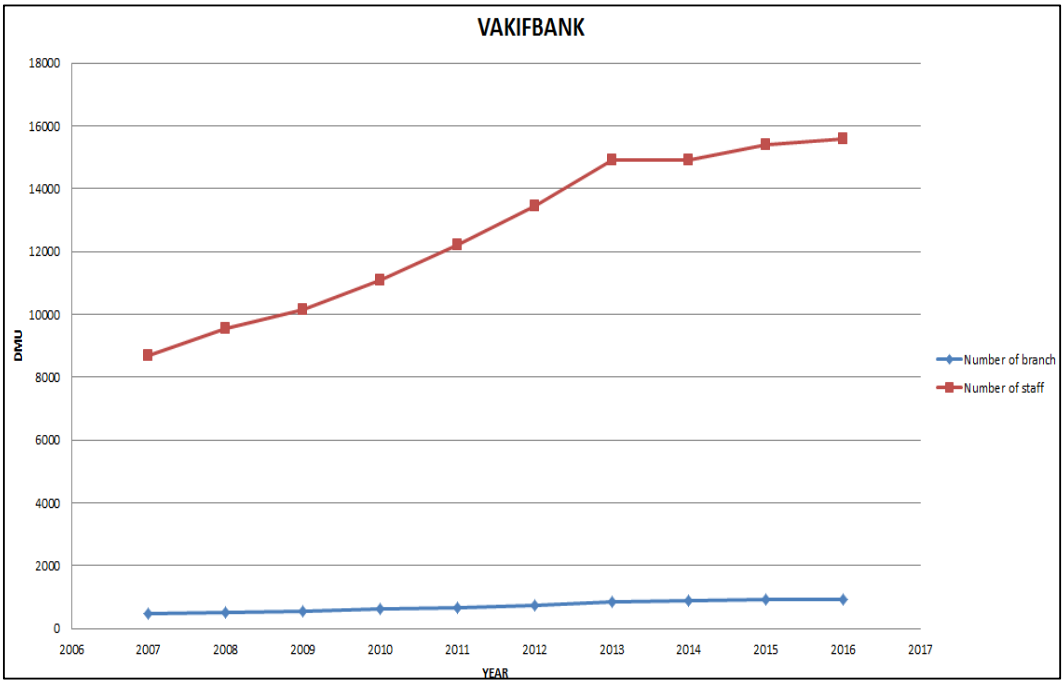

Figure 5.6. Vakıf Bank efficiency

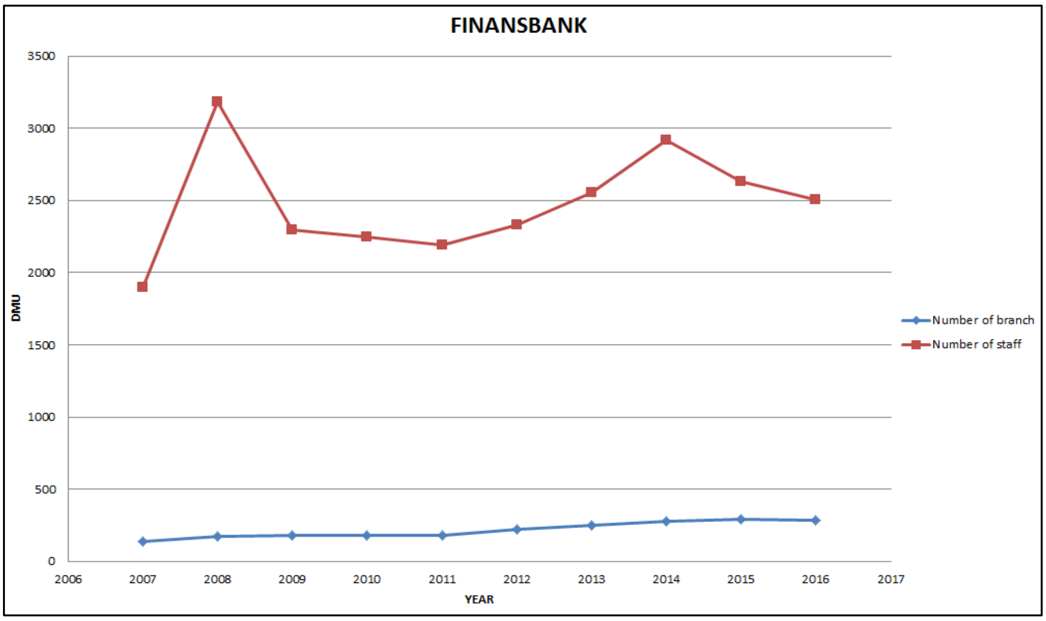

Figure 5.7. Finansbank efficiency

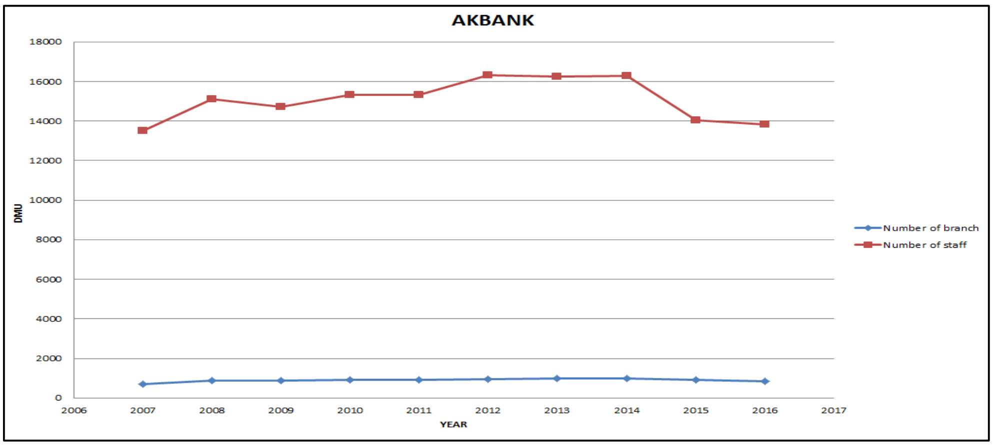

Figure 5.8. Ak Bank efficiency

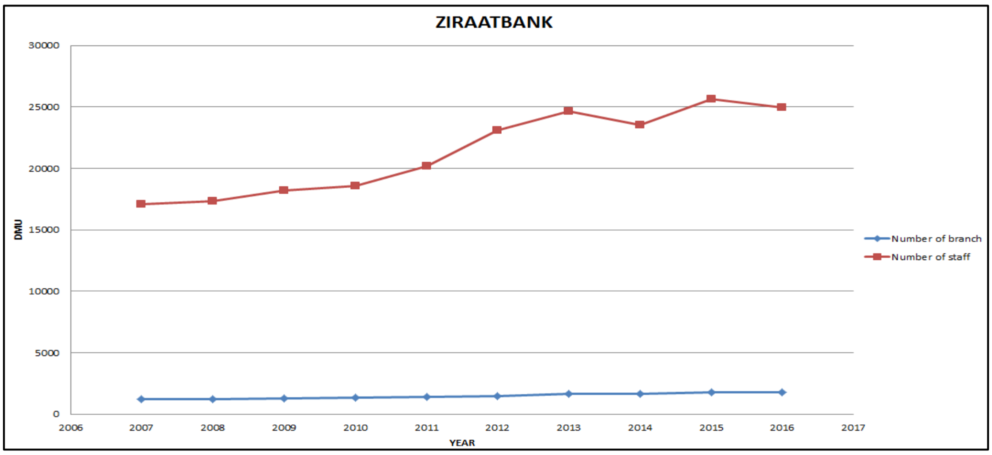

Figure 5.9. Ziraat Bank efficiency

For Vakıf Bank the case of increase is better, Because the number of customers in the corporate banking segment in 2016 increased 8 %. Also the number of commercial customers increased by 4,10 %. And the total activities 16.18 %. As well increased its profit in the year 2016 by 40.05 % compared to the previous year. And Credits increased by 16.8 % in 2016 compared to the previous year.

For Finans Bank the case of decrease is better, because in 2016, the number of mobile branch active customers, compared to the previous year, 74 % growth of 82 % in terms of financial transactions. The number of downloads of all the bank's mobile banking applications rose to (180000) in 2016. And (45000) POS devices which is provided to be reaching 41 % increase in the number of bank POS turnover. The first rankings bank among the participating banks in the number of POS.

And we get a incresing efficiency Figures 5.10 and decresing efficiency Figures 5.11 for fuzzy CCR and BCC models, by using MATLAB R2017a codes in Appendix 21and 22 respectively.

