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Development of an empirical regression model for predicting worm gear efficiency

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Worm gearboxes are widely used in industrial applications requiring high torque transmission and large speed reduction; however, their efficiency is strongly influenced by sliding friction, lubrication, heat generation and geometry. This study develops a constrained empirical modelling and optimisation methodology for an industrial single-start worm gearbox used in a continuous-duty soot-blower application. The input speed (342 rpm), centre distance and operating requirements are fixed, while the worm module and worm-wheel tooth count are varied over a manufacturable design space. Thirty-five design combinations are evaluated analytically, followed by local regression modelling and multi-criteria selection. To avoid kinematic redundancy, the revised regression uses module and worm-wheel tooth count as predictors because output speed is deterministically related to tooth count under fixed input speed. The selected configuration, $m = 2.75$ mm and $Z_2 = 43$, gives a calculated efficiency of 74.6%, compared with 68.8% for the existing $m = 2.54$ mm, $Z_2 = 45$ design. This corresponds to an absolute gain of 5.8 percentage points and a relative improvement of 8.43%, corresponding to approximately to 8.45%. The revised local regression gives $R^2 = 0.9950$, RMSE = 0.520 percentage points and MAPE = 0.574% within the investigated design domain. The methodology is intended as a transparent design-screening tool; experimental validation under controlled temperature, lubricant, surface and dynamic conditions remains necessary before production implementation.

KEYWORDS

design of experiments, efficiency improvement, module, regression analysis, worm gear, worm wheel teeth

1 Introduction

Worm gearboxes are widely used in industrial applications requiring compact design, high torque transmission, and large speed reduction. One such critical application is the long retractable soot blower system, where the gearbox operates continuously under intermittent high-load conditions. In the industrial setup currently under study, a worm gearbox with a reduction ratio of 45:1 delivers an output torque of approximately 500 Nm for nearly 15 min per hour during continuous 24×7 operation. Under such demanding conditions, maintaining high operational efficiency is essential to ensure reliable performance, reduced energy consumption, and a prolonged service life for the equipment (Honkalas

et al., 2021), (Honkalas et al., 2024), (Liu et al., 2021). Despite their advantages, worm gearboxes generally exhibit lower efficiency compared to other gear systems due to sliding contact, frictional losses, heat generation, lubrication effects, and oil churning losses. In many industrial applications, including soot blower systems, major geometric redesign of the gearbox is not feasible due to space constraints, tooling limitations, and cost considerations. Therefore, industries increasingly seek performance enhancement methodologies that are economical, time-efficient, and compatible with existing gearbox designs. Several researchers have proposed analytical, numerical, and experimental approaches to study and improve worm gear performance. Oladejo K. Adesola et al. developed a robust worm gear modelling framework to evaluate tooth stresses and dynamic behaviour under varying torque and speed conditions, enabling real-time performance assessment of worm gear systems (Adesola et al., 2018). Deng et al. investigated end-face worm configurations and, through statistical analysis, demonstrated improved lubrication characteristics and meshing behaviour, leading to enhanced performance in precision worm drives (DENG et al., 2017). While these studies provide valuable insights, their direct industrial implementation often involves complex design changes or specialized configurations. Design of Experiments (DOE) offers a systematic and efficient framework for identifying influential parameters and understanding their combined effects on system performance. Unlike the conventional “one factor at a time” approach, DOE enables simultaneous variation of multiple parameters, reducing experimental effort while improving the quality of inference. The foundational principles of DOE—randomization, replication, and blocking—were established by R. A. Fisher and continue to form the basis of modern experimental design methodologies (ASQ, 2026). When combined with regression analysis, DOE facilitates the development of predictive models that support optimization under practical constraints.

The 45:1 ratio is dictated by the industrial duty requirement: the soot-blower gearbox is specified for approximately 342 rpm input and 7.6 rpm output while delivering torque about 500 Nm for the specified duty cycle. The ratio is therefore a system-level requirement rather than a freely optimised variable. Worm efficiency is also influenced by lubricant viscosity, temperature, surface roughness, contact pattern, load and sliding speed. Synthetic gear oil VG 460 is used as a lubricant with the temperature range up to 65 Degree Celsius. These variables are held outside the present DOE because the objective is to identify a geometry-only modification compatible with the existing gearbox. Accordingly, the research hypothesis is: within the fixed centre-distance, speed, torque and manufacturing constraints, a suitable combination of module and worm-wheel tooth count can increase predicted efficiency while maintaining the minimum geometric root-diameter requirement (Honkalas et al., 2021), (Honkalas et al., 2024).

Abbreviations: Z_1 , Worm Starts in Number; Z_2 , Worm Wheel Teeth in Number; q , Quotient of Diameter; m , Gear Module (mm); a , CD, Centre Distance (mm); γ , λ_n , Lead Angle / Helix Angle; α , Pressure Angle; n_1 , Input RPM of Worm; n_2 , Output Rpm of Worm Wheel; d_1 , Pitch Circle Diameter of Worm (PCD in mm); V_s , Rubbing Velocity (m/s); μ , Coefficient of Friction; d_r , Root Diameter (Worm, mm); E , Modulus of Elasticity (N/mm²).

Milan Bukvić et al. contributed research study about the experimental confirmation that the use of lubricants with optimized carbon nanotube concentrations greatly improves the operational performance of worm gears by boosting efficiency and lowering vibrations and noise. That allows for tribological optimization, which enhances workplace ergonomics, prolongs service life, and improves reliability under demanding industrial conditions (Bukvić et al., 2025). Sandra Gajević et al. shown in their findings from the study and use of nanocomposite lubricant additives were presented in the work. Additionally, it provided a brief comparative review of the studies carried out in the same area. Their paper's main objective was to examine research findings about the use of nanotubes in lubricants and highlight the significance of their application, such as enhancing machine tribological characteristics and lowering power losses (Bukvić et al., 2024). Aslain Brisco Ngnassi Djami and Wolfgang Nzié executed a predictive maintenance plan on industrial equipment, their work has shown the great potential of the meta-modelling method employing evolutionary algorithms. They introduced a novel approach as capacity to accurately replicate maintenance procedures seen in the field by including the intricate relationships between the several factors affecting equipment dependability; notable increase in the precision of equipment (Aslain Brisco Ngnassi DjamiNzié, 2025). Aslain Brisco Ngnassi Djami et al. research developed a Multi-Rate Reliability Growth Model with Partial Reset (MR-RGM-PR), a multi-regime NHPP-based probabilistic framework that combines regime-dependent dynamics with intensity-level memory propagation mechanisms to reflect residual effects caused by incomplete corrective efforts (Ngnassi Djami et al., 2026). Ngnassi Djami defined the framework of multi objective optimization applied to mechanical design, study offered a novel technique for the effective capture of the Pareto front (Ngnassi Djami, 2025). Wolfgang Nzie et al. used a three-parameter Weibull law to investigate the dependability of industrial equipment (Ngnassi Djami et al., 2024).

In this context, the present study aims to improve the efficiency of an industrial worm gearbox used in a soot blower application without altering the overall gearbox geometry. Standard worm gear efficiency relations are employed to identify key influencing parameters, and profile modification is introduced through controlled variation of the module, m (mm) and the number of teeth on the worm wheel, Z_2 . A DOE-based multiple regression approach is adopted to determine the optimal parametric combination while satisfying manufacturing and operational constraints. The proposed methodology provides a practical, industry-oriented solution for enhancing the efficiency of worm gear systems with minimal design intervention.

