PAMM · Proc. Appl. Math. Mech. **15**, 167 – 168 (2015) / **DOI** 10.1002/pamm.201510074

Optimization of two-layered steel/aluminum hollow cylinders under combined load

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Subject of the investigation is a two-layered tube under generalized plane strain subject to a combination of internal pressure and an elevated temperature at the inner surface. Criterion for the permissible stress is the yield criterion by von Mises, and the elastic limits are given as well as a way for a straightforward optimization procedure in respect of the weight of the device is shown.

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1 Introduction

Thick-walled long hollow cylinders are structural elements found frequently in various industrial fields such as power engineering and chemical engineering. Typically, these elements like pressure vessels or fluid conveying tubes are loaded by a radial temperature gradient, by internal pressure or by rotation, and often also by a combination thereof. In some cases, it is advantageous to use a layered tube consisting of two or more different materials; reasons for this might be different chemical and/or thermal requirements at the inner and outer surface, e.g., or the demand for a weight-reduction. Whereas there exist many investigations of the stress fields in thermally loaded layered hollow cylinders for given material data and geometry (e.g.[1]-[3]), the problem of finding the optimum composition for given maximum load (as treated in [4] for shrink-fitted vessels) was rarely addressed, however, in particular for combined loads. Hence, in the present study the elastic limits and the optimum composition of a two-layered tube under generalized plane strain subject to an elevated inner surface temperature and internal pressure are considered, thus supplementing the analogous investigation [5] for the rotation/internal pressure- and rotation/heating-cases. It shall be emphasized that, while the following results are provided particularly for a steel/aluminum hollow cylinder, the approach is applicable to any device of this type with similar ratio of the material data.

2 Elastic limits

Subject of the investigation is a tube consisting of two perfectly bonded layers of steel ($a \leq r \leq b$) and aluminum ($b \leq r \leq c$), respectively; at the inner surface there acts a pressure $-\sigma_r(a) = P$ whereas the outer surface is stress-free. It is presumed that in operation the device subsequently has reached a steady-state temperature distribution with $T(a) = T_0$ and $T(c) = 0$. As criterion for the maximum permissible stress the yield criterion of von Mises is adopted which states that plasticization will occur if at some radius $\sigma_M(r) = \sigma_{y,i}$ with $\sigma_M(r) = \left\{\left[(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2\right]/2\right\}^{1/2}$ where $\sigma_{y,i}$ denotes the uniaxial yield limit of material $i\,(i = St, Al)$. Figures 1a-1c show the analytically calculated elastic limits for combined pressure/temperature-load for three different wall-thicknesses of the layered tube vs. the steel/aluminum interface radius (since tubes of pure aluminum or pure steel are excluded, the dashed lines are to be understood as limits for very thin layers of steel and aluminum, respectively). There exist three possible cases of the onset of yield: ① at the inner surface, $r = a$, in the steel; ② at the interface, $r = b$, in the aluminum; and ③ at the outer surface, $r = c$, in the aluminum. One recognizes that in thick-walled tubes - depending on the load combination - all of these cases can occur (Figs. 1a,b), whereas for smaller wall-thicknesses the onset of yield at the outer surface predominates (Fig. 1c). Above a certain pressure, in all tubes a steel layer with a certain thickness is necessary even without heating (line I, corresponding to case ②). Although in this case a moderate temperature gradient reduces the stresses marginally, the necessary minimum interface radius is then determined by line I, of course. Hence, this line defines a vertical surface, the upper boundary of which is given by the intersection with surface ② or ③. It should nevertheless be emphasized that a parameter combination corresponding to an arbitrary point of this vertical surface does not mean beginning of yield contrarily to points on surfaces ①, ②, and ③. Left of this surface, for not too high temperatures the interface radius is limited only by the fact that at least an arbitrarily thin steel layer is presupposed.

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3 Optimum composition

Since the inner layer of steel is the heavier one, the device is optimized with respect to its weight when for given radii a and c and given maximum load the minimum necessary interface radius b is found. This can be accomplished by the procedure shown schematically in Fig. 1d. That is, one must find the (first) intersection point of load line h with the elastic limit surface or the vertical surfaces defined above. For the thickness-ratios of Figs. 1a-c the optimum value of b/c then can be seen directly, for other radii ratios a straightforward procedure analogous to those provided in [5] can be performed. Hence, the above results also may serve as a sound basis for FE-studies in case of slightly different thermal conditions or somewhat more complex geometrical properties.