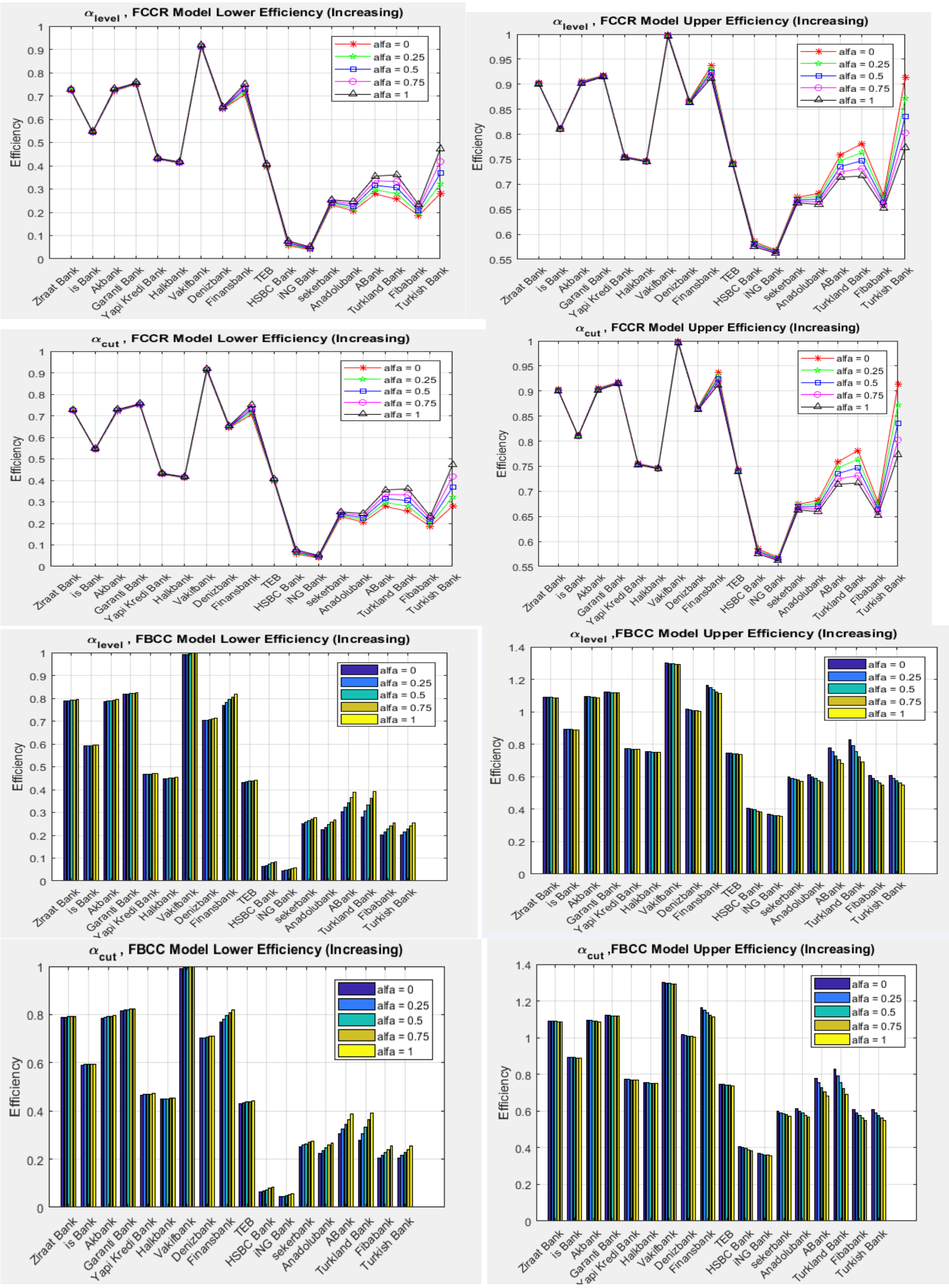

Figure 5.10. Increasing efficiency figures

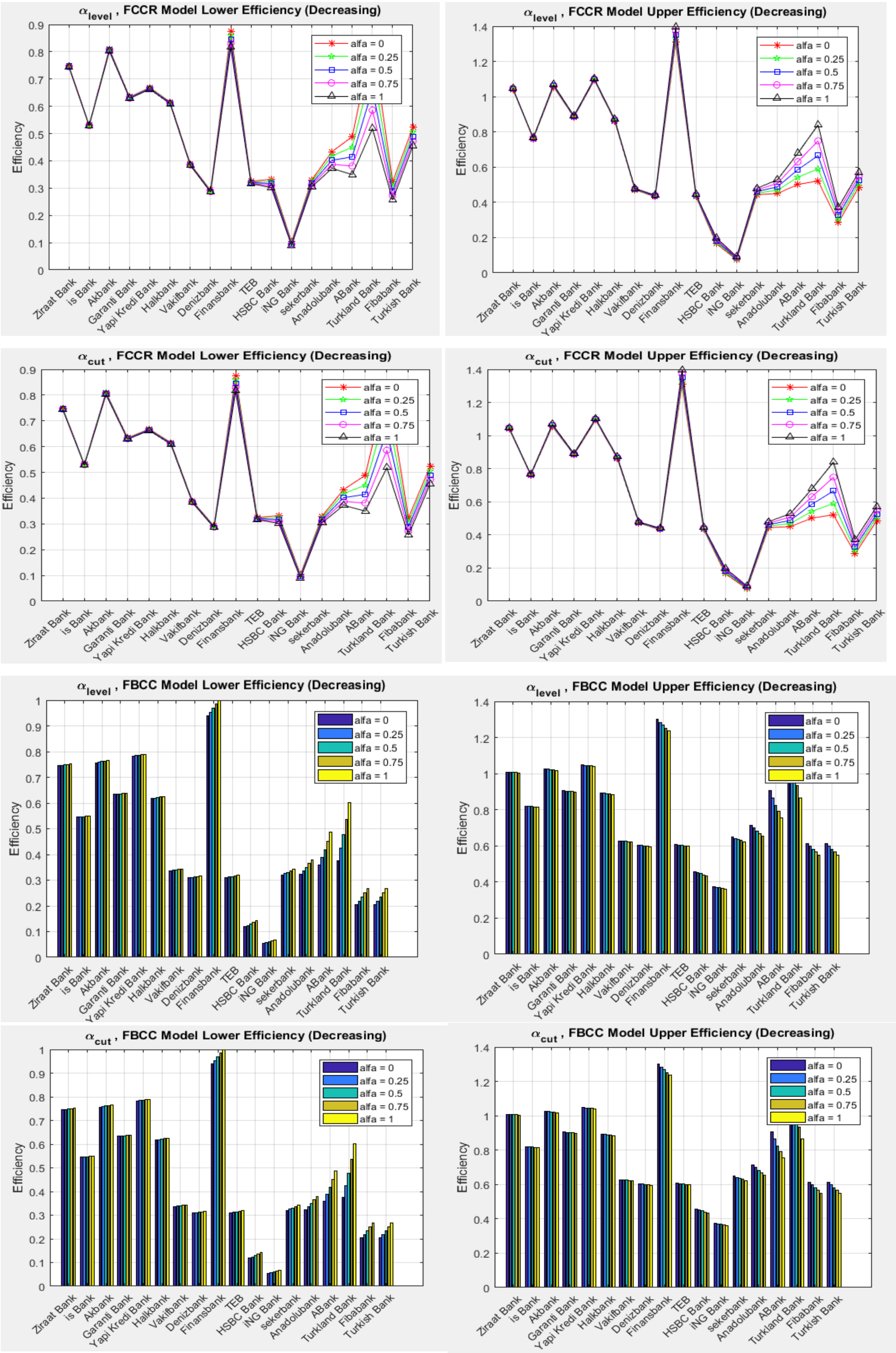

Figure 5.11. Decreasing efficiency figures

6. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

The development of fuzzy DEA with imprecise and ambiguous data has evolved the scope of its application to efficiency measurement in real-life problems. In the real world there are many problems which have fuzzy parameters. In this study, we developed a fuzzy DEA framework with a CCR and BCC model and used an α-cut approach to convert the fuzzy CCR and BCC models into an parametric programming model. Instead of comparing the equality (or inequality) of the two intervals, we defined a variable in the period to meet our limitations and maximize the efficiency value. We presented a real data to show the similarities and dissimilarities between our data and the simulations in case of increasing inputs and decreasing inputs from fuzzy DEA methods in other ways.

In this study we studied the feedforward network algorithm for training the multilayer artificial neural network. We used feedforward learning algorithm to train the feed forward neural network. From our learning curve we see that the validation and test curves are very similar. For avoiding over fitting we used early stopping method. The main goal of early stopping is to divide data into two sets, training and validation. Validation error is not a good estimate of the generalization error. That is why we used early stopping method so that training is stopped when the validation error starts to go up. We performed regression R analysis to measure the correlation between outputs and targets. We created a regression plot for validating the network which shows the relationship between the outputs of the network and the targets. It is observed that the value of R is closer to 1 indicating the accurate prediction. When the data set was trained in feedforward network algorithms the performance obtained was in 329 epochs. Feedforward network algorithm is the most widely used optimization algorithm.

We used proposed models in this study. In order to find the efficiencies of the DMUs to be presented in Table 5.6 and 5.8 increasing efficiency FCCR and FBCC models. On top of that, the optimal values of u_0 for the successful DMUs are reported in Table 5.7 and 5.9 decreasing efficiency FCCR and FBCC models. To look for the efficiency, For example, DMU 7, 4, 9, 3, 1 and 8 respectively successful when α is equal to 0, 0.25, 0.5, 0.75 and 1.0 in caes of increasing see Table (5.6,5.8). And DMU 9, 5, 3 and 1 respectively successful when α is equal to 0, 0.25, 0.5, 0.75 and 1.0 in case of decreasing see Table (5.7 ,5.9).

Proposed approach for solving a fuzzy CCR and fuzzy BCC models in which fuzzy constraints were treated as fuzzy events. They transformed the fuzzy DEA model into a possibility LP problem by using the possibility measures of the fuzzy events. It is noted this approach cannot be applied to computing the expected value of a continuously distributed random variable. What is more, There is no consensus on how fuzzy quantities are order, leave it up to the decision maker to determine how to distinguish the system between facilities. the approach to fuzzy arithmetics using alpha cuts proved to be a simple way to calculate operations fuzzy numbers.

In future research directions our model can be used when the number of DMUs is less used than the blended number of inputs and outputs, and can be used full fuzzy numbers. And this model can be extended to the other subject areas and types of DEA.

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APPENDICES

Appendix-1. Exact input and output data of DMUs 2011

DMUs		Inputs		Outputs		
		Number of branch	**Number of staff**	**Total credit**	**Total income**	**Net profit**
1	Ziraat Bank	1458	24374	71173.26	6209.94	2100.67
2	İş Bank	1201	24887	91620.64	8163.27	2667.49
3	Akbank	927	15339	70213.22	6083.46	2394.53
4	Garanti Bank	914	16773	83532.93	7936.87	3070.58
5	Yapı Kredi Bank	907	14859	67044.87	5821.77	1857.49
6	Halkbank	771	13643	55949.45	5051.93	2045.13
7	Vakıfbank	680	12222	57200.61	4422.16	1226.79
8	Denizbank	588	9772	22196.37	2291.80	873.97
9	Finansbank	522	10837	29867.20	3000.00	848.11
10	TEB	507	9356	25443.64	1793.32	206.68
11	HSBC Bank	330	6155	13662.37	1534.52	240.79
12	ING Bank	322	5232	15264.74	1100.21	79.01
13	Şekerbank	272	3530	8282.22	876.92	118.04
14	Anadolubank	88	1911	3715.39	335.91	85.23
15	ABank	63	1185	4220.59	288.19	28.26
16	Eurobank Tekfen	59	954	2291.94	209.61	37.37
17	Tekstil Bankası	44	880	2461.06	147.24	22.02
18	Citibank	37	2233	2652.96	421.70	5.51
19	Turkland Bank	27	496	1445.38	77.76	3.56
20	Fibabank	21	438	2095.04	91.00	5.19
21	Turkish Bank	20	284	256.57	33.39	0.69

Appendix-2. Fuzzy input data of DMUs

DMUs	Number of branch			Number of staff		
	L	M	U	L	M	U
Ziraat Bank	1453.65	1458	1462.35	24296.14	24374	24451.86
İş Bank	1196.65	1201	1205.35	24809.14	24887	24964.86
Akbank	922.65	927	931.35	15261.14	15339	15416.86
Garanti Bank	909.65	914	918.35	16695.14	16773	16850.86
Yapı Kredi Bank	902.65	907	911.35	14781.14	14859	14936.86
Halkbank	766.65	771	775.35	13565.14	13643	13720.86
Vakıfbank	675.65	680	684.35	12144.14	12222	12299.86
Denizbank	583.65	588	592.35	9694.14	9772	9849.86
Finansbank	517.65	522	526.35	10759.14	10837	10914.86
TEB	502.65	507	511.35	9278.14	9356	9433.86
HSBC Bank	325.65	330	334.35	6077.14	6155	6232.86
ING Bank	317.65	322	326.35	5154.14	5232	5309.86
Şekerbank	267.65	272	276.35	3452.14	3530	3607.86
Anadolubank	83.65	88	92.35	1833.14	1911	1988.86
ABank	58.65	63	67.35	1107.14	1185	1262.86
Eurobank Tekfen	54.65	59	63.35	876.14	954	1031.86
Tekstil Bank	39.65	44	48.35	802.14	880	957.86
Citibank	32.65	37	41.35	2155.14	2233	2310.86
Turkland Bank	22.65	27	31.35	418.14	496	573.86
Fibabank	16.65	21	25.35	360.14	438	515.86
Turkish Bank	15.65	20	24.35	206.14	284	361.86

Appendix-3. Fuzzy output data of DMUs

DMUs	Total credit			Total income			Net profit		
	L	M	U	L	M	U	L	M	U
Ziraat Bank	70857.77	71173.26	71488.75	6181.87	6209.94	6238.01	2090.15	2100.67	2111.20
İş Bank	91305.15	91620.64	91936.12	8135.21	8163.27	8191.34	2656.96	2667.49	2678.01
Akbank	69897.74	70213.22	70528.71	6055.39	6083.46	6111.52	2384.00	2394.53	2405.05
Garanti Bank	83217.44	83532.93	83848.42	7908.81	7936.87	7964.94	3060.05	3070.58	3081.10
Yapı Kredi Bank	66729.38	67044.87	67360.36	5793.71	5821.77	5849.84	1846.96	1857.49	1868.01
Halkbank	55633.96	55949.45	56264.93	5023.87	5051.93	5080.00	2034.61	2045.13	2055.66
Vakıfbank	56885.12	57200.61	57516.10	4394.09	4422.16	4450.22	1216.26	1226.79	1237.31
Denizbank	21880.89	22196.37	22511.86	2263.74	2291.80	2319.87	863.45	873.97	884.50
Finansbank	29551.71	29867.20	30182.68	2971.93	3000.00	3028.07	837.59	848.11	858.64
TEB	25128.16	25443.64	25759.13	1765.25	1793.32	1821.39	196.15	206.68	217.20
HSBC Bank	13346.89	13662.37	13977.86	1506.45	1534.52	1562.58	230.26	240.79	251.31
ING Bank	14949.25	15264.74	15580.22	1072.14	1100.21	1128.27	68.49	79.01	89.54
Şekerbank	7966.74	8282.22	8597.71	848.85	876.92	904.98	107.52	118.04	128.57
Anadolubank	3399.91	3715.39	4030.88	307.85	335.91	363.98	74.70	85.23	95.76
ABank	3905.10	4220.59	4536.07	260.13	288.19	316.26	17.74	28.26	38.79
Eurobank Tekfen	1976.46	2291.94	2607.43	181.54	209.61	237.67	26.84	37.37	47.89
Tekstil Bank	2145.57	2461.06	2776.54	119.18	147.24	175.31	11.49	22.02	32.54
Citibank	2337.48	2652.96	2968.45	393.64	421.70	449.77	-5.02	5.51	16.03
Turkland Bank	1129.89	1445.38	1760.86	77.76	105.83	133.90	-6.96	3.56	14.09
Fibabank	1779.55	2095.04	2410.52	62.93	91.00	119.06	-5.34	5.19	15.71
Turkish Bank	-58.91	256.57	572.06	5.32	33.39	61.46	-9.83	0.69	11.22

Appendix-4. α-cuts bounds of ınput variables

	DMUs	Number of branch		Number of staff	
		Lower bound	**Upper bound**	**Lower bound**	**Upper bound**
1	Ziraat Bank	α1458+(1-α)1453.65	α1458+(1- α)1462.35	α24374+(1- α)24296.14	α24374+(1- α)24451.86
2	İş Bank	α1201+(1- α)1196.65	α1201+(1- α)1205.35	α24887+(1- α)24809.14	α24887+(1- α)24964.86
3	Akbank	α927+(1-α)922.65	α927+(1-α)931.35	α15339+(1- α)15261.14	α15339+(1- α)15416.86
4	Garanti Bank	α914+(1-α)909.65	α914+(1-α)918.35	α16773+(1- α)16695.14	α16773+(1- α)16850.86
5	Yapı Kredi Bank	α907+(1-α)902.65	α907+(1-α)911.35	α14859+(1- α)14781.14	α14859+(1- α)14936.86
6	Halkbank	α771+(1-α)766.65	α771+(1-α)775.35	α13643+(1- α)13565.14	α13643+(1- α)13720.86
7	Vakıfbank	α680+(1-α)675.65	α680+(1-α)684.35	α12222+(1- α)12144.14	α12222+(1- α)12299.86
8	Denizbank	α588+(1-α)583.65	α588+(1-α)592.35	α9772+(1-α)9694.14	α9772+(1-α)9849.86
9	Finansbank	α522+(1-α)517.65	α522+(1-α)526.35	α10837+(1- α)10759.14	α10837+(1- α)10914.86
10	TEB	α507+(1-α)502.65	α507+(1-α)511.35	α9356+(1-α)9278.14	α9356+(1-α)9433.86
11	HSBC Bank	α330+(1-α)325.65	α330+(1-α)334.35	α6155+(1-α)6077.14	α6155+(1-α)6232.86
12	ING Bank	α322+(1-α)317.65	α322+(1-α)326.35	α5232+(1-α)5154.14	α5232+(1-α)5309.86
13	Şekerbank	α272+(1-α)267.65	α272+(1-α)276.35	α3530+(1-α)3452.14	α3530+(1-α)3607.86
14	Anadolubank	α88+(1-α)83.65	α88+(1-α)92.35	α1911+(1-α)1833.14	α1911+(1-α)1988.86
15	ABank	α63+(1-α)58.65	α63+(1-α)67.35	α1185+(1-α)1107.14	α1185+(1-α)1262.86
16	Eurobank Tekfen	α59+(1-α)54.65	α59+(1-α)63.35	α954+(1-α)876.14	α954+(1-α)1031.86
17	Tekstil Bank	α44+(1-α)39.65	α44+(1-α)48.35	α880+(1-α)802.14	α880+(1-α)957.86
18	Citibank	α37+(1-α)32.65	α37+(1-α)41.35	α2233+(1-α)2155.14	α2233+(1-α)2310.86
19	Turkland Bank	α27+(1-α)22.65	α27+(1-α)31.35	α496+(1-α)418.14	α496+(1-α)573.86
20	Fibabank	α21+(1-α)16.65	α21+(1-α)25.35	α438+(1-α)360.14	α438+(1-α)515.86
21	Turkish Bank	α20+(1-α)15.65	α20+(1-α)24.35	α284+(1-α)206.14	α284+(1-α)361.86