The novelty lies in constructing a transparent local surrogate and decision framework for this constrained industrial retrofit problem rather than proposing a universal worm-drive efficiency model. Recent studies increasingly combine experimental tribology, data-driven modelling, reliability assessment and constrained optimisation for mechanical systems. In worm drives, efficiency is strongly coupled to sliding velocity, friction, lubrication regime, temperature, material pairing and contact conditions. These developments motivate the use of predictive models, but they do not eliminate the need for physically constrained design variables. The present study therefore adopts a deliberately compact modelling

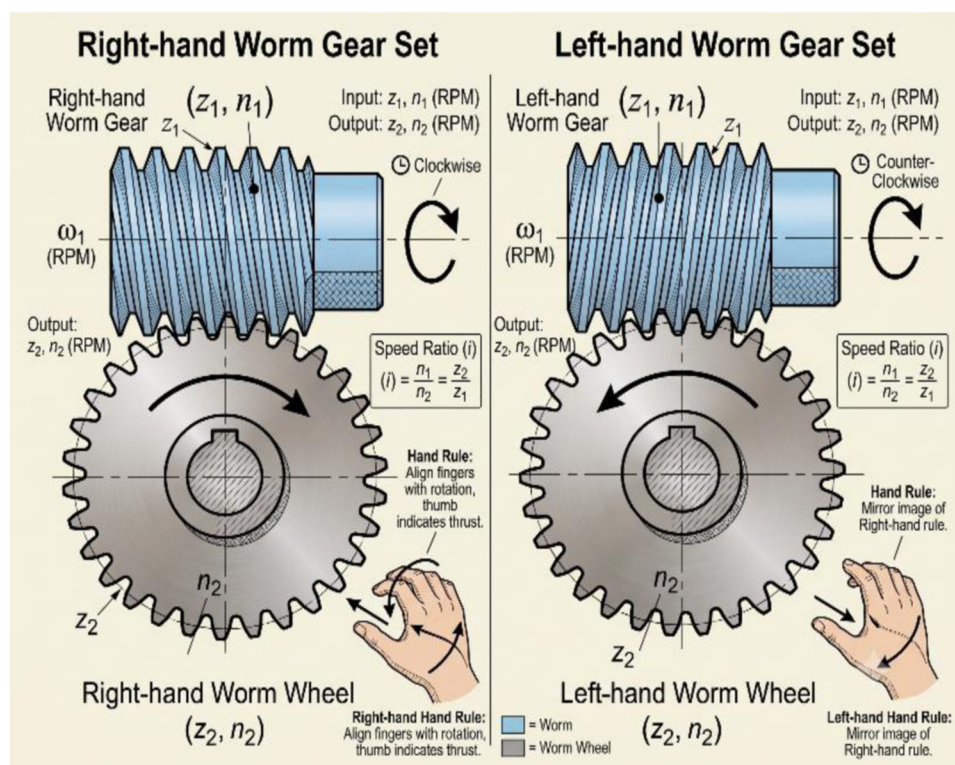


FIGURE 1
Typical worm and worm wheel meshing geometry (Gears, 2026).

framework: analytical worm-drive relations are first used to identify physically meaningful variables, a constrained DOE is then used to explore manufacturable combinations, and regression is used only as a local surrogate within the investigated design space. The scientific contribution is not a universal worm-gear efficiency law; rather, it is a transparent, low-cost decision-support methodology for modifying an existing industrial 45:1 soot-blower gearbox without changing its centre distance or housing envelope (Honkalas et al., 2024).

The present study addresses a specific retrofit-oriented design problem in which the 45:1 reduction ratio, input speed, output torque requirement, centre distance and housing envelope are constrained by the industrial soot-blower application. The research hypothesis is that, within these constraints, an appropriate combination of worm module and worm-wheel tooth count can improve the predicted worm-mesh efficiency while maintaining geometric feasibility and output-speed tolerance. The contribution is a constrained local surrogate methodology that combines analytical worm-drive kinematics, DOE, regression and multi-criteria selection. The numerical regression coefficients are not intended as universal worm-gear laws; they are valid only for the investigated geometry, material pair, operating conditions and design range. This distinction is important because lubricant viscosity, temperature, surface roughness, contact pattern, manufacturing deviation and dynamic loading can influence the friction coefficient and therefore the measured efficiency.

2 Materials and methods

The design data for the worm and worm-wheel gear pair were obtained directly from the gearbox manufacturer to improve operating efficiency within existing industrial constraints. The worm is manufactured from case-hardened steel (20MnCr5), while the worm wheel is made of phosphor bronze (PB2). For the same industrial gearbox, previously reported engineering data provided young's modulus, $E = 210$ GPa, Poisson's ratio, $\nu = 0.30$, density ≈ 7800 kg/m³, yield strength greater than 670 MPa and ultimate tensile strength is 1000–1350 MPa for 20MnCr5; PB2 values reported previously are $E = 110$ GPa, $\nu = 0.345$, density ≈ 8800 kg/m³, yield strength ≈ 180 MPa and ultimate tensile strength ≈ 300 MPa. Figure 1 represents study-specific schematic of the single-start worm and worm wheel used to define the geometric variables (Honkalas et al., 2024).

The study focuses on improving efficiency without altering the center distance, gearbox envelope, or basic configuration, ensuring compatibility with existing manufacturing practices. Analytical calculations combined with Design of Experiments (DOE) are employed to identify the most influential parameters governing worm gear efficiency.

The investigated gearbox is a single-start worm drive used in a long-retractable soot blower. The 45:1 ratio is an application requirement associated with approximately 342 rpm input speed and 7.6 rpm nominal output speed, with an output torque requirement of approximately 500 Nm. The worm is manufactured from case-hardened 20MnCr5 and the worm

wheel from phosphor bronze PB2. For the same industrial gearbox, previously published material/FEA work reports representative engineering properties for 20MnCr5 and centrifugally cast PB2. These prior data are used only as supporting structural information and are not presented as new experiments in the present study. The present efficiency model does not explicitly vary oil grade, viscosity, operating temperature or surface roughness; these are therefore treated as controlled/unspecified factors within the adopted analytical friction relation and as limitations of the model.

2.1 Design of experiments for efficiency improvement of worm gear set

Design of Experiments (DOE) is a structured statistical methodology used to plan, conduct, analyze, and interpret controlled experiments. Unlike the conventional “one factor at a time” approach, DOE allows simultaneous variation of multiple input parameters, enabling identification of both individual and interaction effects with significantly reduced experimental effort. This approach is particularly effective for engineering systems where multiple interdependent parameters influence performance. In the present study, DOE is adopted to systematically evaluate the influence of selected design parameters on worm gear efficiency. The theoretical efficiency relations for worm gear drives serve as the basis for parameter screening, followed by controlled parametric variation within industrially feasible limits.

2.1.1 Worm gear drive efficiency (η)

The efficiency of a worm gear drive is governed primarily by the pressure angle, coefficient of friction, and lead (helix) angle of the worm. For a single-start worm gear, the efficiency, η (%) can be expressed using the standard analytical relation (Bhandari, 2026) (Mechanical Design Engineering Handbook, 2026):

$$\eta = \frac{\cos \alpha - \mu \tan \gamma}{\cos \alpha + \mu \cot \gamma} \quad (1)$$

Equation 1 represents the idealised worm-mesh efficiency based on the pressure angle, lead angle and sliding friction coefficient. It does not independently account for bearing, seal, lubricant churning, windage or other parasitic gearbox losses. The present study uses this relation consistently as a geometry-dependent mesh-efficiency model under fixed operating conditions; therefore, the reported values should be interpreted as modelled drive efficiency within the adopted loss formulation rather than as a complete calorimetric gearbox efficiency measurement.