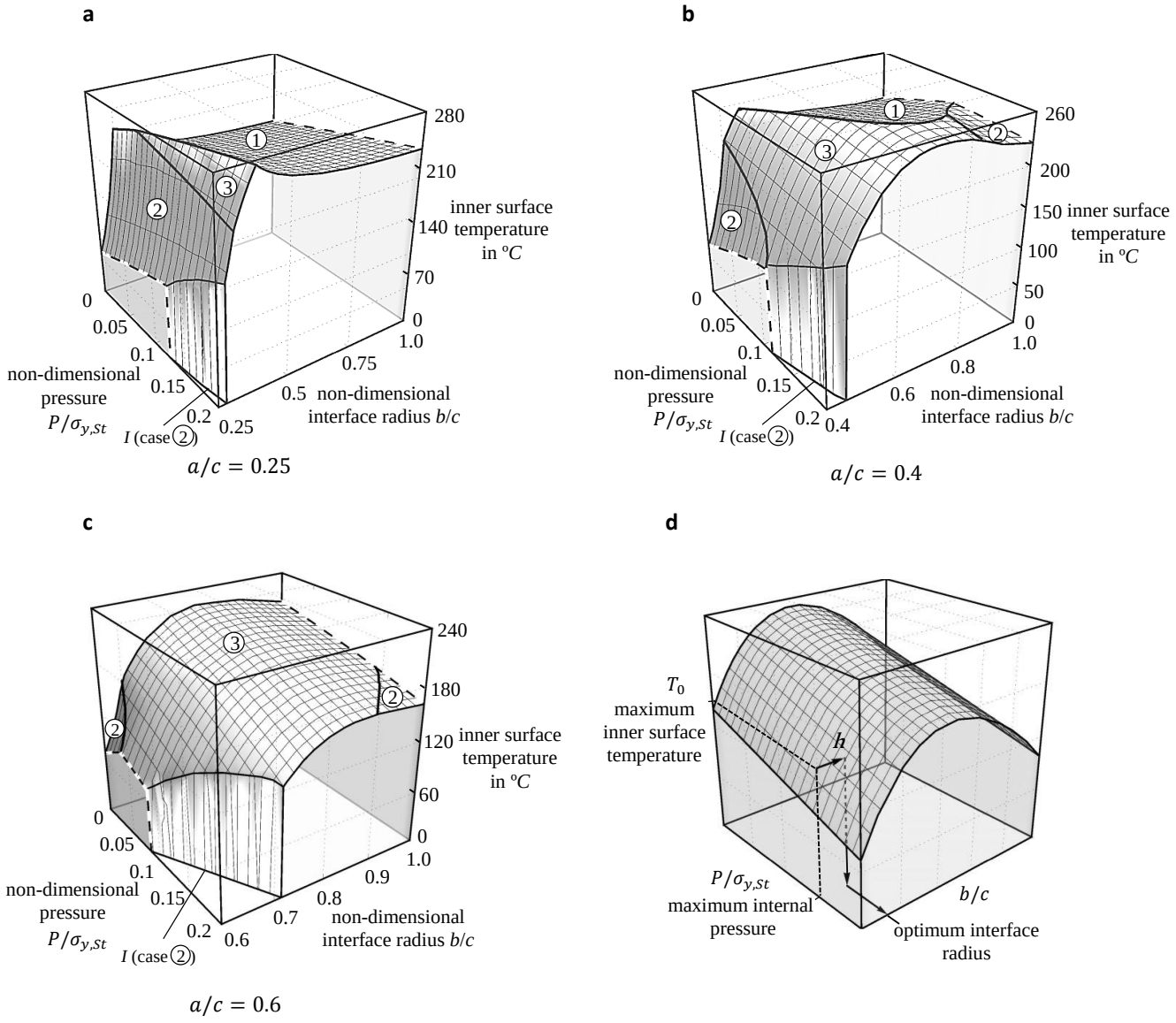


Fig. 1: a-c Elastic limits for three different wall-thicknesses vs. interface radius (material data St/Al: E: 200/70 GPa, σ_y: 430/100 MPa, α: 11.7/23.6 (10^{-6}) 1/°C, thermal conductivity, k: 45/200 W/(m°C)); **d** Scheme of finding the optimum composition

References

[1] Y.-C. Yang and C.-K. Chen, Int. J. Engng. Sci. **24**, 569-581 (1986).
[2] Z.-Y. Lee, C.K. Chen and C.-I. Hung, Acta Mech. **151**, 75-88 (2001).
[3] A.N. Eraslan, J. Thermal Stresses **26**, 167-193 (2003).
[4] G. Yuan, H. Liu and Z. Wang, Chin. J. Mech. Engng. **23**, 582-589 (2010).
[5] T. Apatay and W. Mack, submitted (2015).

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