Appendix-5. α-cuts bounds of output variables

DMUs		Total credit		Total income		Net profit	
		Lower bound	**Upper bound**	**Lower bound**	**Upper bound**	**Lower bound**	**Upper bound**
1	Ziraat Bank	α71173.26+(1-α)70857.77	α71173.26+(1-α)71488.75	α6209.94+(1-α)6181.87	α6209.94+(1-α)6238.01	α2100.67+(1-α)2090.15	α2100.67+(1-α)2111.20
2	İş Bank	α91620.64+(1-α)91305.15	α91620.64+(1-α)91936.12	α8163.27+(1-α)8135.21	α8163.27+(1-α)8191.34	α2667.49+(1-α)2656.96	α2667.49+(1-α)2678.01
3	Akbank	α70213.22+(1-α)69897.74	α70213.22+(1-α)70528.71	α6083.46+(1-α)6055.39	α6083.46+(1-α)6111.52	α2394.53+(1-α)2384	α2394.53+(1-α)2405.05
4	Garanti Bank	α83532.93+(1-α)83217.44	α83532.93+(1-α)83848.42	α7936.87+(1-α)7908.81	α7936.87+(1-α)7964.94	α3070.58+(1-α)3060.05	α3070.58+(1-α)3081.1
5	Yapı Kredi Bank	α67044.87+(1-α)66729.38	α67044.87+(1-α)67360.36	α5821.77+(1-α)5793.71	α5821.77+(1-α)5849.84	α1857.49+(1-α)1846.96	α1857.49+(1-α)1868.01
6	Halkbank	α55949.45+(1-α)55633.96	α55949.45+(1-α)56264.93	α5051.93+(1-α)5023.87	α5051.93+(1-α)5080	α2045.13+(1-α)2034.61	α2045.13+(1-α)2055.66
7	Vakıfbank	α57200.61+(1-α)56885.12	α57200.61+(1-α)57516.1	α4422.16+(1-α)4394.09	α4422.16+(1-α)4450.22	α1226.79+(1-α)1216.26	α1226.79+(1-α)1237.31
8	Denizbank	α22196.37+(1-α)21880.89	α22196.37+(1-α)22511.86	α2291.8+(1-α)2263.74	α2291.8+(1-α)2319.87	α873.97+(1-α)863.45	α873.97+(1-α)884.50
9	Finansbank	α29867.2+(1-α)29551.71	α29867.2+(1-α)30182.68	α3000+(1-α)2971.93	α3000+(1-α)3028.07	α848.11+(1-α)837.59	α848.11+(1-α)858.64
10	TEB	α25443.64+(1-α)25128.16	α25443.64+(1-α)25759.13	α1793.32+(1-α)1765.25	α1793.32+(1-α)1821.39	α206.68+(1-α)196.15	α206.68+(1-α)217.20
11	HSBC Bank	α13662.37+(1-α)13346.89	α13662.37+(1-α)13977.86	α1534.52+(1-α)1506.45	α1534.52+(1-α)1562.58	α240.79+(1-α)230.26	α240.79+(1-α)251.31
12	ING Bank	α15264.74+(1-α)14949.25	α15264.74+(1-α)15580.22	α1100.21+(1-α)1072.14	α1100.21+(1-α)1128.27	α79.01+(1-α)68.49	α79.01+(1-α)89.54
13	Şekerbank	α8282.22+(1-α)7966.74	α8282.22+(1-α)8597.71	α876.92+(1-α)848.85	α876.92+(1-α)904.98	α118.04+(1-α)107.52	α118.04+(1-α)128.57
14	Anadolubank	α3715.39+(1-α)3399.91	α3715.39+(1-α)4030.88	α335.91+(1-α)307.85	α335.91+(1-α)363.98	α85.23+(1-α)74.7	α85.23+(1-α)95.76
15	ABank	α4220.59+(1-α)3905.1	α4220.59+(1-α)4536.07	α288.19+(1-α)260.13	α288.19+(1-α)316.26	α28.26+(1-α)17.74	α28.26+(1-α)38.79
16	Eurobank Tekfen	α2291.94+(1-α)1976.46	α2291.94+(1-α)2607.43	α209.61+(1-α)181.54	α209.61+(1-α)237.67	α37.37+(1-α)26.84	α37.37+(1-α)47.89
17	Tekstil Bank	α2461.06+(1-α)2145.57	α2461.06+(1-α)2776.54	α147.24+(1-α)119.18	α147.24+(1-α)175.31	α22.02+(1-α)11.49	α22.02+(1-α)32.54
18	Citibank	α2652.96+(1-α)2337.48	α2652.96+(1-α)2968.45	α421.7+(1-α)393.64	α421.7+(1-α)449.77	α5.51+(1-α)-5.02	α5.51+(1-α)16.03
19	Turkland Bank	α1445.38+(1-α)1129.89	α1445.38+(1-α)1760.86	α105.83+(1-α)77.76	α105.83+(1-α)133.9	α3.56+(1-α)-6.96	α3.56+(1-α)14.09
20	Fibabank	α2095.04+(1-α)1779.55	α2095.04+(1-α)2410.52	α91+(1- α)62.93	α91+(1-α)119.06	α5.19+(1-α)-5.34	α5.19+(1-α)15.71
21	Turkish Bank	α256.57+(1- α)-58.91	α256.57+(1-α)572.06	α33.39+(1- α)5.32	α33.39+(1-α)61.46	α0.69+(1-α)-9.83	α0.69+(1-α)11.22

Appendix-6. Exact input and output data of DMUs 2007-2016

DMUs		2007					2008				
		Outputs(MillionTL)			Inputs		Outputs(MillionTL)			Inputs	
		Total credit	Total income	Net profit	Number of branch	Number of staff	Total credit	Total income	Net profit	Number of branch	Number of staff
1	Ziraat Bank	21604	3801	2351	1247	17067	30836	4102	2134	2581	17343
2	İş Bank	33980	2960	1702	933	19245	47610	3618	1509	1028	20754
3	Akbank	37016	3375	2041	715	13503	44374	3667	1782	867	15117
4	Garanti Bank	39700	3098	2422	592	13631	52700	3431	1891	722	16292
5	Yapı Kredi Bank	29088	2473	870	675	14242	39554	2824	1261	860	14790
6	Halkbank	18121	17052	1131	585	11457	25836	2126	1018	581	12437
7	Vakıfbank	23470	1676	1031	468	8700	30417	1970	753	525	9567
8	Denizbank	13412	847	341	320	6634	12759	1257	342	400	7376
9	Finansbank	4498	329	147	137	1897	5490	404	161	174	3185
10	TEB	6546	559	130	272	5139	8505	704	164	335	6398
11	HSBC Bank	9350	367	794	437	5733	9751	247	1187	332	6807
12	ING Bank	8511	612	135	364	5921	11044	812	140	365	6355
13	Şekerbank	6089	439	123	235	3826	8041	647	144	250	4088
14	Anadolubank	1764	186	73	76	1724	1958	214	87	77	1726
15	ABank	1864	197	69	39	868	2375	212	42	42	1006
16	Turkland Bank	419	26	0.6	16	390	595	52	0.8	25	457
17	Fibabank	736	66	3.8	16	142	884	49	3.5	18	168
18	Turkish Bank	129	20	1.5	22	274	202	29	10	26	292

DMUs		2009					2010				
		Outputs(MillionTL)			Inputs		Outputs(MillionTL)			Inputs	
		Total credit	Total income	Net profit	Number of Branch	Number of staff	Total credit	Total income	Net profit	Number of branch	Number of staff
1	Ziraat Bank	36725	6068	3511	1305	18198	57443	5356	3713	1379	18602
2	İş Bank	48335	5740	2372	1078	22281	64322	5410	2982	1142	23750
3	Akbank	39718	2713	4593	877	14704	52896	4276	2857	912	15320
4	Garanti Bank	53500	5406	3100	792	15654	69700	4754	3400	869	15210
5	Yapı Kredi Bank	39270	3897	1543	837	14328	54676	3582	2248	867	14407
6	Halkbank	32458	3109	1631	654	12476	44296	3191	2010	768	13413
7	Vakıfbank	34439	3077	1251	545	10153	44836	2730	1157	635	11072
8	Denizbank	14171	1864	605	450	7789	18459	1789	616	500	8573
9	Finansbank	7124	504	171	180	2294	8000	473	206	180	2245
10	TEB	8991	891	210	333	5869	11753	826	300	334	5644
11	HSBC Bank	8939	255	1209	323	6384	9851	241	1094	330	6513
12	ING Bank	11005	1224	187	358	6107	12184	1037	112	322	5862
13	Şekerbank	8955	736	153	256	3937	11369	538	170	260	3484
14	Anadolubank	2430	266	120	86	1851	3027	257	123	86	1834
15	ABank	2726	261	70	46	999	3258	168	34	53	1121
16	Turkland Bank	722	65	2.4	25	464	1003	53	2.5	27	510
17	Fibabank	728	34	17.7	18	165	640	33	14.6	18	292
18	Turkish Bank	233	24	1.2	25	276	415	26	3.4	21	273

DMUs		2011					2012				
		Outputs(MillionTL)			Inputs		Outputs(MillionTL)			Inputs	
		Total credit	Total income	Net profit	Number of branch	Number of staff	Total credit	Total income	Net profit	Number of branch	Number of staff
1	Ziraat Bank	71430	5242	2101	1434	20181	71426	6901	2650	1490	23069
2	İş Bank	91621	5416	2667	1182	24660	106716	6842	3310	1231	24169
3	Akbank	70306	3992	2395	926	15327	87656	5200	2950	961	16303
4	Garanti Bank	90300	5258	3300	918	15465	99500	6420	3400	936	17175
5	Yapı Kredi Bank	70071	3745	2285	906	14855	78789	4948	2088	927	14729
6	Halkbank	56216	3805	2045	770	13616	65894	4467	2595	820	14949
7	Vakıfbank	57201	2897	1227	678	12204	67878	4085	1460	742	13446
8	Denizbank	30947	1880	874	588	9772	38801	2143	812	610	10280
9	Finansbank	8904	575	232	182	2188	12764	792	284	220	2332
10	TEB	25621	1375	207	506	9353	29686	1940	478	508	9285
11	HSBC Bank	14058	240	1079	327	6104	15517	175	1380	335	6119
12	ING Bank	15377	980	79	321	5229	18842	1345	571	318	5316
13	Şekerbank	14490	597	118	272	3530	10085	836	240	272	3565
14	Anadolubank	3733	287	85	88	1911	5059	410	547	91	2024
15	ABank	4337	233	30	63	1185	5201	418	68	63	1278
16	Turkland Bank	1468	70	3.5	27	496	2017	110	12.7	27	524
17	Fibabank	2107	71	5	21	438	3221	131	46	28	612
18	Turkish Bank	363	24	0.7	20	284	396	27	1.5	19	284

DMUs		2013					2014				
		Outputs(MillionTL)			Inputs		Outputs(MillionTL)			Inputs	
		Total credit	Total income	Net profit	Number of Branch	Number of staff	Total credit	Total income	Net profit	Number of branch	Number of staff
1	Ziraat Bank	111048	7738	3330	1636	24639	141915	8606	4051	1682	23525
2	İş Bank	134843	7681	3163	1289	23565	155315	8470	3382	1330	24009
3	Akbank	110670	6173	2942	985	16236	125978	6921	3160	990	16292

Appendix-6. (cont) Exact input and output data of DMUs 2007-2016

4	Garanti Bank	128000	7132	3400	1001	17285	142900	8370	3700	1004	18738
5	Yapı Kredi Bank	100624	5006	3659	948	15679	29857	5973	1844	1002	17454
6	Halkbank	84848	4829	2751	876	14775	101767	5112	2206	899	17288
7	Vakıfbank	86496	4790	1585	857	14923	104344	4652	1753	891	14901
8	Denizbank	56466	2443	1011	688	12817	64564	2838	939	715	13183
9	Finansbank	17448	874	329	250	2554	23057	1098	334	280	2919
10	TEB	34288	1972	535	543	9989	39439	2565	623	550	10139
11	HSBC Bank	19156	45	1303	312	6095	19515	57	1238	295	5730
12	ING Bank	24481	1397	115	329	5775	28046	1705	176	315	6153
13	Şekerbank	13547	867	210	312	4150	14655	1672	224	312	4460
14	Anadolubank	6202	349	488	115	2111	7444	371	648	108	1761
15	ABank	6441	410	76	73	1413	7900	411	137	73	1231
16	Turkland Bank	2668	121	14.4	27	535	3365	181	32	33	641
17	Fibabank	5180	193	44	62	1152	6190	313	74	67	1222
18	Turkish Bank	536	28	0.1	19	269	896	44	5	18	268
		2015					**2016**				
		Outputs(MillionTL)			**Inputs**		**Outputs(MillionTL)**			**Inputs**	
DMUs		**Total credit**	**Total income**	**Net profit**	**Number of branch**	**Number of staff**	**Total credit**	**Total income**	**Net profit**	**Number of branch**	**Number of staff**
1	Ziraat Bank	186813	10508	5162	1786	25618	232644	13448	6576	1786	24932
2	İş Bank	177037	8985	3082	1354	24877	203144	10837	4701	1351	24464
3	Akbank	141763	7337	2995	901	14037	161828	7946	4529	840	13831
4	Garanti Bank	170400	9241	3600	983	19692	200100	11096	5100	971	19689
5	Yapı Kredi Bank	24860	7178	1860	999	18260	30508	8220	2932	935	18364
6	Halkbank	126745	5934	2315	948	17079	158354	7349	2558	963	16921
7	Vakıfbank	122975	5487	1930	918	15390	146619	6967	2703	922	15596
8	Denizbank	77705	4085	859	691	12917	95023	4985	1401	693	12932
9	Finansbank	27012	1404	261	295	2633	25599	1510	296	286	2507
10	TEB	53213	3074	883	532	9917	56364	3437	942	515	9640
11	HSBC Bank	21079	325	1132	281	5060	14634	355	961	87	3145
12	ING Bank	35205	2026	175	297	5601	34817	2222	252	268	5284
13	Şekerbank	16738	1056	103	301	4078	7613	1085	125	273	3611
14	Anadolubank	8112	420	144	106	1711	10784	473	175	106	1784
15	ABank	9306	512	70	69	1116	10489	356	30	53	928
16	Turkland Bank	3963	184	14.5	34	662	3629	190	14	33	608
17	Fibabank	8615	397	82	67	1290	11441	510	115	73	1488
18	Turkish Bank	814	49	4	13	252	1030	53	5	13	252