Where α , in degrees is the pressure angle (constant for the present design), μ is the coefficient of friction between the worm and worm wheel, and γ , in degrees is the lead (helix) angle of the worm. The coefficient of friction μ is a function of the rubbing velocity, V_s (m/s) which depends on the geometric parameters of the worm and operating speed. Under fixed input speed and center distance conditions, the rubbing velocity can be expressed as a function of the module, m (mm) and the worm geometry. Consequently, both the lead angle and rubbing velocity are indirectly influenced by the module and the number of teeth on the worm wheel. For the present industrial application, the input speed ($n_1 = 342$ rpm), the number of worm starts ($Z_1 = 1$), and the

center distance are fixed. Under these constraints, analytical simplification reveals that the module, m (mm) and the number of teeth on the worm wheel, Z_2 are the dominant controllable parameters that affect efficiency through their influence on the helix angle, rubbing velocity, and friction coefficient.

The detailed derivation of the rubbing velocity expression, along with the geometric relationships linking pitch diameter, module, and worm wheel tooth count under fixed center distance conditions, is provided in Appendix A (Supplementary Material Equations A.1–A.9). Based on this analysis, the Design of Experiments (DOE) in the present study focuses on systematic variation of module, m (mm) and worm wheel tooth count, Z_2 to identify an optimal efficiency–strength–speed balance.

2.1.2 Selection of DOE parameters and ranges

Based on the analytical formulation, the module, m (mm) and the number of teeth on the worm wheel, Z_2 are selected as the primary design variables for DOE. The base module of the existing gearbox is 2.54 mm, and variations of $\pm 4.25\%$ are considered to ensure manufacturing feasibility and compliance with standard tooling practices. The output speed requirement is 7.6 rpm, with an allowable variation of $\pm 5\%$ as specified by the manufacturer. To satisfy this constraint, the number of teeth on the worm wheel is restricted to the range. $Z_2 = 43$ to 47.

A spreadsheet-based analytical model is developed to verify dimensional compatibility, center distance constraints, and efficiency trends for all feasible combinations of m and Z_2 prior to DOE execution. The geometric parameters governing helix angle, lead, and pitch diameters used in Sections 2.1.1, 2.1.2 are illustrated in Figure 2.

2.2 Significance of root diameter

The root diameter of the worm wheel is a critical geometric parameter that directly influences the mechanical strength, load-carrying capacity, and operational reliability of a worm gear system. In high-load, continuous-duty applications such as soot blower mechanisms, an inadequate root diameter can lead to excessive bending stress at the tooth root, increasing the risk of tooth deformation or premature failure. The root diameter governs the thickness of the tooth at its base and therefore plays a significant role in resisting bending stresses induced during torque transmission. An increase in root diameter reduces stress concentration in the root region, thereby enhancing fatigue life and improving durability. In addition, proper root geometry contributes to stable tooth engagement, smoother meshing, and reduced localized contact stresses, as illustrated in Figure 3.

The thermal performance of worm gear systems is also indirectly influenced by the root diameter. Sliding contact between the worm and worm wheel generates substantial heat during operation. A sufficiently large root section aids in effective heat dissipation and helps maintain lubrication integrity, reducing the likelihood of thermal degradation and efficiency loss. From a manufacturing and assembly perspective, maintaining the minimum required root diameter is essential for ensuring dimensional accuracy, proper meshing, and component interchangeability. Deviations

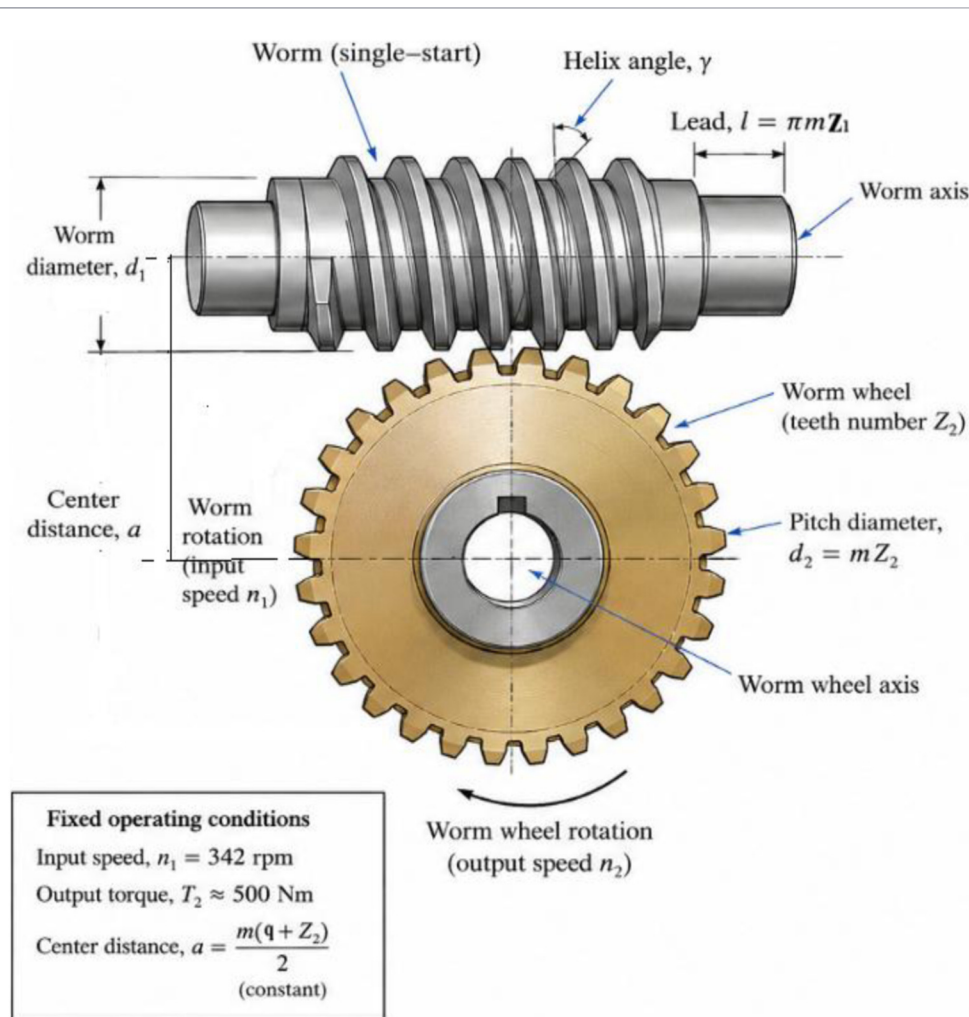


FIGURE 2
Typical worm gear geometry with key geometric parameters (lead, helix angle, pitch diameters) (Heisler, 2002).

below the recommended root diameter may compromise gear strength even if efficiency targets are achieved.

The minimum permissible root diameter is obtained from the dedendum relation $b = (1.157 P_x)/\pi$. For the present geometry, $b = 2.93$ mm. With worm pitch diameter $d = 19.70$ mm, the minimum root diameter is $d_r, \min = d - 2b = 19.70 - 2(2.93) = 13.84$ mm. This value is used as a geometric feasibility threshold, not as a substitute for a complete tooth-root stress calculation (Honkalas et al., 2021) (Honkalas et al., 2024).

