IE306 Operations Research III Formula Sheet

1) $\boldsymbol{X}$, is a random variable which is suitable of the Poisson distribution with the $\boldsymbol{\lambda}$,

a) $P(X = t) = f(t) = \lambda e^{-\lambda t}$

b) $P(X > t) = 1 - F(t) = e^{-\lambda t}$

c) $E[X] = \frac{1}{\lambda}$

2) Exponential distribution has no-memory property.
$P(X > t + s | X > t) = P(X > s)$,

3) T is a random variable which is suitable of the Poisson distribution with the $\boldsymbol{\lambda}$, (Pure birth model)

n arrivals occur in that period is $p_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$

4) N *is total customers in queue* and μ is service frequency. The probability that n customers remain in queue in that period is (Pure death model) $p_n(t) = \frac{(\mu t)^{N-n} e^{-\mu t}}{(N-n)!}$

5) Generalized Poisson Queueing Model;

a) Steady-state probability of *n* customers in the system;

$$p_n = \left(\frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_1\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_2\mu_1}\right)p_0$$

b) There is no customer in the system,

$$p_0 = 1 - \sum_{n=1}^{\infty} p_n$$

6) The steady-state performance values;

a) $L_s = \sum_{n=1}^{\infty} np_n = \lambda_{eff} W_s$

b) $L_q = \sum_{n=(c+1)}^{\infty} (n-c)p_n = \lambda_{eff} W_q$

c) $W_s = W_q + \frac{1}{\mu}$

d) $L_s = L_q + \frac{\lambda_{eff}}{\mu}$

e) $\overline{c} = L_s - L_q = \frac{\lambda_{eff}}{\mu}$

f) $\rho = \frac{\lambda}{\mu}$

g) $\lambda = \lambda_{eff} + \lambda_{lost}$

7) (M/M/1):(GD/∞/∞) ;

a) $p_n = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$

b) $p_0 = 1 - \rho$

c) $L_s = \frac{\rho}{1-\rho}$

d) $W_s = \frac{L_s}{\lambda} = \frac{1}{\mu(1-\rho)}$

e) $W_q = W_s - \frac{1}{\mu} = \frac{\rho}{\mu(1-\rho)}$

f) $L_q = \lambda W_q = \frac{\rho^2}{(1-\rho)}$

g) $\overline{c} = L_s - L_q = \rho$

8) (M/M/1):(GD/N/∞) ;

a) $p_n = \begin{cases} \rho^n p_0 & n \le N \\ 0 & n > N \end{cases}$

b) $p_0 = \begin{cases} \frac{(1-\rho)\rho^n}{1-\rho^{N+1}} & \rho \ne 1 \\ \frac{1}{N+1} & \rho = 1 \end{cases}$

c) $\lambda_{lost} = \lambda p_N$

d) $\lambda_{eff} = \lambda(1 - p_{N)}$

e) $L_s = \begin{cases} \sum_{n=0}^{N} np_n = \frac{\rho\{1-(N+1)\rho^n + N\rho^{n+1}\}}{(1-\rho)(1-\rho^{N-1})} & \rho \ne 1 \\ \frac{N}{2} & \rho = 1 \end{cases}$

f) $W_s = \frac{L_s}{\lambda_{eff}}$

g) $W_q = W_s - \frac{1}{\mu}$

h) $L_q = \lambda_{eff} W_q$

i) $\overline{c} = L_s - L_q = \rho$

9) (M/M/c):(GD/∞/∞) ;

a) $p_n = \begin{cases} \frac{\lambda^n}{n!\mu^n} p_0 & n \le c \\ \frac{\lambda^n}{c!c^{n-c}\mu^n} p_0 & n > c \end{cases}$

b) $p_0 = \left\{\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!}\left(\frac{1}{1-\frac{\rho}{c}}\right)\right\}^{-1} \qquad \frac{\rho}{c} < 1$

c) $L_q = \sum_{n=c}^{\infty} (n-c)p_n = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} p_0$

d) $L_s = L_q + \rho$

e) $\lambda_{eff} = \lambda, \quad W_s = \frac{L_s}{\lambda}, \quad W_q = \frac{L_q}{\lambda}$

10) (M/M/c):(GD/N/∞) ; c≤N

a) $p_n = \begin{cases} \frac{\rho^n}{n!} p_0 & 0 \le n \le c \\ \frac{\rho^n}{c!c^{n-c}} p_0 & c < n \le N \end{cases}$

b) $p_0 = \begin{cases} \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c\left(1-\left(\frac{\rho}{c}\right)^{N-c+1}\right)}{c!\left(1-\frac{\rho}{c}\right)}\right]^{-1} & \frac{\rho}{c} \ne 1 \\ \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!}(N-c+1)\right]^{-1} & \frac{\rho}{c} = 1 \end{cases}$

c) $L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2}\left\{1 - \left(\frac{\rho}{c}\right)^{N-c+1} - (N-c+1)\left(1-\frac{\rho}{c}\right)\left(\frac{\rho}{c}\right)^{N-c}\right\} p_0$

d) $\frac{\rho}{c} = 1$ için $L_q = \frac{\rho^c (N-c)(N-c+1)}{2c!} p_0$

e) $\lambda_{lost} = \lambda p_N$

f) $\lambda_{eff} = \lambda - \lambda_{lost}, \quad W_s = \frac{L_s}{\lambda_{eff}}, \quad W_q = \frac{L_q}{\lambda_{eff}}$

11) (M/M/∞):(GD/∞/∞) – Self Service Model

a) $p_n = \frac{e^{-\rho}\rho^n}{n!} \qquad n = 0, 1, 2, \ldots$

b) $L_s = \rho, \quad L_q = W_q = 0$