Appendix-7. Simulation Data in Case of Increase

DMU		Outputs			Inputs	
		Total Credit	Total Income	Net Profit	Number of branch	Number of staff
1	Ziraat Bank	189533.50	11180.21	5166.91	1796	25132
2	İş Bank	177282.00	9364.92	3901.02	1361	24664
3	Akbank	175072.40	7225.73	2590.61	850	14031
4	Garanti Bank	134022.20	7584.48	3429.60	981	19889
5	Yapı Kredi Bank	51278.89	5599.32	2869.66	945	18564
6	Halkbank	190156.70	9007.67	3351.42	973	17121
7	Vakıfbank	94930.14	5732.22	2667.91	932	15796
8	Denizbank	43843.06	3103.34	1530.47	713	13132
9	Finansbank	23607.45	822.59	507.46	296	2707
10	TEB	56170.89	2873.80	1338.74	525	9840
11	HSBC Bank	21382.62	1033.14	248.33	97	3345
12	ING Bank	126081.80	4207.77	1364.46	278	5484
13	Şekerbank	15348.14	1017.26	535.19	283	3811
14	Anadolubank	7048.02	317.61	149.90	116	1984
15	ABank	7768.25	260.47	17.32	63	1128
16	Turkland Bank	4085.51	164.78	41.06	43	808
17	Fibabank	6144.77	302.94	69.64	83	1688
18	Turkish Bank	1407.05	92.35	87.69	23	452

Appendix-8. Fuzzy Input Data of DMUs in Case of Increase

DMUs		Number of branch			Number of staff		
		L	M	U	L	M	U
1	Ziraat Bank	1790.812	1796	1801.188	25045.590	25132	25218.410
2	İş Bank	1355.812	1361	1366.188	24577.590	24664	24750.410
3	Akbank	844.811	850	855.188	13944.590	14031	14117.410
4	Garanti Bank	975.811	981	986.188	19802.590	19889	19975.410
5	Yapı Kredi Bank	939.811	945	950.188	18477.590	18564	18650.410
6	Halkbank	967.811	973	978.188	17034.590	17121	17207.410
7	Vakıfbank	926.811	932	937.188	15709.590	15796	15882.410
8	Denizbank	707.811	713	718.188	13045.590	13132	13218.410
9	Finansbank	290.811	296	301.188	2620.587	2707	2793.413
10	TEB	519.811	525	530.188	9753.587	9840	9926.413
11	HSBC Bank	91.811	97	102.188	3258.587	3345	3431.413
12	ING Bank	272.811	278	283.188	5397.587	5484	5570.413
13	Şekerbank	277.811	283	288,188	3724.587	3811	3897.413
14	Anadolubank	110.811	116	121.188	1897.587	1984	2070.413
15	ABank	57.811	63	68.188	1041.587	1128	1214.413
16	Turkland Bank	37.811	43	48.188	721.587	808	894.412
17	Fibabank	77.811	83	88.188	1601.587	1688	1774.413
18	Turkish Bank	17.811	23	28.188	365.587	452	538.412

Appendix-9. Fuzzy output data of dmus in case of ıncrease

	DMUs	Total credit			Total income			Net profit		
		L	M	U	L	M	U	L	M	U
1	Ziraat Bank	188393.4	189533.5	190673.5	11131.6	11180.2	11228.7	5150.8	5166.9	5183.0
2	İş Bank	176141.9	177282.0	178422.0	9316.3	9364.9	9413.4	3884.9	3901.0	3917.1
3	Akbank	173932.3	175072.4	176212.4	7177.2	7225.7	7274.2	2574.5	2590.6	2606.7
4	Garanti B.	132882.2	134022.2	135162.3	7535.9	7584.4	7633.0	3413.5	3429.6	3445.6
5	YapıKrediB.	50138.8	51278.8	52418.9	5550.7	5599.3	5647.8	2853.5	2869.6	2885.7
6	Halkbank	189016.7	190156.7	191296.8	8959.1	9007.6	9056.2	3335.3	3351.4	3367.5
7	Vakıfbank	93790.0	94930.1	96070.1	5683.6	5732.2	5780.7	2651.8	2667.9	2684.0
8	Denizbank	42703.0	43843.0	44983.1	3054.8	3103.3	3151.8	1514.3	1530.4	1546.5
9	Finansbank	22467.4	23607.4	24747.5	774.0	822.5	871.1	491.3	507.4	523.5
10	TEB	55030.8	56170.8	57310.9	2825.2	2873.8	2922.3	1322.6	1338.7	1354.8
11	HSBC Bank	20242.5	21382.6	22522.6	984.6	1033.1	1081.6	232.2	248.3	264.4
12	ING Bank	124941.8	126081.8	127221.9	4159.2	4207.7	4256.2	1348.3	1364.4	1380.5
13	Şekerbank	14208.0	15348.1	16488.1	968.7	1017.2	1065.7	519.0	535.1	551.2
14	Anadolu B..	5907.9	7048.0	8188.0	269.0	317.6	366.1	133.8	149.9	166.0
15	Abank	6628.2	7768.2	8908.3	211.9	260.4	309.0	1.2	17.3	33.4
16	Turkland B.	2945.4	4085.5	5225.5	116.2	164.7	213.3	24.9	41.0	57.1
17	Fibabank	5004.7	6144.7	7284.8	254.4	302.9	351.4	53.5	69.6	85.7
18	Turkish B.	267.0	1407.0	2547.1	43.8	92.3	140.8	71.6	87.6	103.7

Appendix-10. α-cuts bounds of ınput variables in case of ıncrease

DMUs		Number of branch		Number of staff	
		Lower bound	Upper bound	Lower bound	Upper bound
1	Ziraat Bank	α1796+(1-α)1790.81	α1796+(1-α)1801.18	α25132+(1-α)25045.59	α25132+(1-α)25218.41
2	İş Bank	α1361+(1-α)1355.81	α1361+(1-α)1366.18	α24664+(1-α)24577.59	α24664+(1-α)24750.41
3	Ak Bank	α850+(1-α)844.81	α850+(1-α)855.18	α14031+(1-α)13944.59	α14031+(1-α)14117.41
4	Garanti Bank	α981+(1-α)975.81	α981+(1-α)986.18	α19889+(1-α)19802.59	α19889+(1-α)19975ç41
5	Yapı Kredi Bank	α945+(1-α)939.81	α945+(1-α) 950.18	α18564+(1-α)18477.59	α18564+(1-α)18650.41
6	Halk Bank	α973+(1-α)967.81	α973+(1-α)978.18	α17121+(1-α)17034.59	α17121+(1-α)17207.41
7	Vakıf Bank	α932+(1-α)926.81	α932+(1-α)937.18	α15796+(1-α)15709.59	α15796+(1-α)15882.41
8	Deniz Bank	α713+(1-α)707.81	α713+(1-α)718.18	α13132+(1-α)13045.59	α13132+(1-α)13218.41
9	Finans Bank	α296+(1-α)290.81	α296+(1-α)301.18	α2707+(1-α)2620.59	α2707+(1-α)2793.41
10	TEB	α525+(1-α)519.81	α525+(1-α)530.18	α9840+(1-α)9753.59	α9840+(1-α)9926.41
11	HSBC Bank	α 97+(1-α)91.81	α 97+(1-α)102.18	α3354+(1-α)3258.59	α3354+(1-α)3431.41
12	ING Bank	α278+(1-α)272.81	α278+(1-α)283.18	α5484+(1-α)5397.59	α5484+(1-α)5570.41
13	Şeker Bank	α283+(1-α)277.81	α283+(1-α)288.18	α3811+(1-α)3724.59	α3811+(1-α)3897.41
14	Anadolu Bank	α116+(1-α)110.81	α116+(1-α)121.18	α1984+(1-α)1879.59	α1984+(1-α)2070.41
15	ABank	α63+(1-α)57.81	α63+(1-α)68.18	α1128+(1-α)1041.59	α1128+(1-α)894.41
16	Turkland Bank	α43+(1-α)77.81	α43+(1-α)88.18	α808+(1-α)1601.59	α808+(1-α)1774.41
17	Fiba Bank	α83+(1-α)77.81	α83+(1-α)88.18	α1688+(1-α)1601.59	α1688+(1-α)1774.41
18	Turkish Bank	α23+(1-α)17.81	α23+(1-α)28.18	α452+(1-α)365.59	α452+(1-α)538.41

Appendix-11. α-cuts bounds of output variables in case of ıncrease

DMUs		Total credit		Total income		Net profit	
		Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
1	Ziraat Bank	α189533.5+(1-α)188393.4	α189533.5+(1-α) 190673.5	α11180.2+(1 - α)11131,6	α11180.2+(1-α)11228.7	α5166.9+(1-α)5150.8	α5166.9+(1-α)5183.0
2	İş Bank	α177282+(1-α)176141.9	α177282+(1-α)178422.0	α9364.9+(1-α)9316.3	α9364.9+(1-α)9413.4	α3901.0+(1-α)3884.9	α3901.0+(1-α)3917.1
3	Ak Bank	α175072.4+(1-α)173932.3	α175072.4+(1-α)176212.4	α7225.7+(1-α)7177.2	α7225.7+(1-α)7274.2	α2590.6+(1-α)2574.5	α2590.6+(1-α)2606.7
4	Garanti Bank	α134022.2+(1-α)132882.2	α134022.2+(1-α)135162.3	α7584.4+(1-α)7535.9	α7584.4+(1-α)7633.0	α3429.6+(1-α)3413.5	α3429.6+(1-α)3445.6
5	Yapı Kredi Bank	α51278.8+(1-α)50138.8	α51278.8+(1-α)52418.9	α5599.3+(1-α)5550.7	α5599.3+(1-α)5647.8	α2869.6+(1-α)2853.5	α2869.6+(1-α)2885.7
6	Halk Bank	α190156.7+(1-α)189016.7	α190156.7+(1-α)191296.8	α9007.6+(1-α)8959.1	α9007.6+(1-α)9056.2	α3351.4+(1-α)3335.3	α3351.4+(1-α)3367.5
7	Vakıf Bank	α94930.1+(1-α)93790.0	α94930.1+(1-α)96070.1	α5732.2+(1-α)5683.6	α5732.2+(1-α)5780.7	α2667.9+(1-α)2651.8	α2667.9+(1-α)2684.0
8	Deniz Bank	α43843.0+(1-α)42703.0	α43843.0+(1-α)44983.1	α3103.3+(1-α)3054.8	α3103.3+(1-α)3151.8	α1530.4+(1-α)1514.3	α1530.4+(1-α)1546.5
9	Finans Bank	α23607.5+(1-α)22467.4	α23607.5+(1-α)24747.5	α822.5+(1-α)774.0	α822.5+(1 - α)871.1	α507.4+(1-α)491.3	α507.4+(1-α)523.5
10	TEB	α56170.8+(1-α)55030.8	α56170.8+(1-α)57310.9	α2873.8+(1-α)2825.2	α2873.8+(1-α)2922.3	α1338.7+(1-α)1322.6	α1338.7+(1-α)1354.8
11	HSBC Bank	α21382.6+(1-α)20242.5	α21382.6+(1-α)22522.6	α1033.1+(1-α)984.6	α1033.1+(1-α)1081.6	α248.3+(1-α)232.2	α248.3+(1-α)264.4
12	ING Bank	α126081.8+(1-α)124941.8	α126081.8+(1-α)127221.9	α4207.7+(1-α)4159.2	α4207.7+(1-α)4256.2	α1364.4+(1-α)1348.3	α1364.4+(1-α)1380.5
13	Şeker Bank	α15348.1+(1-α)14208.0	α15348.1+(1-α)16488.1	α1017.2+(1-α)968.7	α1017.2+(1-α)1065.7	α535.1+(1-α)519.0	α535.1+(1-α)551.2
14	Anadolu Bank	α7048.0+(1-α)5907.9	α7048.0+(1-α)8188.0	α317.6+(1-α)269.0	α317.6+(1 - α)366.1	α149.9+(1-α)133.8	α149.9+(1-α)166.0
15	ABank	α7768.2+(1-α)6628.2	α7768.2+(1-α)8908.3	α260.4+(1-α)211.9	α260.4+(1 - α)309.0	α17.3+(1-α)1.2	α17.3+(1-α)33.4
16	Turkland Bank	α4085.5+(1-α)2945.4	α4085.5+(1-α)5225.5	α164.7+(1-α)116.2	α164.7+(1 - α)213.3	α41.0+(1-α)24.9	α41.0+(1-α)57.1
17	Fiba Bank	α6144.7+(1-α)5004.7	α6144.7+(1-α)7284.8	α302.9+(1-α)254.4	α302.9+(1-α)351.4	α69.6+(1-α)53.5	α69.6+(1-α)85.7
18	Turkish Bank	α1407.0+(1-α)267.0	α1407.0+(1-α)2547.1	α92.3+(1-α)43.8	α92.3+(1-α)140.8	α87.6+(1-α)71.6	α87.6+(1-α)103.7

Appendix-12. Simulation data in case of decrease

DMU		Outputs			İnputs	
		Total Credit	Total Income	Net Profit	Number of branch	Number of staff
1	Ziraat Bank	274608.2	13373.05	5527.191	1776	24732
2	İş Bank	201051.1	9686.557	3911.644	1341	24264
3	Akbank	237662.3	8740.667	2808.742	830	13631
4	Garanti Bank	391208.5	14582.15	4629.555	961	19489
5	Yapı Kredi Bank	53003.24	6022.194	3126.083	925	18164
6	Halkbank	123679.5	7384.696	3126.365	953	16721
7	Vakıfbank	55398.44	938.7804	1735.941	912	15396
8	Denizbank	65488.18	3902.446	1772.324	683	12732
9	Finansbank	28919.12	859.5991	484.5278	276	2307
10	TEB	43107.35	2349.634	1149.122	505	9440
11	HSBC Bank	9546.542	685.084	142.8136	77	2945
12	ING Bank	37511.63	253.2767	459.6176	258	5084
13	Şekerbank	12948.16	860.0882	477.7954	263	3411
14	Anadolubank	9863.912	322.2226	104.6069	96	1584
15	ABank	4677.524	157.3182	39.98249	43	728
16	Turkland Bank	1350.317	79.04383	88.74368	23	408
17	Fibabank	5177.426	239.976	11.28389	63	1288
18	Turkish Bank	56.0816	44.35766	121.5243	3	52