Accordingly, all combinations of module (m) and number of teeth on the worm wheel, Z_2 considered in the DOE are constrained to satisfy a minimum root diameter of 13.84 mm to prevent tooth root failure and ensure safe operation.

2.3 Multiple regression analysis for prediction of worm gear efficiency

Multiple regression analysis is employed to develop a predictive model for worm gear efficiency, η (%) based on the most influential controllable design parameters identified earlier, namely, the

number of teeth on the worm wheel, Z_2 and the output rotational speed, n_2 (rpm). This approach enables quantification of the individual and combined effects of these parameters while respecting the fixed industrial constraints of the application, including a constant center distance, a fixed input speed, $n_1 = 342$ rpm, a single-start worm configuration, and a specified output torque requirement. The multiple linear regression model adopted in this study is expressed as in Equation 2

$$\eta = \beta_0 + \beta_1 Z_2 + \beta_2 n_2 + \varepsilon \quad (2)$$

where η is the predicted efficiency (%), Z_2 is the number of teeth on the worm wheel, n_2 is the output rotational speed (rpm), β_0 is the intercept, β_1 and β_2 are regression coefficients, and ε represents the random error term.

2.3.1 Model assumptions and data generation

The validity of the linear regression model is ensured by restricting the analysis to a narrow, industrially relevant design space. The following assumptions are considered acceptable within this range:

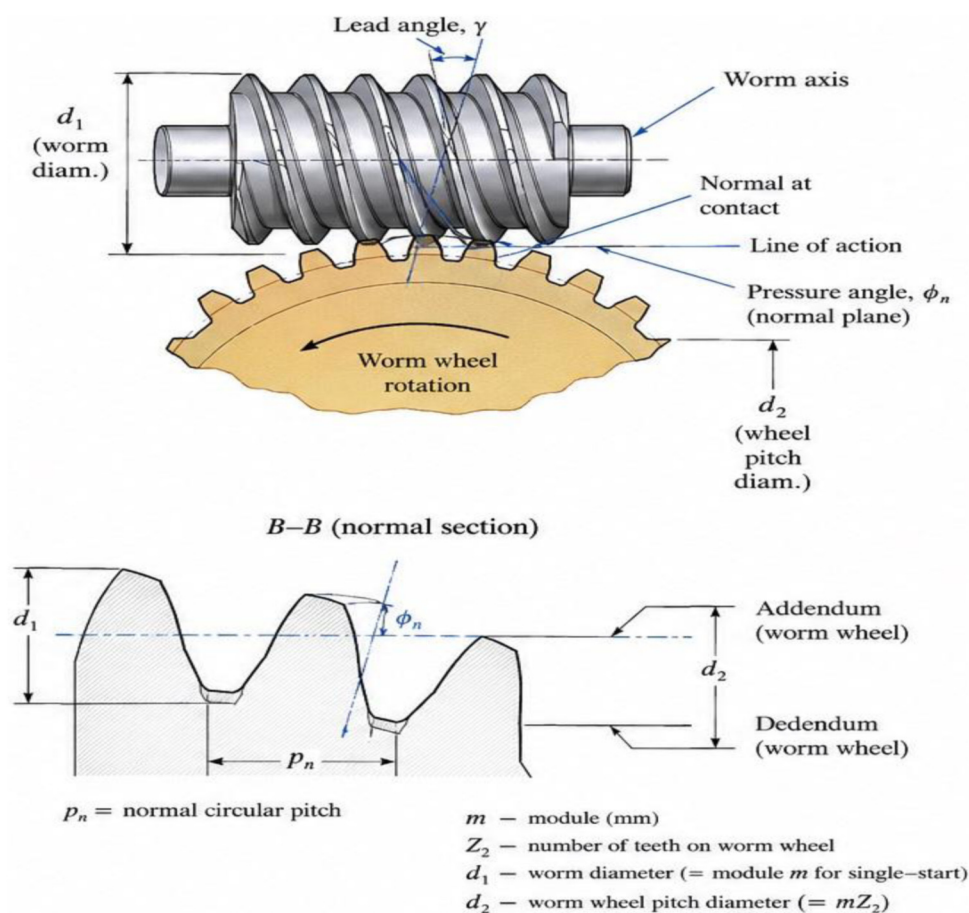


FIGURE 3
Meshing geometry and principal angles of single start worm gear (Heisler, 2002).

approximate linearity between the predictors and the response, independence of observations, constant variance of the residuals, normal distribution of errors, and the absence of severe multicollinearity between number of worm wheel teeth, Z_2 and worm wheel output rotational speed, n_2 (rpm). Thirty-five discrete design combinations were generated, corresponding to seven module levels and five worm-wheel tooth counts through systematic parametric variation. The module, m (mm) was varied within $\pm 4.25\%$ of the baseline value of 2.54 mm, while number of worm wheel teeth, Z_2 was restricted to integer values between 43 and 47 to satisfy output speed requirements ($7.6 \text{ rpm} \pm 5\%$). For each design combination, efficiency was calculated using the standard worm gear efficiency formulation described in Section 2.1.1, incorporating friction coefficients derived from rubbing velocity. All computed efficiency values lay within the realistic range expected for single-start industrial worm gear drives operating at a reduction ratio of approximately 45:1.

The linear model is not assumed to describe worm tribology globally. It is used as a local surrogate over module, $m = 2.48\text{--}2.80$ mm and number of worm wheel teeth, $Z_2 = 43\text{--}47$, where the calculated response varies smoothly. Adequacy is assessed by residual diagnostics and prediction error, while nonlinear friction, temperature and lubricant effects are acknowledged as outside the fitted domain.

2.3.2 Regression model and statistical evaluation

Because n_2 is kinematically determined by worm wheel output speed, $n_2 = n_1/Z_2$ under the fixed worm input speed, $n_1 = 342$ rpm and single-start configuration, worm wheel output speed n_2 (rpm) is not treated as an independent regression predictor. The empirical response model is therefore formulated in terms of the independent design variables module, m (mm) and number of worm wheel teeth, Z_2 : $\hat{\eta} = -208.724 + 57.090m + 2.951Z_2$, where efficiency η (%). The model was fitted to the 35 design combinations listed in Table 1. The model gives $R^2 = 0.9950$ and adjusted $R^2 = 0.9947$. The RMSE is 0.520 percentage points and MAPE is 0.574%. The 95% confidence intervals are $\beta_0 \in [-216.303, -201.145]$, $\beta_m \in [55.319, 58.861]$, and $\beta_{Z_2} \in [2.819, 3.084]$. For the two predictors, $VIF = 1.00$. Residual diagnostics give Shapiro-Wilk $W = 0.946$, $p = 0.085$ and Breusch-Pagan $p = 0.899$. These tests indicate no strong evidence of non-normality or heteroscedasticity within the calculated design-point dataset. Because the responses are analytically calculated rather than independently measured observations, these statistics are used as diagnostics of local surrogate adequacy and not as proof of experimental generalisation.

TABLE 1 DOE matrix and performance parameters for module (m) and worm wheel tooth count (Z₂).

