AIP Conference Proceedings

Integral type contractions in partial metric spaces

Cite as: AIP Conference Proceedings **2086**, 030031 (2019); https://doi.org/10.1063/1.5095116
Published Online: 02 April 2019

Vildan Ozturk, and Duran Turkoglu

View Online Export Citation

ARTICLES YOU MAY BE INTERESTED IN

Some results for a certain class of holomorphic functions at the boundary of the unit disc
AIP Conference Proceedings **2086**, 030030 (2019); https://doi.org/10.1063/1.5095115

Normality and quotient of crossed modules within group with operations
AIP Conference Proceedings **2086**, 030032 (2019); https://doi.org/10.1063/1.5095117

Global convergence of conjugate gradient method in unconstrained optimization problems
AIP Conference Proceedings **2086**, 030029 (2019); https://doi.org/10.1063/1.5095114



AIP Publishing

© 2019 Author(s).

Integral Type Contractions In Partial Metric Spaces

Vildan Ozturk[1,a,b] and Duran Turkoglu[2,c]

[1]*Department Of Mathematics, Faculty Of Science And Arts, University Of Ankara Haci Bayram Veli, Ankara, Turkey. vildanozturk84@gmail.com*

[2]*Department Of Mathematics, Faculty Of Science, University Of Gazi, Ankara, Turkey. dturkoglu@gazi.edu.tr*

[a]Corresponding author: vildanozturk84@gmail.com
[b)]URL: vildanozturk84@gmail.com
[c)]dturkoglu@gazi.edu.tr

Abstract. It is known that history of mathematics is old as history of humanity. Mathematics covered a distance significantly from ancient age to now. Recently, there are many important works for modern mathematics([6],[8]).

Let X be a nonempty set and $f : X \to X$ be a mapping. If $f(x) = x$, for some $x \in X$, then x is fixed point of f. Banach fixed point theorem was introduced in 1922 in complete metric spaces as "(X, d) be a complete metric space and $f : X \to X$ be a self-mapping. If there exists $0 \leq k < 1$ such that $d(fx, fy) \leq kd(x, y)$ for all $x, y \in X$. Then f has unique fixed point"([1]).

Partial metric spaces were introduced by Matthews (1994) as a generalisation of usual metric spaces where the self distance for any point need not be equal to zero. In this work, we define generalized integral type $F-$contractions and prove common fixed point theorems for four mappings satisfying these types contractions in partial metric spaces.

Keywords: fixed point, F-contaction, integral type contraction, partial metric.
PACS Code: 02.40.Pc, 02.30.Sa.

INTRODUCTION

Let X be a nonempty set and $p : X \times X \to [0, \infty)$ satisfy

(PM1) $x = y \Leftrightarrow p(x, x) = p(y, y) = p(x, y)$,
(PM2) $p(x, x) \leq p(x, y)$,
(PM3) $p(x, y) = p(y, x)$,
(PM4) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$

for all x, y and $z \in X$, Then the pair (X, p) is called a partial metric space (in short PMS) and p is called a partial metric on X ([7]).

Let (X, p) be a PMS. Then, the functions $d_p, d_m : X \times X \to [0, \infty)$ given by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y), \tag{1}$$

$$d_m(x, y) = \max\{p(x, y) - p(x, x), p(x, y) - p(y, y)\} \tag{2}$$

are (usual) metrics on X. It is clear that d_p and d_m are equivalent ([7]).

Definition ([7])

(i) A sequence $\{x_n\}$ in a PMS (X, p) converges to $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

(ii) A sequence $\{x_n\}$ in a PMS (X, p) is called a Cauchy sequence if and only if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists (and finite).

(iii) A PMS (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

International Conference of Mathematical Sciences (ICMS 2018)
AIP Conf. Proc. 2086, 030031-1–030031-4; https://doi.org/10.1063/1.5095116
Published by AIP Publishing. 978-0-7354-1816-5/$30.00

(iv) A mapping $f : X \to X$ is said to be continuous at $x_0 \in X$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(B(x_0, \delta)) \subset B(f(x_0), \varepsilon)$.

Lemma ([7])

(i) A sequence $\{x_n\}$ is Cauchy in a PMS (X, p) if and only if $\{x_n\}$ is Cauchy in a metric space $\left(X, d_p\right)$.

(ii) A PMS (X, p) is complete if and only if the metric space $\left(X, d_p\right)$ is complete. Moreover,

$$\lim_{n\to\infty} d_p(x, x_n) = 0 \Leftrightarrow p(x, x) = \lim_{n\to\infty} p(x, x_n) = \lim_{n,m\to\infty} p(x_n, x_m) \tag{3}$$

where x is a limit of $\{x_n\}$ in $\left(X, d_p\right)$.

Remark ([5]) Let (X, p) be a PMS. Therefore,

(i) if $p(x,y) = 0$, then $x = y$;

(ii) if $x \neq y$, then $p(x,y) > 0$.

Lemma ([4]) Assume $x_n \to z$ as $n \to \infty$ in a PMS (X, p) such that $p(z,z) = 0$. Then $\lim_{n\to\infty} p(x_n, y) = p(z, y)$ for every $y \in X$.

In literature, there are many generalisations of Banach contraction principle in metric and generalized metric spaces. One of them is integral type contraction which was defined by Brianciari ([2]. On the other hand Wardowski introduced $F-$contraction in metric spaces as a generalization of Banach contraction principle ([9]). In this work, we will introduce generalized integral type $F-$contraction in partial metric spaces and prove common fixed point theorems.