Appendix-13. Fuzzy ınput data of dmus in case of decrease

DMUs		Number of branch			Number of staff		
		L	M	U	L	M	U
1	Ziraat Bank	1770.813	1776	1781.187	24645.590	24732	24818.410
2	İş Bank	1335.813	1341	1346.187	24177.590	24264	24350.410
3	Akbank	824.813	830	835.187	13544.590	13631	13717.410
4	Garanti Bank	955.813	961	930.187	19402.590	19489	19575.410
5	Yapı Kredi Bank	919.813	925	930.187	18077.590	18164	18250.410
6	Halkbank	947.813	953	958.187	16634.590	16721	16807.410
7	Vakıfbank	906.813	912	917.187	15309.590	15396	15482.410
8	Denizbank	677.813	683	688.187	12645.590	12732	12818.410
9	Finansbank	270.813	276	281.187	2220.590	2307	2393.413
10	TEB	499.813	505	510.187	9353.590	9440	9526.413
11	HSBC Bank	71.813	77	82.187	2858.590	2945	3031.413
12	ING Bank	252.813	258	263.187	4997.590	5084	5170.413
13	Şekerbank	257.813	263	268.187	3324.590	3411	3497.413
14	Anadolubank	90.813	96	101.187	1497.590	1584	1670.413
15	ABank	37.813	43	48.187	641.590	728	814.413
16	Turkland Bank	17.813	23	28.187	321.590	408	494.413
17	Fibabank	57.813	63	68.187	1201.590	1288	1374.413
18	Turkish Bank	-2.186	3	8.187	-34.412	52	138.413

Appendix-14. Fuzzy output data of dmus in case of decrease

DMUs		Total credit			Total income			Net profit		
		L	M	U	L	M	U	L	M	U
1	Ziraat Bank	273468.100	274608.200	275748.200	13324.520	13373.050	13421.580	5509.392	5527.191	5544.991
2	İş Bank	199911.000	201051.100	202191.100	9638.028	9686.557	9735.086	3893.845	3911.644	3929.444
3	Ak Bank	236522.200	237662.300	238802.300	8692.138	8740.667	8789.196	2790.942	2808.742	2826.541
4	Garanti B.	390068.400	391208.500	392348.500	14533.620	14582.150	14630.680	4611.756	4629.555	4647.355
5	YapıKredi Bank	51863.190	53003.240	54143.290	5973.665	6022.194	6070.724	3108.284	3126.083	3143.883
6	HalkBank	122539.400	123679.500	124819.500	7336.167	7384.696	7433.225	3108.565	3126.365	3144.165
7	VakıfBank	54258.390	55398.440	56538.490	890.251	938.780	987.309	1718.141	1735.941	1753.741
8	DenizBank	64348.130	65488.180	66628.230	3853,916	3902.446	3950.975	1754.524	1772.324	1790.124
9	Finans B.	27779.070	28919.120	30059.170	811.069	859.599	908.128	466.728	484.527	502.327
10	TEB	41967.300	43107.350	44247.400	2301.105	2349.634	2398.164	1131.322	1149.122	1166.921
11	HSBC Bank	8406.492	9546.542	10686.590	636.554	685.084	733.613	125.014	142.813	160.613
12	ING Bank	36371.580	37511.630	38651.680	204.747	253.276	301.806	441.818	459.617	477.417
13	ŞekerBank	11808.110	12948.160	14088.210	811.558	860.088	908.617	459.995	477.795	495.595
14	Anadolu Bank	8723.862	9863.912	11003.96	273.693	322.222	370.752	86.807	104.606	122.406
15	ABank	3537.474	4677.524	5817.573	108.788	157.318	205.847	22.182	39.982	57.782
16	Turkland Bank	210.267	1350.317	2490.367	30.514	79.043	127.573	70.944	88.743	106.543
17	FibaBank	4037.376	5177.426	6317.476	191.446	239.976	288.505	-6.515	11.283	29.083
18	Turkish Bnak	-1083.970	56.081	1196.132	-4.171	44.357	92.887	103.724	121.524	139.323

Appendix-15. α-cuts bounds of ınput variables in case of decrease

DMUs		Number of branch		Number of staff	
		Lower bound	Upper bound	Lower bound	Upper bound
1	Ziraat Bank	α1796+(1-α)1771	α1796+(1- α)1781	α24732+(1-α)24646	α24732+(1- α)24818
2	İş Bank	α1341+(1- α)1336	α1341+(1- α)1346	α24264+(1-α)24178	α24264+(1- α)24350
3	Akbank	α830+(1-α)825	α830+(1-α)835	α13631+(1-α)13545	α13631+(1- α)13717
4	Garanti Bank	α961+(1-α)956	α961+(1-α)930	α19489+(1-α)19403	α19489+(1- α)19575
5	Yapı Kredi Bank	α925+(1-α)920	α925+(1-α)930	α18164+(1-α)18078	α18164+(1- α)18250
6	Halkbank	α 953+(1-α)948	α 953+(1-α)985	α16721+(1-α)16635	α16721+(1- α)16807
7	Vakıfbank	α 912+(1-α)907	α 912+(1-α)917	α15396+(1-α)15310	α15396+(1- α)15482
8	Denizbank	α 683+(1-α)678	α 683+(1-α)688	α12732+(1-α)12646	α 12732+(1-α)12818
9	Finansbank	α 276+(1-α)271	α 276+(1-α)281	α 2307+(1- α)2221	α 2307+(1- α)2393
10	TEB	α505+(1-α)500	α505+(1-α)510	α9440+(1-α)9354	α9440+(1-α)9526
11	HSBC Bank	α 77+(1-α)72	α 77+(1-α)82	α 2945+(1-α)28586	α 2945+(1-α)3031
12	ING Bank	α 258+(1-α)253	α 258+(1-α)263	α 5084+(1-α)4998	α 5084+(1-α)5170
13	Şekerbank	α263+(1-α)258	α263+(1-α)268	α3411+(1-α)3325	α3411+(1-α)3497
14	Anadolubank	A96+(1-α)91	A96+(1-α)101	α1584+(1-α)1498	α1584+(1-α)1670
15	ABank	α 43+(1-α)38	α 43+(1-α)48	α 728+(1-α)642	α 728+(1-α)814
16	Turkland Bank	α23+(1-α)18	α23+(1-α)28	α408+(1-α)322	α408+(1-α)494
17	Fibabank	α 63+(1-α)58	α 63+(1-α)68	α 1288+(1-α)1202	α 1288+(1-α)1374
18	Turkish Bank	α 3+(1-α)(-2)	α 3+(1-α)8	α 52+(1-α)(-34)	α 52+(1-α)138

Appendix-16. α-cuts bounds of output variables in case of decrease

DMUs		Total credit		Total income		Net profit	
		Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
1	Ziraat Bank	α 274608.2+ (1-α)273468.1	α 274608.2+ (1-α)275748.2	α 13373.0+ (1-α)13324.5	α 13373.0+ (1-α)13421.5	α 5527.1+ (1-α)5509.3	α 5527.1+ (1-α)5544.9
2	İş Bank	α 201051.1+ (1-α)19911.0	α 201051.1+ (1-α)202191.1	α 9686.5+ (1-α)9638.0	A9686.5+(1-α)9735.0	α 3911.6+ (1-α)3893.8	α 3911.6+ (1-α)3929.4
3	Akbank	α 237662.3+ (1- α)236522.2	α 237662.3+ (1- α)238802.3	α 8740+(1-α)8692.1	α8740+(1-α)8789.1	α2808.7+ (1- α)2790.9	α2808.7+ (1- α)2826.5
4	Garanti Bank	α 391208.5+ (1- α)390068.4	α 391208.5+ (1- α)392348.5	α 14582.1+ (1- α)14533.6	α 14582.1+ (1- α)14630.6	α 4629.5+ (1- α)4611.7	α 4629.5+ (1- α)4647.3
5	Yapı Kredi Bank	α 53003.2+ (1- α)51863.1	α 53003.2+ (1- α)54143.2	α 6022.1+ (1- α)5973.6	α 6022.1+ (1- α)6070.7	α 3126.0+ (1- α)3108.2	α 3126.0+ (1- α)3143.8
6	Halkbank	α 123678.5+ (1- α)122539.4	α123678.5+ (1- α)124819.5	α 7384.6+ (1- α)7336.1	α7384.6+ (1- α)7433.2	α 3126.3+ (1- α)3108.5	α3126.3+ (1- α)3144.1
7	Vakıfbank	α55398.4+ (1- α)54258.3	α55398.4+ (1- α)56538.4	α 938.7+ (1- α)890.2	α938.7+ (1- α)987.3	α1735.9+ (1- α)1718.1	α1735.9+ (1- α)1753.7
8	Denizbank	α 65488.1+ (1- α)64348.1	α 65488.1+ (1- α)66628.2	α 3902.4+ (1- α)3853.9	α3902.4+ (1- α)3950.9	α 1772.3+ (1- α)1754.5	α1772.3+ (1- α)1790.1
9	Finansbank	α28919.2+ (1- α)27779.0	α28919.2+ (1- α)30059.1	α 859.5+ (1- α)811.0	α859.5+ (1- α)908.1	α 484.5+ (1- α)466.7	α484.5+ (1- α)502.3
10	TEB	α 43107.3+ (1- α)41967.3	α43107.3+ (1- α)44247.4	α 2349.6+ (1- α)2301.1	α2349.6+ (1- α)2398.1	α 1149.1+ (1- α)1131.3	α1149.1+ (1- α)1166.9
11	HSBC Bank	α 9546.5+ (1- α)8406.4	α9546.5+(1-α)10686.5	α685.0+ (1- α)636.5	α685.0+ (1- α)733.6	α 142.8+ (1- α)125.0	α142.8+ (1- α)160.6
12	ING Bank	α37511.6+ (1- α)36371.5	α37511.6+ (1- α)38651.6	α253.2+ (1- α)204.7	α253.2+ (1- α)301.8	α459.6+ (1- α)441.8	α459.6+ (1- α)477.4
13	Şekerbank	α12948.1+ (1- α)11808.1	α12948.1+ (1- α)14088.2	α860.0+ (1- α)811.5	α860.0+ (1- α)908.6	α 477.7+ (1- α)459.9	α477.7+ (1- α)495.5
14	Anadolubank	α 9863.9+ (1- α)8723.8	α 9863.9+ (1- α)11003.9	α322.2+ (1- α)273.6	α322.2+ (1- α)370.7	α104.6+ (1- α)86.8	α104.6+ (1- α)122.4
15	ABank	α 4677.5+ (1- α)3537.4	α 4677.5+ (1- α)5817.5	α 157.3+ (1- α)108.7	α 157.3+ (1- α)205.8	α 39.9+ (1- α)22.1	α 39.9+ (1- α)57.7
16	Turkland Bank	α1350.3+ (1- α)210.2	α1350.3+ (1- α)2490.3	α 79.0+ (1- α)30.5	α 79.0+ (1- α)127.5	α88.7+ (1-α)70.9	α88.7+ (1-α)106.5
17	Fibabank	α 5177.4 + (1- α)4037.3	α 5177.4+ (1- α)6317.4	α239.9+ (1- α)191.4	α239.9+ (1- α)288.5	α 11.2+ (1-α)(-6.5)	α 11.2+ (1-α)29.0
18	Turkish Bank	α 56.0+ (1- α)(-1083.9)	α 56.0+ (1- α)1196.1	α 44.3+ (1- α)(-4.1)	α 44.3+ (1- α)92.8	α 121.5+ (1-α)103.7	α 121.5+ (1-α)139.3

Appendix-17. FCCR model program codes

```
% EaU
for Alfa = [0 0.5 0.75 1]
    N = 4; % DMU Number
    g = 1; % input Number
    c = 1;  % output Number
    d = g+c; % Variable Number
    % a_cut1
    A_a_cut1 = [11+ Alfa,14-2*Alfa];
    B_a_cut1 = [30,30];
    C_a_cut1 = [40,40];
    D_a_cut1 = [45+2*Alfa,55-3*Alfa];
    % a_cut2
    A_a_cut2 = [10,10];
    B_a_cut2 = [12+ Alfa,16-2*Alfa];
    C_a_cut2 = [11,11];
    D_a_cut2 = [12+3*Alfa,22-3*Alfa];

    F = [A_a_cut2;
         B_a_cut2;
         C_a_cut2;
         D_a_cut2];

    Aeq = [0 D_a_cut1(1)];
    beq = 1;
    % U-L
    A = [A_a_cut2(2) -A_a_cut1(1);
         B_a_cut2(2) -B_a_cut1(1);
         C_a_cut2(2) -C_a_cut1(1);
         D_a_cut2(2) -D_a_cut1(1);
         % L-U
         A_a_cut2(1) -A_a_cut1(2);
         B_a_cut2(1) -B_a_cut1(2);
         C_a_cut2(1) -C_a_cut1(2);
         D_a_cut2(1) -D_a_cut1(2)];

    b = [0;0;0;0;0;0;0;0];

    lb = zeros(1,2);

    % FCCR
    for j = 1:N
        f = [0 F(j,2)];
        FCCRSKOR(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR(j)=f*y1;
        for k = 1:d
            FCCRWEIGHT(j,k) = y1(k);
        end
    end
    EaU(:,Alf) = FCCRSKOR';
    Alf = Alf + 1;
end
```

Appendix-17. (cont) FCCR model program codes

```
Alf = 1;
% EaL
for Alfa = [0 0.5 0.75 1];
    N = 4; % DMU Number
    g = 1; % input Number
    c = 1;  % output Number
    d = g+c; % Variable Number
    % a_cut1
    A_a_cut1 = [11+ Alfa,14-2*Alfa];
    B_a_cut1 = [30,30];
    C_a_cut1 = [40,40];
    D_a_cut1 = [45+2*Alfa,55-3*Alfa];
    % a_cut2
    A_a_cut2 = [10,10];
    B_a_cut2 = [12+ Alfa,16-2*Alfa];
    C_a_cut2 = [11,11];
    D_a_cut2 = [12+3*Alfa,22-3*Alfa];

    F = [A_a_cut2;
         B_a_cut2;
         C_a_cut2;
         D_a_cut2];

    Aeq = [0 D_a_cut1(2)];
    beq = 1;
    % L-U
    A = [A_a_cut2(1) -A_a_cut1(2);
         B_a_cut2(1) -B_a_cut1(2);
         C_a_cut2(1) -C_a_cut1(2);
         D_a_cut2(1) -D_a_cut1(2);
         % U-L
         A_a_cut2(2) -A_a_cut1(1);
         B_a_cut2(2) -B_a_cut1(1);
         C_a_cut2(2) -C_a_cut1(1);
         D_a_cut2(2) -D_a_cut1(1)];

    b = [0;0;0;0;0;0;0;0];

    lb = zeros(1,2);

    % FCCR
    for j = 1:N
        f = [0 F(j,1)];
        FCCRSKOR(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR(j)=f*y1;
        for k = 1:d
            FCCRWEIGHT(j,k) = y1(k);
        end
    end
    EaL(:,Alf) = FCCRSKOR';
    Alf = Alf + 1;
end
```