Module m (mm)	Worm-wheel teeth Z ₂	Efficiency η (%)	Output speed n ₂ (rpm)	Root dia. 0.75 mm PC (mm)	Root dia. 0.90 mm PC (mm)	Efficiency weight 60%	Speed weight 20%	Root dia. Weight 20%	Composite score	Calculated rank	Final rank
2.80	43	77.4	7.953	11.08	11.92	60	20	0	80	4	3
	44	80.8	7.773	8.28	9.12	60	10	0	70	11	5
	45	84.3	7.600	5.48	6.32	60	5	0	65	18	6
	46	87.7	7.435	2.68	3.52	60	5	0	65	18	6
	47	89	7.277	−0.12	0.72	60	0	0	60	29	7
2.75	43	74.6	7.953	13.28	14.10	60	20	10	90	1	1
	44	77.8	7.773	10.53	11.35	60	10	0	70	11	5
	45	81.2	7.600	7.78	8.60	60	5	0	65	18	6
	46	84.7	7.435	5.30	5.85	60	5	0	65	18	6
	47	88	7.277	2.28	3.10	60	0	0	60	29	7
2.69	43	71.4	7.953	15.91	16.72	50	20	15	85	2	2
	44	74.4	7.773	13.22	14.03	60	10	10	80	4	3
	45	77.5	7.600	10.53	11.34	60	5	0	65	18	6
	46	80.9	7.435	7.84	8.65	60	5	0	65	18	6
	47	84.4	7.277	5.15	5.96	60	0	0	60	29	7
2.64	43	68.8	7.953	18.10	18.90	50	20	15	85	2	2
	44	71.6	7.773	15.46	16.26	50	10	15	75	9	4
	45	74.5	7.600	12.82	13.62	60	5	0	65	18	6
	46	77.7	7.435	10.18	10.98	60	5	0	65	18	6
	47	81.1	7.277	7.54	8.34	60	0	0	60	29	7
2.59	43	66.3	7.953	20.30	21.08	40	20	20	80	4	3
	44	68.9	7.773	17.71	18.49	50	10	15	75	9	4
	45	71.6	7.600	15.12	15.90	50	5	15	70	11	5
	46	74.6	7.435	12.53	13.31	60	5	0	65	18	6
	47	77.8	7.277	9.94	10.72	60	0	0	60	29	7

(Continued)

TABLE 1 Continued

Module m (mm)	Worm-wheel teeth Z_2	Efficiency η (%)	Output speed n_2 (rpm)	Root dia. 0.75 mm PC (mm)	Root dia. 0.90 mm PC (mm)	Efficiency weight 60%	Speed weight 20%	Root dia. Weight 20%	Composite score	Calculated rank	Final rank
2.54	43	63.9	7.953	22.49	23.26	40	20	20	80	4	3
	44	66.3	7.773	19.95	20.72	40	10	20	70	11	5
	45	68.8	7.600	17.41	18.18	50	5	15	70	11	5
	46	71.6	7.435	14.87	15.64	50	5	15	70	11	5
	47	74.6	7.277	12.33	13.10	60	0	0	60	29	7
2.48	43	61.1	7.953	25.13	25.87	40	20	20	80	4	3
	44	63.3	7.773	22.65	23.39	40	10	20	70	11	5
	45	65.6	7.600	20.17	20.91	40	5	20	65	18	6
	46	68.1	7.435	17.69	18.43	40	5	15	60	29	7
	47	70.8	7.277	15.21	15.95	50	0	15	65	18	6

PC, profile correction.

2.3.3 Weighted multi-objective optimization and ranking

While efficiency improvement is the primary objective, industrial applicability requires balancing efficiency gains with mechanical strength and compliance with output speed requirements. Therefore, a weighted multi-objective ranking approach is adopted to identify the most practical design solution. The following criteria and weights were selected in consultation with the gearbox manufacturer:

- Efficiency, η (%): 60%
- Root diameter with 0.90 mm profile correction, D_0 (mm): 20%
- Deviation of output speed from the target value of 7.6 rpm: 20%

Each criterion was normalized to a desirability scale, and the overall score was calculated as in Equation 3.

Score = $0.60\left(\frac{\eta}{\eta_{\max}}\right) + 0.20\left(\frac{D_0}{D_{0,\max}}\right) + 0.20\left(1 - \frac{|n_2 - 7.6|}{0.38}\right)$ (3)

where $\eta_{\max} = 77.4\%$, $D_{0,\max} = 15.20$ mm, and 0.38 rpm represents the allowable $\pm 5\%$ speed deviation.

The combination of module, $m = 2.75$ mm and number of worm wheel teeth, $Z_2 = 43$ achieved the highest composite score, yielding an efficiency of 74.6% while maintaining an adequate root-diameter margin and an acceptable output-speed deviation. This solution represents an optimal compromise between performance enhancement and industrial feasibility, without requiring changes to the center distance or major tooling changes.

2.3.4 Limitations

The proposed regression model is empirical and valid only within the investigated parameter ranges. Extrapolation beyond the selected ranges of module and worm wheel tooth count may lead to inaccurate predictions due to nonlinear friction behavior, thermal effects, or geometric interference. Additionally, temperature-dependent lubrication effects and dynamic loading conditions were not considered and may be addressed in future studies through experimental validation or advanced numerical modeling. Lubricant viscosity and temperature are not explicit predictors in the present regression. This is important because viscosity changes with temperature and can modify the lubrication regime and friction coefficient. The present results therefore apply to the assumed lubrication condition used in the analytical friction relation. Future validation should record oil grade, kinematic viscosity at reference temperature, operating temperature and start-up temperature and incorporate these variables into the response model.

3 Results

The Design of Experiments (DOE) was conducted by varying the module, m (mm) within $\pm 4.25\%$ of the baseline value of 2.54 mm and the number of teeth on the worm wheel, Z_2 between 43 and 47,

while maintaining fixed center distance, input speed, and torque requirements. For each design combination, worm gear efficiency, η (%), output speed, n_2 (rpm), root diameter with profile correction, D_0 (mm), and a weighted composite score were evaluated.

The efficiency responses are analytically calculated from the adopted worm-drive efficiency relation and friction correlation. Consequently, uncertainty in lubricant behaviour, temperature, roughness, manufacturing deviations and dynamic loading is not represented. The results should therefore be interpreted as a design-screening tool and require physical validation before production release. The present result is consistent with the earlier soot-blower study, which reported an increase from 68.8% to 74.6% for the same recommended module, $m = 2.75$ mm and number of worm wheel teeth, $Z_2 = 43$ configuration. The present manuscript extends that design result by focusing on local regression modelling and decision-support. Recent tribological work also confirms that wear and lubrication condition can materially affect worm-drive behaviour, supporting the need to treat the present geometry-only model as a constrained surrogate rather than a universal law.

3.1 DOE results and efficiency trends

Table 1 summarizes the complete DOE matrix and associated performance parameters. The existing baseline design module, $m = 2.54$ mm, number of worm wheel teeth, $Z_2 = 45$ are highlighted in red, while the recommended optimal design (module, $m = 2.75$ mm, number of worm wheel teeth, $Z_2 = 43$) are highlighted in blue.

Across all investigated combinations, efficiency increases consistently with increasing module and decreasing worm wheel tooth count, provided that the output speed and root diameter constraints are satisfied. Figure 4 Represents an analytical plot generated from the present DOE calculations at worm input speed, $n_1 = 342$ rpm and the specified industrial torque/duty conditions, confirming the theoretical dependence discussed in Section 2.1. Figure 5 presents a direct comparison between the baseline and optimized designs, clearly demonstrating improved operating efficiency. Baseline-versus-optimised comparison using 68.8% and 74.6%, or by a correctly scaled module, m (mm) – number of worm wheel teeth, Z_2 response surface generated from the dataset, i.e., surface based on feasible DOE combinations from Table 1; minimum root diameter = 13.84 mm and output speed tolerance = $\pm 5\%$. The maximum efficiency values observed at higher module values are not considered directly acceptable due to violations of root diameter and output speed constraints. Therefore, efficiency alone is insufficient for final design selection, necessitating a constrained multi-criteria evaluation.