Definition ([9]) Let a mapping $F : (0, \infty) \to \mathbb{R}$ satisfies:

F1. F is strictly increasing, i.e. for all $\alpha, \beta \in (0, \infty)$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$;

F2. For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0 \Leftrightarrow \lim_{n\to\infty} F(\alpha_n) = -\infty$;

F3. There exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

We denote by $\mathcal{F}$, the set of all functions satisfying the conditions (F1)-(F3).

Definition ([7]) A mapping $T : X \to X$ is said to be $F-$contraction if there exists $\tau > 0$ such that

$$\forall x, y \in X, (d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y))). \tag{4}$$

Theorem ([9]) Let (X, d) be a complete metric space and let $T : X \to X$ be an $F-$contraction. Then T has a unique fixed point in X.

Example ([9]) Let $F : \mathbb{R}^+ \to \mathbb{R}$ be given by $F(\alpha) = \ln \alpha$. F satisfies (F1), (F2), and (F3). Each mapping $T : X \to X$ is an F-contraction such that for all x, y in X and $Tx \neq Ty$,

$$d(Tx, Ty) \leq e^{-\tau} d(x, y) \tag{5}$$

It is clear that for $x, y \in X$ such that $Tx = Ty$ the inequality $d(Tx, Ty) \leq e^{-\tau} d(x, y)$ also holds i.e. T is a Banach contraction.

Definition ([3]) The mappings $f, g : X \to X$ are said to be weakly compatible if f and g commute at each coincidence point i.e. $fx = gx$ for some $x \in X$.

MAIN THEOREM

Theorem Let (X, p) be a complete partial metric space and f, g, S and T are four self mappings on X satisfying $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$. Suppose there exists $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $p(fx, gy) > 0$, the following holds:

$$\tau + F\left(\int_0^{p(fx,gy)} \varphi(t)dt\right) \leq F\left(\int_0^{M(x,y)} \varphi(t)dt\right) \tag{6}$$

where

$$M(x,y) = \max\left\{p(Sx, Ty), p(fx, Sx), p(gy, Ty), \frac{p(Sx, gy) + p(fx, Ty)}{2}\right\} \tag{7}$$

and $\varphi : [0, \infty) \to [0, \infty)$ is a Lebesque-integrable mapping which is summable on each compact subset of $[0, \infty)$ nonnegative and for each $\mu > 0$

$$\int_0^{\mu} \varphi(t)dt > 0, \tag{8}$$

If,

(i) $f(X), g(X), S(X), T(X)$ is closed,

(ii) F is continuous,

(iii) $\{f, S\}$ and $\{g, T\}$ are weakly compatible,

then f, g, S and T have a unique common fixed point.

Corollary Let (X, p) be a complete partial metric space and f and g are two self mappings on X. Suppose there exists $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $p(fx, gy) > 0$, the following holds:

$$\tau + F\left(\int_0^{p(fx,gy)} \varphi(t)dt\right) \leq F\left(\int_0^{M(x,y)} \varphi(t)dt\right) \tag{9}$$

where,

$$M(x,y) = \max\left\{p(x,y), p(x, fx), p(y, gy), \frac{p(y,fx))+\rho(x,gy)}{2}\right\}. \tag{10}$$

and $\varphi : [0, \infty) \to [0, \infty)$ is a Lebesque integrable mapping which is summable, nonnegative and for each $\mu > 0$

$$\int_0^{\mu} \varphi(t)dt > 0, \tag{11}$$

If,

i) $f(X)$ or $g(X)$ is closed,

ii) F is continuous,

then the pairs f and g have a unique common fixed point.

Corollary Let (X, p) be a complete partial metric space and f and g are two self mappings on X. Suppose there exists $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $p(fx, gy) > 0$, the following holds:

$$\tau + F\left(\int_0^{p(fx,gy)} \varphi(t)dt\right) \leq F\left(\int_0^{p(x,y)} \varphi(t)dt\right). \tag{12}$$

and $\varphi : [0, \infty) \to [0, \infty)$ is a Lebesque integrable mapping which is summable, nonnegative and for each $\mu > 0$

$$\int_0^{\mu} \varphi(t)dt > 0, \tag{13}$$

If,

(i) $f(X)$ or $g(X)$ is closed

(ii) F is continuous,

then the pairs f and g have a unique common fixed point.

ACKNOWLEDGMENTS

The authors would like to thank the referees for their careful reading and their varuable comments and suggestions to improve their results.

REFERENCES

[1] S. Banach, Sur les ope'rations dansles ensembles abstraits et leur application aux e'quationsint e'grales, Fundamenta Mathematicae, 3, 133-181, JFM48.0201.01 (1922)
[2] A. Branciari, A fixed point theorem formappings satisfying a general contractive condition of integral type, International Journal of Mathematics and Mathematical Sciences, 29(9), 531-536 (2002).
[3] G. Jungck, Compatible mappings and common fixed points, Int. J. Math. Math. Sci., 9, 771-779 (1986).
[4] E. Karapinar, Generalizations of Caristi Kirk's theorem on partial metric spaces, Fixed Point Theory and Applications, 2011, Article ID 4 (2011).
[5] E. Karapinar and U. Yuksel, Some common fixed point theorems in partial metric spaces, Journal of Applied Mathematics, 2011, Article ID 263621 (2011).
[6] V.J. Katz, A history of mathematics, An Introduction, Second Edition, Addison Wesley, USA (1998).
[7] S. G. Matthews, Partial Metric topology, in Proceedings of the 8th Summer Conference at Queens College,S. Andima, Ed., 728 of Papers on General Topology and Applications, Annals of the New YorkAcademy of Sciences, ,183–197 (1994).
[8] R. Ozturk, Eskicag'da aritmetik ve Eucleides (Oklid) algoritmasi, Tarih Okulu Dergisi, 11, XXXVI,676–694 (2018)
[9] D. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces. Fixed Point Theory Appl. 2012, Article ID 94 (2012).