Appendix-18. FBCC model program codes

```
% EaU
for Alfa = [0 0.5 0.75 1]
    N = 4; % DMU Number
    g = 1; % input Number
    c = 1;  % output Number
    d = g+c; % Variable Number
    % a_cut1
    A_a_cut1 = [11+ Alfa,14-2*Alfa];
    B_a_cut1 = [30,30];
    C_a_cut1 = [40,40];
    D_a_cut1 = [45+2*Alfa,55-3*Alfa];
    % a_cut2
    A_a_cut2 = [10,10];
    B_a_cut2 = [12+ Alfa,16-2*Alfa];
    C_a_cut2 = [11,11];
    D_a_cut2 = [12+3*Alfa,22-3*Alfa];

    F = [A_a_cut2;
         B_a_cut2;
         C_a_cut2;
         D_a_cut2];

    Aeq = [0 D_a_cut1(1) 0];
    beq = 1;
    % U-L
    A = [A_a_cut2(2) -A_a_cut1(1) -1;
         B_a_cut2(2) -B_a_cut1(1) -1;
         C_a_cut2(2) -C_a_cut1(1) -1;
         D_a_cut2(2) -D_a_cut1(1) -1;
    % L-U
         A_a_cut2(1) -A_a_cut1(2) -1;
         B_a_cut2(1) -B_a_cut1(2) -1;
         C_a_cut2(1) -C_a_cut1(2) -1;
         D_a_cut2(1) -D_a_cut1(2) -1];

    b = [0;0;0;0;0;0;0;0];

    lb = zeros(1,2);

    % FBCC
    for j = 1:N
        f = [0 F(j,2) 0];
        FBCCSKOR(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR(j)=f*y1;
        for k = 1:d
            FBCCWEIGHT(j,k) = y1(k);
        end
    end
    EaU(:,Alf) = FBCCSKOR';
    Alf = Alf + 1;
end
```

Appendix-18. (cont) FBCC model program codes

```
Alf = 1;
% EaL
for Alfa = [0 0.5 0.75 1];
    N = 4; % DMU Number
    g = 1; % input Number
    c = 1; % output Number
    d = g+c; % Variable Number
    % a_cut1
    A_a_cut1 = [11+ Alfa,14-2*Alfa];
    B_a_cut1 = [30,30];
    C_a_cut1 = [40,40];
    D_a_cut1 = [45+2*Alfa,55-3*Alfa];
    % a_cut2
    A_a_cut2 = [10,10];
    B_a_cut2 = [12+ Alfa,16-2*Alfa];
    C_a_cut2 = [11,11];
    D_a_cut2 = [12+3*Alfa,22-3*Alfa];

    Aeq = [0 D_a_cut1(2) 0];
    beq = 1;
    % L2U
    A = [A_a_cut2(1) -A_a_cut1(2) -1;
         B_a_cut2(1) -B_a_cut1(2) -1;
         C_a_cut2(1) -C_a_cut1(2) -1;
         D_a_cut2(1) -D_a_cut1(2) -1;
    % U2L
         A_a_cut2(2) -A_a_cut1(1) -1;
         B_a_cut2(2) -B_a_cut1(1) -1;
         C_a_cut2(2) -C_a_cut1(1) -1;
         D_a_cut2(2) -D_a_cut1(1) -1];

    b = [0;0;0;0;0;0;0;0];
    lb = zeros(1,2);

    % FBCC
    for j = 1:N
        f = [0 F(j,1) 0];
        FBCCSKOR(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR(j)=f*y1;
        for k = 1:d
            FBCCWEIGHT(j,k) = y1(k);
        end
    end
    EaL(:,Alf) = FBCCSKOR';
    Alf = Alf + 1;
end
```

Appendix-19. FCCR program codes for application DMUs of banks

```
Alfa = 0;
Alfa = 1;

Banks={'Ziraat Bank','is Bank','Akbank','Garanti Bank','Yapi
KrediBank','Halkbank','Vakifbank','Denizbank','Finansbank','TEB','HSBC
Bank','iNG Bank','sekerbank','Anadolubank','ABank','Eurobank
Tekfen','Tekstil Bankasi','Citibank','Turkland Bank','Fibabank','Turkish
Bank'};

Ziraat bank = 1;
İŞ bank = 2;
    N = 21; % DMU Number
    g = 3;  % input Number
    c = 2;  % output Number
    d = g+c; % Variable Number
for Alfa = [0 0.5 0.75 1]

inputData = [Alfa*1458+(1-Alfa)*1453.65        Alfa*1458+(1-
Alfa)*1462.35 Alfa*24374+(1- Alfa)*24296.14   Alfa*24374+(1-
Alfa)*24451.86
Alfa*1201+(1- Alfa)*1196.65       Alfa*1201+(1- Alfa)*1205.35
Alfa*24887+(1- Alfa)*24809.14   Alfa*24887+(1- Alfa)*24964.86
Alfa*927+(1-Alfa)*922.65          Alfa*927+(1-Alfa)*931.35
Alfa*15339+(1- Alfa)*15261.14   Alfa*15339+(1- Alfa)*15416.86
Alfa*914+(1-Alfa)*909.65          Alfa*914+(1-Alfa)*918.35
Alfa*16773+(1- Alfa)*16695.14   Alfa*16773+(1- Alfa)*16850.86
Alfa*907+(1-Alfa)*902.65          Alfa*907+(1-Alfa)*911.35
Alfa*14859+(1- Alfa)*14781.14   Alfa*14859+(1- Alfa)*14936.86
Alfa*771+(1-Alfa)*766.65          Alfa*771+(1-Alfa)*775.35
Alfa*13643+(1- Alfa)*13565.14   Alfa*13643+(1- Alfa)*13720.86
Alfa*680+(1-Alfa)*675.65          Alfa*680+(1-Alfa)*684.35
Alfa*12222+(1- Alfa)*12144.14   Alfa*12222+(1- Alfa)*12299.86
Alfa*588+(1-Alfa)*583.65          Alfa*588+(1-Alfa)*592.35
Alfa*9772+(1-Alfa)*9694.14       Alfa*9772+(1-Alfa)*9849.86
Alfa*522+(1-Alfa)*517.65          Alfa*522+(1-Alfa)*526.35
Alfa*10837+(1- Alfa)*10759.14   Alfa*10837+(1- Alfa)*10914.86
Alfa*507+(1-Alfa)*502.65          Alfa*507+(1-Alfa)*511.35
Alfa*9356+(1-Alfa)*9278.14        Alfa*9356+(1-Alfa)*9433.86
Alfa*330+(1-Alfa)*325.65          Alfa*330+(1-Alfa)*334.35
Alfa*6155+(1-Alfa)*6077.14        Alfa*6155+(1-Alfa)*6232.86
Alfa*322+(1-Alfa)*317.65          Alfa*322+(1-Alfa)*326.35
Alfa*5232+(1-Alfa)*5154.14        Alfa*5232+(1-Alfa)*5309.86
Alfa*272+(1-Alfa)*267.65          Alfa*272+(1-Alfa)*276.35
Alfa*3530+(1-Alfa)*3452.14        Alfa*3530+(1-Alfa)*3607.86
Alfa*88+(1-Alfa)*83.65            Alfa*88+(1-Alfa)*92.35
Alfa*1911+(1-Alfa)*1833.14       Alfa*1911+(1-Alfa)*1988.86
Alfa*63+(1-Alfa)*58.65            Alfa*63+(1-Alfa)*67.35
Alfa*1185+(1-Alfa)*1107.14       Alfa*1185+(1-Alfa)*1262.86
Alfa*59+(1-Alfa)*54.65           Alfa*59+(1-Alfa)*63.35
Alfa*954+(1-Alfa)*876.14         Alfa*954+(1-Alfa)*1031.86
Alfa*44+(1-Alfa)*39.65          Alfa*44+(1-Alfa)*48.35
Alfa*880+(1-Alfa)*802.14         Alfa*880+(1-Alfa)*957.86
Alfa*37+(1-Alfa)*32.65          Alfa*37+(1-Alfa)*41.35
Alfa*2233+(1-Alfa)*2155.14       Alfa*2233+(1-Alfa)*2310.86
Alfa*27+(1-Alfa)*22.65          Alfa*27+(1-Alfa)*31.35
Alfa*496+(1-Alfa)*418.14         Alfa*496+(1-Alfa)*573.86
Alfa*21+(1-Alfa)*16.65          Alfa*21+(1-Alfa)*25.35
```

Appendix-19. (cont) FCCR program codes for application DMUs of banks

```
Alfa*438+(1-Alfa)*360.14        Alfa*438+(1-Alfa)*515.86
Alfa*20+(1-Alfa)*15.65         Alfa*20+(1-Alfa)*24.35
Alfa*284+(1-Alfa)*206.14        Alfa*284+(1-Alfa)*361.86];

OutputData = [Alfa*71173.26+(1- Alfa)*70857.77  Alfa*71173.26+(1-
Alfa)*71488.75    Alfa*6209.94+(1- Alfa)*6181.87  Alfa*6209.94+(1-
Alfa)*6238.01  Alfa*2100.67+(1- Alfa)*2090.15  Alfa*2100.67+(1-
Alfa)*2111.2    Alfa*91620.64+(1- Alfa)*91305.15  Alfa*91620.64+(1-
Alfa)*91936.12    Alfa*8163.27+(1- Alfa)*8135.21  Alfa*8163.27+(1-
Alfa)*8191.34  Alfa*2667.49+(1- Alfa)*2656.96  Alfa*2667.49+(1-
Alfa)*2678.01    Alfa*70213.22+(1- Alfa)*69897.74  Alfa*70213.22+(1-
Alfa)*70528.71    Alfa*6083.46+(1- Alfa)*6055.39  Alfa*6083.46+(1-
Alfa)*6111.52  Alfa*2394.53+(1- Alfa)*2384     Alfa*2394.53+(1-
Alfa)*2405.05    Alfa*83532.93+(1- Alfa)*83217.44  Alfa*83532.93+(1-
Alfa)*83848.42    Alfa*7936.87+(1- Alfa)*7908.81  Alfa*7936.87+(1-
Alfa)*7964.94  Alfa*3070.58+(1- Alfa)*3060.05  Alfa*3070.58+(1-
Alfa)*3081.1    Alfa*67044.87+(1- Alfa)*66729.38  Alfa*67044.87+(1-
Alfa)*67360.36    Alfa*5821.77+(1- Alfa)*5793.71  Alfa*5821.77+(1-
Alfa)*5849.84  Alfa*1857.49+(1- Alfa)*1846.96  Alfa*1857.49+(1-
Alfa)*1868.01    Alfa*55949.45+(1- Alfa)*55633.96  Alfa*55949.45+(1-
Alfa)*56264.93    Alfa*5051.93+(1- Alfa)*5023.87  Alfa*5051.93+(1-
Alfa)*5080     Alfa*2045.13+(1- Alfa)*2034.61  Alfa*2045.13+(1-
Alfa)*2055.66    Alfa*57200.61+(1- Alfa)*56885.12  Alfa*57200.61+(1-
Alfa)*57516.1     Alfa*4422.16+(1- Alfa)*4394.09  Alfa*4422.16+(1-
Alfa)*4450.22  Alfa*1226.79+(1- Alfa)*1216.26  Alfa*1226.79+(1-
Alfa)*1237.31    Alfa*22196.37+(1- Alfa)*21880.89  Alfa*22196.37+(1-
Alfa)*22511.86    Alfa*2291.8+(1- Alfa)*2263.74   Alfa*2291.8+(1-
Alfa)*2319.87   Alfa*873.97+(1- Alfa)*863.45    Alfa*873.97+(1-
Alfa)*884.5
Alfa*29867.2+(1- Alfa)*29551.71   Alfa*29867.2+(1- Alfa)*30182.68
Alfa*3000+(1- Alfa)*2971.93     Alfa*3000+(1- Alfa)*3028.07
Alfa*848.11+(1- Alfa)*837.59    Alfa*848.11+(1- Alfa)*858.64
Alfa*25443.64+(1- Alfa)*25128.16  Alfa*25443.64+(1- Alfa)*25759.13
Alfa*1793.32+(1- Alfa)*1765.25  Alfa*1793.32+(1- Alfa)*1821.39
Alfa*206.68+(1- Alfa)*196.15    Alfa*206.68+(1- Alfa)*217.2
Alfa*13662.37+(1- Alfa)*13346.89  Alfa*13662.37+(1- Alfa)*13977.86
Alfa*1534.52+(1- Alfa)*1506.45  Alfa*1534.52+(1- Alfa)*1562.58
Alfa*240.79+(1- Alfa)*230.26    Alfa*240.79+(1- Alfa)*251.31
Alfa*15264.74+(1- Alfa)*14949.25  Alfa*15264.74+(1- Alfa)*15580.22
Alfa*1100.21+(1- Alfa)*1072.14  Alfa*1100.21+(1- Alfa)*1128.27
Alfa*79.01+(1- Alfa)*68.49      Alfa*79.01+(1- Alfa)*89.54
Alfa*8282.22+(1- Alfa)*7966.74    Alfa*8282.22+(1- Alfa)*8597.71
Alfa*876.92+(1- Alfa)*848.85    Alfa*876.92+(1- Alfa)*904.98
Alfa*118.04+(1- Alfa)*107.52    Alfa*118.04+(1- Alfa)*128.57
Alfa*3715.39+(1- Alfa)*3399.91    Alfa*3715.39+(1- Alfa)*4030.88
Alfa*335.91+(1- Alfa)*307.85    Alfa*335.91+(1- Alfa)*363.98
Alfa*85.23+(1- Alfa)*74.7       Alfa*85.23+(1- Alfa)*95.76
Alfa*4220.59+(1- Alfa)*3905.1     Alfa*4220.59+(1- Alfa)*4536.07
Alfa*288.19+(1- Alfa)*260.13    Alfa*288.19+(1- Alfa)*316.26
Alfa*28.26+(1- Alfa)*17.74      Alfa*28.26+(1- Alfa)*38.79
Alfa*2291.94+(1- Alfa)*1976.46    Alfa*2291.94+(1- Alfa)*2607.43
Alfa*209.61+(1- Alfa)*181.54    Alfa*209.61+(1- Alfa)*237.67
Alfa*37.37+(1- Alfa)*26.84      Alfa*37.37+(1- Alfa)*47.89
Alfa*2461.06+(1- Alfa)*2145.57    Alfa*2461.06+(1- Alfa)*2776.54
Alfa*147.24+(1- Alfa)*119.18    Alfa*147.24+(1- Alfa)*175.31
Alfa*22.02+(1- Alfa)*11.49      Alfa*22.02+(1- Alfa)*32.54
```