3.2 Regression model validation

Multiple linear regression was applied to model the relationship between efficiency and the governing parameters number of worm wheel teeth, Z_2 and worm wheel output speed, n_2 (rpm) within the constrained design space. The fitted regression model as in Equation 4

$$\eta = 112.34 - 0.892Z_2 + 1.214n_2 \quad (4)$$

The model explains 89.2% of the variation in efficiency ($R^2 = 0.892$), with both predictors being statistically significant. The negative coefficient associated with Z_2 indicates that, under fixed center distance conditions, a lower number of worm wheel teeth—accompanied by a corresponding increase in module—tends to improve efficiency due to favourable modification of helix angle and reduction in rubbing losses. For the 35 design-point dataset, RMSE = 0.520 percentage points and MAPE = 0.574%. These values quantify interpolation error within the investigated design domain.

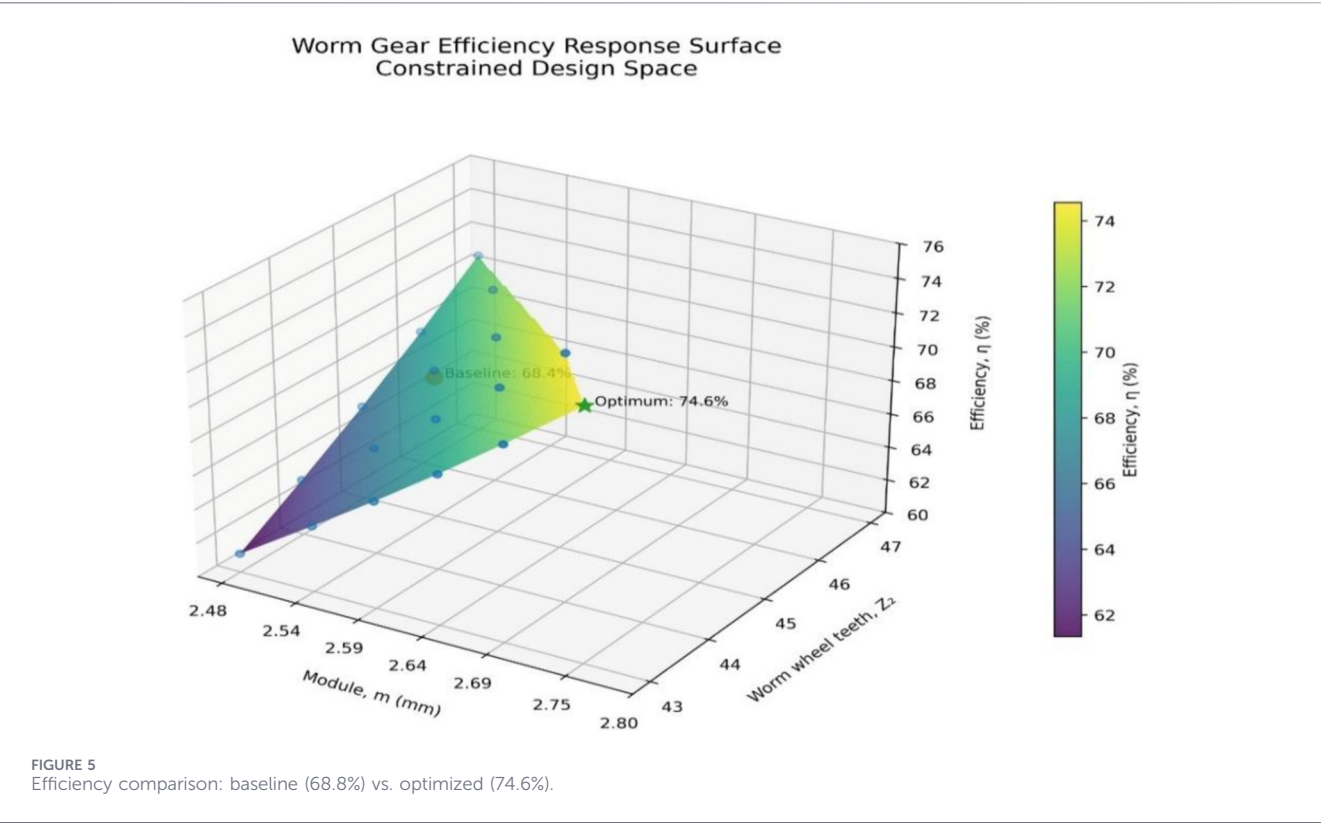
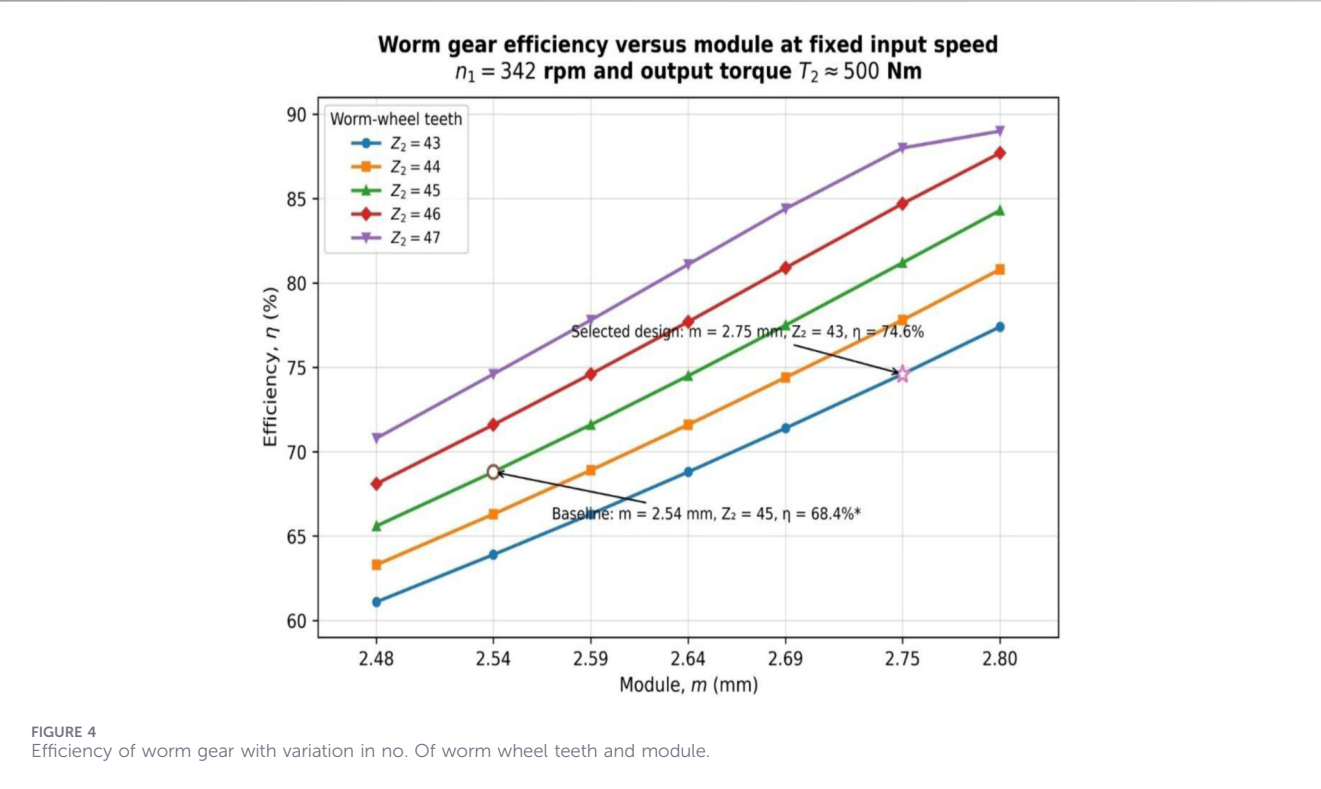
Figure 6 presents observed versus predicted efficiency for the 35 design combinations and Residuals versus fitted efficiency. The predicted efficiency surface obtained from the regression model, illustrating a clear efficiency peak in the region around module, $m \approx 2.75$ mm and number of worm wheel teeth, $Z_2 = 43$.

3.3 Multi-objective ranking and optimal design selection

To ensure industrial feasibility, 60/20/20 weighting scheme is defined in Section 2.3.3. This approach enables identification of a design solution that balances performance improvement with mechanical integrity and operational compliance. Among all evaluated combinations, the design with module, $m = 2.75$ mm and number of worm wheel teeth, $Z_2 = 43$ achieved the highest composite score. This configuration satisfies the minimum root diameter requirement (13.84 mm), maintains output speed within the allowable tolerance, and achieves an efficiency of 74.6%. Compared to the baseline design efficiency of 68.8%, the recommended configuration provides an absolute efficiency improvement of 5.8 percentage points, corresponding to a relative improvement of approximately 8.45%, without requiring modification of the center distance or major manufacturing changes.

4 Discussion

The proposed methodology demonstrates that targeted parametric optimization of the module, m (mm) and worm wheel tooth count, Z_2 , without altering the center distance or overall gearbox envelope, can deliver a meaningful improvement in operating efficiency. The optimized configuration improves efficiency from 68.8% to 74.6%, corresponding to an absolute gain of 5.8 percentage points and a relative improvement of approximately 8.45%. Such an improvement is particularly significant for continuous-duty applications such as soot blowers, where even modest efficiency gains can reduce energy consumption, lower operating temperatures, extend lubricant life, and improve equipment reliability over prolonged periods. The observed efficiency improvement is primarily governed by three interacting physical mechanisms. First, the increase in module from 2.54 mm to 2.75 mm results in a favourable change in the helix (lead) angle, γ in degrees. This modification alters the force distribution at the meshing interface, reducing the proportion of sliding friction relative to rolling contact, thereby lowering frictional power losses. Second, the adjusted geometry shifts the rubbing velocity, V_s (m/s) into an operating regime where the empirical coefficient of friction, μ decreases noticeably for well-lubricated steel–bronze gear pairs, further enhancing efficiency. Third, combining a slightly



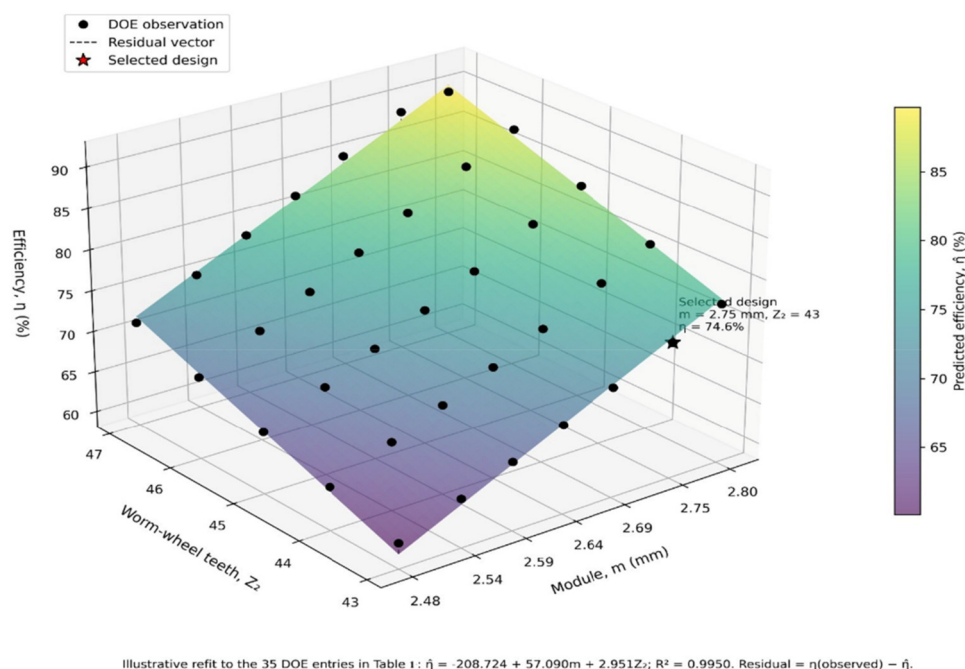


FIGURE 6
Predicted efficiency surface vs. module (m) and Z_2 from the regression model.

larger module with a reduced number of worm wheel teeth, $Z_2 = 43$ instead of 45, improves load sharing and contact conditions, thereby reducing localized contact stresses and contributing to smoother power transmission.

These efficiency-oriented modifications introduce inherent design trade-offs. In particular, reducing number of worm wheel teeth, Z_2 marginally reduces the root-diameter margin compared with higher-tooth-count configurations, potentially affecting bending fatigue resistance under shock loading or misalignment. This limitation is effectively mitigated by adopting a higher-profile correction factor (0.90 mm instead of the existing 0.75 mm), which increases the root fillet radius and the effective tooth thickness. As a result, adequate tooth root strength is restored without compromising the efficiency gain achieved. Reported efficiencies for single-start worm gear drives with reduction ratios near 45:1 typically range from 65% to 78% under comparable lubrication and operating conditions [9, V.B. Bhandari, and related studies]. The achieved efficiency of 74.6% places the optimized design in the upper performance quartile of this range, validating the effectiveness of the regression-guided DOE approach. Importantly, this level of performance is achieved without resorting to more complex and costly methods such as detailed finite element-based profile optimization or advanced lubrication modeling, making the proposed methodology particularly attractive for industrial implementation. From a manufacturing perspective, the recommended design changes are highly feasible. Increasing the module to 2.75 mm requires only a standard tooling update, such as a revised hobbing cutter for the worm wheel. Reducing the worm wheel tooth count to $Z_2 = 43$ remains well within the capabilities of conventional hobbing machines and does not require changes to fixturing. Crucially, no modification to the worm shaft geometry,

housing, bearings, or center distance is necessary, minimizing requalification effort and production disruption.

The present article focuses on efficiency modelling and constrained parametric selection; it does not introduce a new FEA campaign. Structural feasibility is therefore treated as a constraint rather than as a newly measured response. For the same industrial soot-blower gearbox and the same recommended configuration (module, $m = 2.75$ mm, number of worm wheel teeth, $Z_2 = 43$, profile correction = 0.90 mm), the authors' preceding study reported analytical and FEA checks of the worm/worm-wheel assembly and material validation for 20MnCr5 and PB2. Those results included a maximum worm von Mises stress of 907.49 MPa, worm-wheel von Mises stress of 143.81 MPa and analytical worm bending stress of 214.40 MPa under the stated loading framework. These values are cited as prior structural validation and are not presented as new measurements in the current study. The present study therefore limits its claim to geometric feasibility and refers the reader to the prior FEA work for detailed stress validation (Honkalas et al., 2024).