Appendix-19. (cont) FCCR program codes for application DMUs of banks

```
Alfa*2652.96+(1- Alfa)*2337.48    Alfa*2652.96+(1- Alfa)*2968.45
Alfa*421.7+(1- Alfa)*393.64     Alfa*421.7+(1- Alfa)*449.77
Alfa*5.51+(1-Alfa)*- 5.02       Alfa*5.51+(1- Alfa)*16.03
Alfa*1445.38+(1- Alfa)*1129.89    Alfa*1445.38+(1- Alfa)*1760.86
Alfa*105.83+(1- Alfa)*77.76     Alfa*105.83+(1- Alfa)*133.9
Alfa*3.56+(1-Alfa)*- 6.96       Alfa*3.56+(1- Alfa)*14.09
Alfa*2095.04+(1- Alfa)*1779.55    Alfa*2095.04+(1- Alfa)*2410.52
Alfa*91+(1- Alfa)*62.93         Alfa*91+(1- Alfa)*119.06
Alfa*5.19+(1-Alfa)*- 5.34       Alfa*5.19+(1- Alfa)*15.71
Alfa*256.57+(1- Alfa)*-58.91      Alfa*256.57+(1- Alfa)*572.06
Alfa*33.39+(1- Alfa)*5.32       Alfa*33.39+(1- Alfa)*61.46
Alfa*0.69+(1-Alfa)*- 9.83       Alfa*0.69+(1- Alfa)*11.22];

    input1 = inputData(:,1:2);
    input2 = inputData(:,3:4);
    Output1 = OutputData(:,1:2);
    Output2 = OutputData(:,3:4);
    Output3 = OutputData(:,5:6);

    % FCCR ( Lower)
    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1)-input1(j,2)-
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR1(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR1(j)=f*y1;
        for k = 1:d
            FCCRWEiGHT(j,k) = y1(k);
        end
    end
    EaU1(:,Alf) = FCCRSKOR1';
    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2)-input1(j,1)-
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR2(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR2(j)=f*y1;
        for k = 1:d
            FCCRWEiGHT(j,k) = y1(k);
        end
    end
    EaU2(:,Alf) = FCCRSKOR2';
    EaU = EaU1 + EaU2;
    Alf = Alf + 1;
End
```

Appendix-19. (cont) FCCR program codes for application DMUs of banks

```
    % FCCR (upper)
    for j = 1:N
        F = [Output1(j,2) Output2(j,2) Output3(j,2)];
        Aeq = [zeros(1,3) input1(j,1) input2(j,1)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2)-input1(j,1)-
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR(j)=f*y1;
        for k = 1:d
            FCCRWEiGHT(j,k) = y1(k);
        end
    end
    EaU(:,Alf) = FCCRSKOR';
    Alf = Alf + 1;
end
EaU = 0.5 + EaU;

    for j = 1:N
        F = [Output1(j,2) Output2(j,2) Output3(j,2)];
        Aeq = [zeros(1,3) input1(j,1) input2(j,1)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1)-input1(j,2)-
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR(j)=f*y1;
        for k = 1:d
            FCCRWEiGHT(j,k) = y1(k);
        end
    end
    EaU(:,Alf) = FCCRSKOR';
    Alf = Alf + 1;
end
```

Appendix-20. FBCC program codes for application DMUs of banks

```
    %BCC lower
    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1) -input1(j,2)-
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FBCCSKOR1(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR1(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU1(:,Alf) = FBCCSKOR1';

    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2) -input1(j,1)-
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FBCCSKOR2(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR2(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU2(:,Alf) = FBCCSKOR2';
    EaU = EaU1 + EaU2;
    Alf = Alf + 1;
end


    % FBCC upper
    for j = 1:N
        F = [Output1(j,2) Output2(j,2) Output3(j,2)];
        Aeq = [zeros(1,3) input1(j,1) input2(j,1)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2)-input1(j,1)-
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FBCCSKOR1(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR1(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        End
```

Appendix-20. (cont) FBCC program codes for application DMUs of banks

```
    end
    EaU1(:,Alf) = FBCCSKOR1';

    for j = 1:N
        F = [Output1(j,2) Output2(j,2) Output3(j,2)];
        Aeq = [zeros(1,3) input1(j,1) input2(j,1)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1)-input1(j,2)-
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR2(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR2(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU2(:,Alf) = FBCCSKOR2';
    EaU = EaU1 + EaU2;
    Alf = Alf + 1;
```

Appendix-21. FCCR program codes for simulated DMUs of banks

```
Alfa = 0;
Alfa = 1;

Banks = {'Ziraat Bank','is Bank','Akbank','Garanti Bank','Yapi Kredi
Bank','Halkbank','Vakifbank','Denizbank','Finansbank','TEB','HSBC
Bank','iNG Bank','sekerbank','Anadolubank','ABank','Turkland
Bank','Fibabank','Turkish Bank'};

Ziraat bank = 1;
İs bank = 2;

N = 18; % DMU Number
g = 3;  % input Number
c = 2;  % output Number
d = g+c; % Variable Number

for Alfa = [0 0.25 0.5 0.75 1]

inputData = [Alfa*1458+(1-Alfa)*1453.65     Alfa*1458+(1- Alfa)*1462.35
Alfa*24374+(1- Alfa)*24296.14   Alfa*24374+(1- Alfa)*24451.86
Alfa*1201+(1- Alfa)*1196.65    Alfa*1201+(1- Alfa)*1205.35 Alfa*24887+(1-
Alfa)*24809.14   Alfa*24887+(1- Alfa)*24964.86 Alfa*927+(1-Alfa)*922.65
Alfa*927+(1-Alfa)*931.35    Alfa*15339+(1- Alfa)*15261.14
Alfa*15339+(1- Alfa)*15416.86   Alfa*914+(1-Alfa)*909.65
Alfa*914+(1-Alfa)*918.35    Alfa*16773+(1- Alfa)*16695.14
Alfa*16773+(1- Alfa)*16850.86
Alfa*907+(1-Alfa)*902.65       Alfa*907+(1-Alfa)*911.35    Alfa*14859+(1-
Alfa)*14781.14   Alfa*14859+(1- Alfa)*14936.86   Alfa*771+(1-Alfa)*766.65
Alfa*771+(1-Alfa)*775.35    Alfa*13643+(1- Alfa)*13565.14
Alfa*13643+(1- Alfa)*13720.86    Alfa*680+(1-Alfa)*675.65
Alfa*680+(1-Alfa)*684.35    Alfa*12222+(1- Alfa)*12144.14
Alfa*12222+(1- Alfa)*12299.86
Alfa*588+(1-Alfa)*583.65       Alfa*588+(1-Alfa)*592.35    Alfa*9772+(1-
Alfa)*9694.14      Alfa*9772+(1-Alfa)*9849.86   Alfa*522+(1-Alfa)*517.65
Alfa*522+(1-Alfa)*526.35    Alfa*10837+(1- Alfa)*10759.14
Alfa*10837+(1- Alfa)*10914.86 Alfa*507+(1-Alfa)*502.65      Alfa*507+(1-
Alfa)*511.35    Alfa*9356+(1-Alfa)*9278.14      Alfa*9356+(1-
Alfa)*9433.86   Alfa*330+(1-Alfa)*325.65       Alfa*330+(1-Alfa)*334.35
Alfa*6155+(1-Alfa)*6077.14      Alfa*6155+(1-Alfa)*6232.86
Alfa*322+(1-Alfa)*317.65       Alfa*322+(1-Alfa)*326.35    Alfa*5232+(1-
Alfa)*5154.14      Alfa*5232+(1-Alfa)*5309.86   Alfa*272+(1-Alfa)*267.65
Alfa*272+(1-Alfa)*276.35    Alfa*3530+(1-Alfa)*3452.14      Alfa*3530+(1-
Alfa)*3607.86   Alfa*88+(1-Alfa)*83.65         Alfa*88+(1-Alfa)*92.35
Alfa*1911+(1-Alfa)*1833.14      Alfa*1911+(1-Alfa)*1988.86
Alfa*63+(1-Alfa)*58.65         Alfa*63+(1-Alfa)*67.35      Alfa*1185+(1-
Alfa)*1107.14      Alfa*1185+(1-Alfa)*1262.86    Alfa*59+(1-Alfa)*54.65
Alfa*59+(1-Alfa)*63.35      Alfa*954+(1-Alfa)*876.14        Alfa*954+(1-
Alfa)*1031.86    Alfa*44+(1-Alfa)*39.65         Alfa*44+(1-Alfa)*48.35
Alfa*880+(1-Alfa)*802.14        Alfa*880+(1-Alfa)*957.86
Alfa*37+(1-Alfa)*32.65         Alfa*37+(1-Alfa)*41.35      Alfa*2233+(1-
Alfa)*2155.14      Alfa*2233+(1-Alfa)*2310.86];

OutputData = [Alfa*71173.26+(1- Alfa)*70857.77  Alfa*71173.26+(1-
Alfa)*71488.75    Alfa*6209.94+(1- Alfa)*6181.87  Alfa*6209.94+(1-
Alfa)*6238.01  Alfa*2100.67+(1- Alfa)*2090.15  Alfa*2100.67+(1-
Alfa)*2111.2  Alfa*91620.64+(1- Alfa)*91305.15  Alfa*91620.64+(1-
Alfa)*91936.12    Alfa*8163.27+(1- Alfa)*8135.21  Alfa*8163.27+(1-
```

Appendix-21. (cont) FCCR program codes for simulated DMUs of banks

```
Alfa)*8191.34  Alfa*2667.49+(1- Alfa)*2656.96  Alfa*2667.49+(1-
Alfa)*2678.01   Alfa*70213.22+(1- Alfa)*69897.74  Alfa*70213.22+(1-
Alfa)*70528.71    Alfa*6083.46+(1- Alfa)*6055.39  Alfa*6083.46+(1-
Alfa)*6111.52  Alfa*2394.53+(1- Alfa)*2384     Alfa*2394.53+(1-
Alfa)*2405.05   Alfa*83532.93+(1- Alfa)*83217.44  Alfa*83532.93+(1-
Alfa)*83848.42    Alfa*7936.87+(1- Alfa)*7908.81  Alfa*7936.87+(1-
Alfa)*7964.94  Alfa*3070.58+(1- Alfa)*3060.05  Alfa*3070.58+(1-
Alfa)*3081.1   Alfa*67044.87+(1- Alfa)*66729.38  Alfa*67044.87+(1-
Alfa)*67360.36    Alfa*5821.77+(1- Alfa)*5793.71  Alfa*5821.77+(1-
Alfa)*5849.84  Alfa*1857.49+(1- Alfa)*1846.96  Alfa*1857.49+(1-
Alfa)*1868.01   Alfa*55949.45+(1- Alfa)*55633.96  Alfa*55949.45+(1-
Alfa)*56264.93    Alfa*5051.93+(1- Alfa)*5023.87  Alfa*5051.93+(1-
Alfa)*5080     Alfa*2045.13+(1- Alfa)*2034.61  Alfa*2045.13+(1-
Alfa)*2055.66   Alfa*57200.61+(1- Alfa)*56885.12  Alfa*57200.61+(1-
Alfa)*57516.1     Alfa*4422.16+(1- Alfa)*4394.09  Alfa*4422.16+(1-
Alfa)*4450.22  Alfa*1226.79+(1- Alfa)*1216.26  Alfa*1226.79+(1-
Alfa)*1237.31   Alfa*22196.37+(1- Alfa)*21880.89  Alfa*22196.37+(1-
Alfa)*22511.86    Alfa*2291.8+(1- Alfa)*2263.74   Alfa*2291.8+(1-
Alfa)*2319.87   Alfa*873.97+(1- Alfa)*863.45   Alfa*873.97+(1-
Alfa)*884.5
Alfa*29867.2+(1- Alfa)*29551.71   Alfa*29867.2+(1- Alfa)*30182.68
Alfa*3000+(1- Alfa)*2971.93     Alfa*3000+(1- Alfa)*3028.07
Alfa*848.11+(1- Alfa)*837.59    Alfa*848.11+(1- Alfa)*858.64
Alfa*25443.64+(1- Alfa)*25128.16  Alfa*25443.64+(1- Alfa)*25759.13
Alfa*1793.32+(1- Alfa)*1765.25  Alfa*1793.32+(1- Alfa)*1821.39
Alfa*206.68+(1- Alfa)*196.15    Alfa*206.68+(1- Alfa)*217.2
Alfa*13662.37+(1- Alfa)*13346.89  Alfa*13662.37+(1- Alfa)*13977.86
Alfa*1534.52+(1- Alfa)*1506.45  Alfa*1534.52+(1- Alfa)*1562.58
Alfa*240.79+(1- Alfa)*230.26    Alfa*240.79+(1- Alfa)*251.31
Alfa*15264.74+(1- Alfa)*14949.25  Alfa*15264.74+(1- Alfa)*15580.22
Alfa*1100.21+(1- Alfa)*1072.14  Alfa*1100.21+(1- Alfa)*1128.27
Alfa*79.01+(1- Alfa)*68.49      Alfa*79.01+(1- Alfa)*89.54
Alfa*8282.22+(1- Alfa)*7966.74    Alfa*8282.22+(1- Alfa)*8597.71
Alfa*876.92+(1- Alfa)*848.85    Alfa*876.92+(1- Alfa)*904.98
Alfa*118.04+(1- Alfa)*107.52    Alfa*118.04+(1- Alfa)*128.57
Alfa*3715.39+(1- Alfa)*3399.91    Alfa*3715.39+(1- Alfa)*4030.88
Alfa*335.91+(1- Alfa)*307.85    Alfa*335.91+(1- Alfa)*363.98
Alfa*85.23+(1- Alfa)*74.7       Alfa*85.23+(1- Alfa)*95.76
Alfa*4220.59+(1- Alfa)*3905.1     Alfa*4220.59+(1- Alfa)*4536.07
Alfa*288.19+(1- Alfa)*260.13    Alfa*288.19+(1- Alfa)*316.26
Alfa*28.26+(1- Alfa)*17.74      Alfa*28.26+(1- Alfa)*38.79
Alfa*2291.94+(1- Alfa)*1976.46    Alfa*2291.94+(1- Alfa)*2607.43
Alfa*209.61+(1- Alfa)*181.54    Alfa*209.61+(1- Alfa)*237.67
Alfa*37.37+(1- Alfa)*26.84      Alfa*37.37+(1- Alfa)*47.89
Alfa*2461.06+(1- Alfa)*2145.57    Alfa*2461.06+(1- Alfa)*2776.54
Alfa*147.24+(1- Alfa)*119.18    Alfa*147.24+(1- Alfa)*175.31
Alfa*22.02+(1- Alfa)*11.49      Alfa*22.02+(1- Alfa)*32.54
Alfa*2652.96+(1- Alfa)*2337.48    Alfa*2652.96+(1- Alfa)*2968.45
Alfa*421.7+(1- Alfa)*393.64     Alfa*421.7+(1- Alfa)*449.77
Alfa*5.51+(1-Alfa)*- 5.02       Alfa*5.51+(1- Alfa)*16.03];

    input1 = inputData(:,1:2);
    input2 = inputData(:,3:4);
    Output1 = OutputData(:,1:2);
    Output2 = OutputData(:,3:4);
    Output3 = OutputData(:,5:6);
```

Appendix-21. (cont) FCCR program codes for simulated DMUs of banks

```
    % FCCR Lower
    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1) -input1(j,2) -
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR1(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR1(j)=f*y1;
        for k = 1:d
            FCCRWEiGHT(j,k) = y1(k);
        end
    end
    EaU1(:,Alf) = FCCRSKOR1';

    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2) -input1(j,1) -
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR2(j) = 0;

        y1 = linprog(f,A,b,Aeq,beq,lb);
        FCCRSKOR2(j)=f*y1;
        for k = 1:d
            FCCRWEiGHT(j,k) = y1(k);
        end
    end
    EaU2(:,Alf) = FCCRSKOR2';
    EaU = EaU1 + EaU2;
    Alf = Alf + 1;
end

figure
bar(EaU)
grid on
ylabel(' Efficiency ')
title('\alpha_c_u_t , DMU')
legend('alfa = 0','alfa = 0.25','alfa = 0.5','alfa = 0.75','alfa = 1')
xticks([1:N])
xticklabels(Banks)
xtickangle(45)

figure
plot(EaU(:,1),'-*r')
hold on
plot(EaU(:,2),'-pg')
plot(EaU(:,3),'-sb')
```

Appendix-21. (cont) FCCR program codes for simulated DMUs of banks

```
plot(EaU(:,4),'-om')
plot(EaU(:,5),'-.k')

grid on
ylabel(' Efficiency ')
title('\alpha_c_u_t , DMU')
legend('alfa = 0','alfa = 0.25','alfa = 0.5','alfa = 0.75','alfa = 1')
xticks([1:N])
xticklabels(Banks)
xtickangle(45)


fprintf('Decreasing icin\n')
fprintf('---------------\n')
load Data2018Decreasing
TestDecreasing = Data2018Decreasing';
CevapDecreasing = net(TestDecreasing);
CevapDecreasing = abs(CevapDecreasing*Max);

TotalCreditDecreasing = CevapDecreasing(1,:)';
TotalIncomeDecreasing = CevapDecreasing(2,:)';
NetProfitDecreasing = CevapDecreasing(3,:)';

T =
table(TotalCreditDecreasing,TotalIncomeDecreasing,NetProfitDe
creasing,'RowNames',Name)
Decreasing =
[TotalCreditDecreasing,TotalIncomeDecreasing,NetProfitDecreas
ing];
xlswrite('Decreasing.xlsx',Decreasing)

fprintf('Increasing icin\n')
fprintf('---------------\n')
load Data2018Increasing
TestIncreasing = Data2018Increasing';
CevapIncreasing = net(TestIncreasing);
CevapIncreasing = abs(CevapIncreasing*Max);

TotalCreditIncreasing = CevapIncreasing(1,:)';
TotalIncomeIncreasing = CevapIncreasing(2,:)';
NetProfitIncreasing = CevapIncreasing(3,:)';

T =
table(TotalCreditIncreasing,TotalIncomeIncreasing,NetProfitIn
creasing,'RowNames',Name)
Increasing =
[TotalCreditIncreasing,TotalIncomeIncreasing,NetProfitIncreas
ing];
xlswrite('Increasing.xlsx',Increasing)
```

Appendix-22. FBCC program codes for simulated DMUs of banks

```
input1 = inputData(:,1:2);
input2 = inputData(:,3:4);
Output1 = OutputData(:,1:2);
Output2 = OutputData(:,3:4);
Output3 = OutputData(:,5:6);

    %BCC lower
    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1) -input1(j,2)-
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FBCCSKOR1(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR1(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU1(:,Alf) = FBCCSKOR1';

    for j = 1:N
        F = [Output1(j,1) Output2(j,1) Output3(j,1)];
        Aeq = [zeros(1,3) input1(j,2) input2(j,2)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2) -input1(j,1)-
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FBCCSKOR2(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR2(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU2(:,Alf) = FBCCSKOR2';
    EaU = EaU1 + EaU2;
    Alf = Alf + 1;
end



    % FBCC upper
    for j = 1:N
        F = [Output1(j,2) Output2(j,2) Output3(j,2)];
        Aeq = [zeros(1,3) input1(j,1) input2(j,1)];
        beq = ones(1,1);
        A = [Output1(j,2) Output2(j,2) Output3(j,2)-input1(j,1)-
input2(j,1)];
        b = zeros(1,1);
        lb = zeros(1,5);
```

Appendix-22. (cont) FBCC program codes for simulated DMUs of banks

```
        f = [F(2) F(2) F(2) F(2) F(2)];
        FBCCSKOR1(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR1(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU1(:,Alf) = FBCCSKOR1';

    for j = 1:N
        F = [Output1(j,2) Output2(j,2) Output3(j,2)];
        Aeq = [zeros(1,3) input1(j,1) input2(j,1)];
        beq = ones(1,1);
        A = [Output1(j,1) Output2(j,1) Output3(j,1)-input1(j,2)-
input2(j,2)];
        b = zeros(1,1);
        lb = zeros(1,5);
        f = [F(2) F(2) F(2) F(2) F(2)];
        FCCRSKOR2(j) = 0;
        y1 = linprog(f,A,b,Aeq,beq,lb);
        FBCCSKOR2(j)=f*y1;
        for k = 1:d
            FBCCWEiGHT(j,k) = y1(k);
        end
    end
    EaU2(:,Alf) = FBCCSKOR2';
    EaU = EaU1 + EaU2;
    Alf = Alf + 1;


fprintf('Decreasing icin\n')
fprintf('---------------\n')
load Data2018Decreasing
TestDecreasing = Data2018Decreasing';
CevapDecreasing = net(TestDecreasing);
CevapDecreasing = abs(CevapDecreasing*Max);

TotalCreditDecreasing = CevapDecreasing(1,:)';
TotalIncomeDecreasing = CevapDecreasing(2,:)';
NetProfitDecreasing = CevapDecreasing(3,:)';

T =
table(TotalCreditDecreasing,TotalIncomeDecreasing,NetProfitDe
creasing,'RowNames',Name)
Decreasing =
[TotalCreditDecreasing,TotalIncomeDecreasing,NetProfitDecreas
ing];
xlswrite('Decreasing.xlsx',Decreasing)

fprintf('Increasing icin\n')
fprintf('---------------\n')
```

Appendix-22. (cont) FBCC program codes for simulated DMUs of banks

```
load Data2018Increasing
TestIncreasing = Data2018Increasing';
CevapIncreasing = net(TestIncreasing);
CevapIncreasing = abs(CevapIncreasing*Max);

TotalCreditIncreasing = CevapIncreasing(1,:)';
TotalIncomeIncreasing = CevapIncreasing(2,:)';
NetProfitIncreasing = CevapIncreasing(3,:)';

T =
table(TotalCreditIncreasing,TotalIncomeIncreasing,NetProfitIn
creasing,'RowNames',Name)
Increasing =
[TotalCreditIncreasing,TotalIncomeIncreasing,NetProfitIncreas
ing];
xlswrite('Increasing.xlsx',Increasing)
```

Appendix-23. Feedforward neural network program codes to simulation DMUs of banks

```
Name = {'Ziraat Bank';'Is Bank';'Akbank';'Garanti Bank';'Yapi
KrediBank';'Halkbank';'Vakifbank';'Denizbank';'Finansbank';'T
EB';'HSBCBank';'INGBank';'Sekerbank';'Anadolubank';'ABank';'T
urkland Bank';'Fibabank';'Turkish Bank'};

load Data

DataInput2007 = Data2007(:,4:5);
DataInput2008 = Data2008(:,4:5);
DataInput2009 = Data2009(:,4:5);
DataInput2010 = Data2010(:,4:5);
DataInput2011 = Data2011(:,4:5);
DataInput2012 = Data2012(:,4:5);
DataInput2013 = Data2013(:,4:5);
DataInput2014 = Data2014(:,4:5);
DataInput2015 = Data2015(:,4:5);
DataInput2016 = Data2016(:,4:5);

DataOutput2007 = Data2007(:,1:3);
DataOutput2008 = Data2008(:,1:3);
DataOutput2009 = Data2009(:,1:3);
DataOutput2010 = Data2010(:,1:3);
DataOutput2011 = Data2011(:,1:3);
DataOutput2012 = Data2012(:,1:3);
DataOutput2013 = Data2013(:,1:3);
DataOutput2014 = Data2014(:,1:3);
DataOutput2015 = Data2015(:,1:3);
DataOutput2016 = Data2016(:,1:3);

DataInput = [DataInput2007',DataInput2008',DataInput2009',...
             DataInput2010',DataInput2011',DataInput2012',...
             DataInput2013',DataInput2014',DataInput2015',...
             DataInput2016'];

DataOutput =
[DataOutput2007',DataOutput2008',DataOutput2009',...

DataOutput2010',DataOutput2011',DataOutput2012',...

DataOutput2013',DataOutput2014',DataOutput2015',...
              DataOutput2016'];

DataNumber = size(DataInput,2);
TrainNumber = round(0.75*DataNumber);
[TrainALL TestALL] =
RandomizationData(DataNumber,TrainNumber);
InputTrain = DataInput(:,TrainALL);
TargetTrain = DataOutput(:,TrainALL);
```

Appendix-23. (cont) Feedforward neural network program codes to simulation DMUs of banks

```
InputTrain =
[InputTrain,InputTrain,InputTrain,InputTrain,InputTrain,Input
Train,InputTrain,InputTrain,InputTrain,InputTrain,InputTrain,
InputTrain];
TargetTrain =
[TargetTrain,TargetTrain,TargetTrain,TargetTrain,TargetTrain,
TargetTrain,TargetTrain,TargetTrain,TargetTrain,TargetTrain,T
argetTrain,TargetTrain];

Max = max(TargetTrain(:));
TargetTrain = TargetTrain/Max;

InputTest = DataInput(:,TestALL);
TargetTest = DataOutput(:,TestALL);

net = feedforwardnet([49 15 7]);
net =
train(net,InputTrain,TargetTrain,'useParallel','yes','showRes
ources','yes');
a = net(InputTest);
a = a*Max;
Yontem = 'MLP';

figure
subplot(311)
plot(TargetTest(1,:),'r')
hold on
plot(a(1,:),'b')
grid on
legend('Original',Yontem)
title('Total credit')

subplot(312)
plot(TargetTest(2,:),'r')
hold on
plot(a(2,:),'b')
grid on
legend('Original',Yontem)
title('Total income')

subplot(313)
plot(TargetTest(3,:),'r')
hold on
plot(a(3,:),'b')
grid on
legend('Original',Yontem)
title('Net profit')
```

CURRICULUM VITAE

Personal Information

Surname Name	: Jarjies, Oday Abdulrahman
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Date of birth, and place	: 1973, Iraq
Marital Status	: Married And I Have Three Children
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Education

Degree	School/ Program	Graduation Date
Ph.D	Gazi University / Statistics	Ongoing
Diploma	Ankara University TOMR / Turkish Language	2011
MS.c	Mosul University / Statistics	2001
Undergraduate	Mosul University / Statistics	1997
High School	Almustakbal High School	1991

Work Experience

Year	Place of Work	Position
2008-2010	Mosul University	Assistant Dean
2003 -2008	Mosul University	Director of Scientific Affairs

Foreign Language

English and Turkish

Native language

Arabic

Papers published and accepted for publication in refereed journals

Oday J., and Kubais S. F., (2001). Probability model for customers in closed queue networks. *Iraqi Journal of Statistical Sciences*, Iraq.

Oday J., (2002). Potential distribution of customers in queue networks with multiple levels and service channels. *Proceedings of the Thirteenth Scientific Conference of the Iraqi Society for Statistical Sciences*, Iraq.

Oday J., (2004). Use of network surveillance to estimate the prevalence of thalassemia. *Iraqi Journal of Statistical Sciences*, Iraq.

Oday J., (2008). Employment of transport models to solve problems aimed at maximizing the objective function. *The Second International Scientific Conference of Mathematics*, Syria.

Oday J., (2009). Event scheduling in simulation of intermittent accident systems with experimentation on queuing systems. *Iraqi Journal of Statistical Sciences*, Iraq.

Oday J., and İhsan A., (2013). Using the Data Envelopment Analysis (DEA) CCR model with constant returns to scale to measure the technical efficiency of oil refineries. *At the International Data Envelope Analysis Conference in the city of Samsun*, Turkey.

Oday J., Hulya B., and Kubra D., (2015). Using of Fractional Factorial Design (rk-p) in Data Envelopment Analysis of the selection of outputs and inputs, *Hacettepe Journal of Mathematics and Statistics*, 44 (3) , 679 – 687.

Oday J., and Hasan B., (2017). Interval Linear Programming And Fuzzy DEA-BCC Models With Ranking Of DMU New Approach, *Adıyaman University Journal of Science In Turkey*, 7(1), 60-88.

Oday J., and Hasan B., (2018). Parametric Linear Programming And Fuzzy DEA - Neural Approach Models With application on DMU banks. *Accepted for publication in the International Journal of Science and Research (IJSR) In India*, 7(5), 36-72.

Participation in scientific and non-scientific societies

1. Iraqi Society for Statistical Sciences Member 2002.
2. The Association of Iraqi Businessmen in Turkey Founder and Vice - Chairman of the Board of Directors 2016.
3. Association of Iraqi Community Affairs in Turkey Founder and Director of the Education Committee 2017.

Hobbies

Reading and swimming.

GAZİ GELECEKTİR..