The present study does have limitations. The efficiency values are derived from analytical calculations and regression-based prediction rather than full-scale experimental testing. Analytical validation within the design-point dataset is done. No new test-rig measurements are presented in this article. Experimental validation of the underlying industrial gearbox has been reported previously; the present contribution focuses on regression-based local prediction and constrained selection. Assumptions of constant operating temperature, ideal lubrication conditions, and steady loading were adopted, while dynamic effects such as vibration, misalignment, load fluctuation and thermal expansion can alter the contact pattern, local sliding conditions and lubricant film

TABLE 2 Statistical results table.

Metric	Revised value	Interpretation	Action
Design points	35	7 module levels × 5 tooth-count levels	Replace n = 15
Predictors	M, Z ₂	Independent design variables	Remove n ₂ as predictor
Model	$\hat{\eta} = -208.724 + 57.090m + 2.951Z_2$	η in %, m in mm	Use in revised manuscript
R ²	0.9950	Local fit to Table 1	Report with limitation
Adjusted R ²	0.9947	Adjusted local fit	Report
RMSE	0.520 percentage points	Prediction error within design domain	Add
MAPE	0.574%	Mean absolute percentage error	Add
VIF(m)	1.00	No collinearity in m–Z ₂ grid	Report
VIF(Z ₂)	1.00	No collinearity in m–Z ₂ grid	Report
Shapiro–Wilk	W = 0.946, p = 0.085	No strong evidence against normality	Report cautiously
Breusch–Pagan	p = 0.899	No strong evidence of heteroscedasticity	Report cautiously
95% CI β ₀	[−216.303, −201.145]	Coefficient interval	Report
95% CI β _m	[55.319, 58.861]	Coefficient interval	Report
95% CI β _{Z₂}	[2.819, 3.084]	Coefficient interval	Report

thickness. These effects may change frictional losses and therefore measured efficiency; however, the magnitude is application-specific and is not quantified here because no dynamic test data were collected. Start-up and low-temperature operation may involve higher lubricant viscosity, different film formation and transient frictional behaviour. Because the present analysis assumes steady operating conditions, the model should not be extrapolated to cold-start efficiency without additional thermal-lubrication measurements.

Future work should focus on experimental validation using a dynamometer-based test rig to confirm the predicted efficiency improvements under realistic soot blower duty cycles. Extension of the model to incorporate temperature-dependent lubricant viscosity, thermal effects, and additional surface parameters such as roughness and oil grade would further enhance predictive accuracy. A detailed cost–benefit analysis evaluating energy savings, payback period, and lifecycle impact under continuous industrial operation would also be valuable.

The efficiency improvement is physically consistent with the geometry-dependent sliding losses represented by the adopted analytical model. Increasing the module changes the worm lead and helix angle, while reducing the worm-wheel tooth count changes the worm pitch diameter under fixed centre distance. These changes modify rubbing velocity and therefore the friction coefficient used by the analytical relation. The optimum is not selected from efficiency alone: geometric feasibility and output-speed tolerance must also be satisfied. The recommended m = 2.75 mm, Z₂ = 43 configuration provides 74.6% calculated efficiency and an output speed of approximately 7.95 rpm, which remains within the stated ±5% tolerance. However, the present study does not quantify service-life extension, start-up losses, thermal behaviour, vibration effects or lubricant degradation. Those quantities require dedicated experiments.

TABLE 3 Baseline/optimum consistency table.

Design	m (mm)	Z ₂	η (%)	n ₂ (rpm)
Existing	2.54	45	68.8	7.6
Recommended	2.75	43	74.6	7.953

4.1 Effect on energy saving, environmental sustainability and energy cost effectiveness

The modified worm and worm wheels calculated efficiency is 74.6% which is 8.5% higher than existing worm gear set. Further analytical calculations of worm gear set on basis of energy consumption is carried out to check total energy conservation of modified worm type gear than existing worm type gear (Honkalas et al., 2024) (Yasar Demirel, 2018).

A. Calculated Efficiency of Existing worm gear = 68.8%
power consumption = 1/efficiency = 1/0.688 = 1.45 KWh
B. Calculated Efficiency of modified worm gear = 74.6%
power consumption = 1/efficiency = 1/0.746 = 1.34 KWh
Total power conservation = Consumption of ‘A’ – Consumption of ‘B’
Total power conservation = 1.45 KWh - 1.34 KWh = 0.11 kwh
Energy saving = Rated power x Load Factor x Total power consumption
Where, Rated power = 0.56 KW, Load Factor = 1 (for full load condition),
Energy saving = 0.56 x 1 x 0.11 = 0.616 KWh
Annual Energy Saving = (Energy saving) x (Annual operating Hours)
For 365 days = 24-hour x 365 days = 8760 hours
Therefore, Annual Energy Saving = 0.616 x 8760 = 5396.16 KWh that is 5.40 MWh per year per motor.

4.1.1 Calculation of actual cost saving with proposed worm gear

As per earlier calculations of annual energy saving with proposed worm gear used in Soot Blower Gear Motor is 5396.16 KWh that is 5.40 MWh per year per motor. Considering average commercial rate of electricity supply in India that is Rs. 15/ KWh. Therefore, Total Electricity Cost Saving = $5396.16 \times 15 = \text{Rs.}80942.40$ per motor.

Overall, the presented approach provides a practical, low-risk methodology for improving worm gearbox efficiency under strict industrial constraints. By combining analytical insight with DOE and regression-based optimization, the methodology offers a balanced solution that aligns well with the requirements of gearbox manufacturers and end-users in power generation and heavy process industries.

5 Conclusion

The study supports the hypothesis within the investigated design domain: module and worm-wheel tooth count are effective controllable variables for improving calculated worm-drive efficiency under fixed centre distance and operating constraints. The revised analysis uses 35 design points and avoids the deterministic dependence between Z_2 and n_2 by modelling efficiency as a function of m and Z_2 . The selected configuration, $m = 2.75$ mm and $Z_2 = 43$, gives 74.6% calculated efficiency compared with 68.8% for the existing $m = 2.54$ mm and $Z_2 = 45$ design, corresponding to a 5.8 percentage-point absolute gain and an 8.43% relative improvement. The local regression has $R^2 = 0.9950$, RMSE = 0.520 percentage points and MAPE = 0.574% within the investigated domain. These results support the use of the methodology as a design-screening and decision-support tool, but not as a universal predictive law. The model is limited to the stated single-start geometry, material pair and operating assumptions. Experimental validation under controlled lubricant, temperature, surface and dynamic conditions is required before production implementation or service-life claims are made. The results confirm that meaningful efficiency gains can be achieved through simple geometric adjustments and statistical modeling, thereby avoiding the need for costly redesign or complex numerical simulations. Overall, the proposed methodology offers a low-risk, cost-effective pathway for enhancing worm gearbox performance and can be readily extended to similar continuous-duty industrial applications in which efficiency improvements under strict design constraints are required. Tables 2, 3 represents statical results and baseline/consistency values of worm gear respectively.

Data availability statement

The original contributions presented in the study are included in the article/[Supplementary Material](#), further inquiries can be directed to the corresponding authors.

Author contributions

RH: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review and editing. BD: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review and editing. PP: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review and editing. SS: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review and editing. LC: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review and editing.

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Conflict of interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Generative AI statement

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Supplementary material

The Supplementary Material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/fmech.2026.1973503/full#supplementary-material>