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Generalized Random Spreading Performance Analysis of CDMA over GWSSUS Fading Channels

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Abstract

Signal-to-interference ratio (SIR) experienced by users is the major quality and capacity indicator of CDMA systems. In this paper, we analyze the SIRs achievable with diversity-reception linear multiuser receivers over GWSSUS time-variant fading uplink DS-CDMA channels based on finite-dimensional generalized random spreading performance evaluation methodology. Through the results of random matrix theory on the non-asymptotic eigenvalue distributions of finite-size random matrices, SIR statistics with linear RAKE and decorrelating multiuser receivers are derived in terms of the key system parameters. Besides deciphering the performance expectations for finite-size systems independent of the specific choice and assignment of spreading sequences, the analysis of the derived average SIR expressions at the asymptotic wideband regime also further validate the predictions of previous large-system results on the deterministic SIRs with multiuser receivers at wideband limits.

1. INTRODUCTION

In this paper, we analyze the SIRs achievable with diversity-reception linear multiuser receivers over GWSSUS time-variant fading uplink DS-CDMA channels based on finite-dimensional generalized random spreading performance evaluation methodology. In most of the recent significant contributions on the random spreading performance analysis of CDMA systems, the channel models attacked are mainly either flat-fading or Gaussian channels that do not provide diversity intrinsically. Furthermore, majority of these contributions concentrate on large-system limits, where the asymptotic eigenvalue random variables (rv) of the crosscorrelation matrices converge to certain distributions - Marcenko-Pastur law - as the number of users K and the processing gain L tend to infinity with a fixed finite ratio, and exploit this *asymptotic limiting theory* to examine the achievable SIR and capacity/spectral efficiency.

Complementary to the asymptotic limiting theory based analysis in this sequel, we present and employ a finite-dimensional approach for generalized random spreading analysis of multichannel diversity-reception CDMA systems based on the non-asymptotic results of random matrix theory for the eigenvalue distributions of random matrices with finite sizes. Though our analysis throughout the sequel is mainly finite-dimensional, by letting system dimensionality parameters to grow without bound, the average SIR expressions derived also exhibit behavior that SIRs with linear multiuser receivers asymptotically converge to deterministic constants at the wideband/large-system limit in close accordance with those predicted and presented by previous works based on asymptotic limiting theory. Thus, this observation further contributes to bridge the gap between the asymptotic and non-asymptotic random matrix theoretic performance analysis approaches.

The sequel is organized as follows. Next in Section II., the system model considered and the preliminaries are presented. Statistical characterization of SIRs and the derivation of mean SIRs with diversity-reception linear multiuser receivers are pursued in Section III. Numerical results and discussions are then presented in Section IV. followed by conclusions in Section V.

2. SYSTEM MODEL and PRELIMINARIES

We consider a chip/symbol synchronous uplink DS-

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CDMA system of W_s Hz system bandwidth with K equal-rate equal-power users randomly located in a single isolated cell and a central base-station receiver with a single receive antenna. Denoting the channel-symbol signalling period by T_{cs} and the chip period by T_c, the channel-symbols at the transmitters are assumed to be spread with normalized spherical complex-Gaussian random spreading sequences of processing gain $L = \frac{T_{cs}}{T_c}$:

$$c_k = \frac{1}{\sqrt{L}}[c_{k1}\, c_{k2} ... c_{kL}]^T \tag{1}$$

where each chip corresponds to a circularly-symmetric complex-Gaussian, zero-mean, unity-variance random variable. Hence, the realization of the spreading sequences in each channel-symbol signalling period T_{cs} is asymptotically unit Euclidean-norm via law of large numbers as $L \to \infty$, i.e. desired signal loss due to random spreading sequences with randomly fluctuating energies vanishes at the asymptotic wideband limit.

The unit-energy pulse-shaped and spread signals of all users occupying the same bandwidth are transmitted over the time-varying waveform channel. We are considering statistically identical time-varying Gaussian wide-sense-stationary uncorrelated-scattering (GWSSUS) channels for each user with *multipath delay spread* T_m, where $T_m << T_{cs}$ is assumed for negligible ISI and the optimality of one-shot detection. The inverse of the multipath delay spread reveals the *coherence bandwidth, i.e.* $(\Delta f)_c = \frac{1}{T_m}$, identifying the frequency-selectivity of the channel, and the *multipath diversity order* D resolvable by each user in conjunction with the spread system bandwidth W_s is then given by:

$$D = \lceil T_m W_s \rceil \simeq \frac{T_m}{T_c} \tag{2}$$

As common exercise for chip-synchronous systems, let's assume the propagation delays of each path of each user are integer multiples of the chip period with the first path of each user being at zero propagation delay. Then, by successive chip-matched/code-matched filtering at the appropriate delays for each path at the receiver side, the discrete-time equivalent of the multichannel diversity-reception DS-CDMA system in an arbitrary channel-symbol signalling interval is obtained as:

$$\mathbf{y} = \mathbf{RHs} + \mathbf{w} \tag{3}$$

where $\mathbf{y} = [y_{11}\ y_{12} \cdots y_{1D}\ y_{21} \cdots y_{KD}]^T \in C^{KD\times 1}$ is the stacked sufficient statistics vector for each path of each user, $\mathbf{R} = \mathbf{S}^H\mathbf{S} \in C^{KD\times KD}$ is the Hermitian crosscorrelation matrix with $\mathbf{S} \in C^{L\times KD}$ being the matrix of the chip-shifted spreading sequences for each path of each user with the first path of each user at zero-offset. $\mathbf{H} = diag\{\mathbf{h}_1, \mathbf{h}_2, .., \mathbf{h}_K\} \in C^{KD\times K}$ is the block-diagonal channel coefficients matrix of all users with $\mathbf{h}_k = [h_{k1}\, h_{k2} ... h_{kD}]^T \in C^{D\times 1}$ being the channel coefficients vector of kth user and $\mathbf{s} \in C^{K\times 1}$ is the channel-symbols vector of all K users. Furthermore, $\mathbf{w} \in C^{KD\times 1}$ is the zero-mean complex-Gaussian background noise vector having crosscovariance matrix $\sigma_n^2\mathbf{R}$, where the noise variance $\sigma_n^2 = N_0 \times W_s$ - in watts - and N_0 (watts/Hz) is the background AWGN power spectral density.

Keeping the unity path-power assumption for a total of D propagation paths, the analytic pdf of the norm-squared of the circularly-symmetric stochastic CIR under unity path-power D-path GWSSUS fading model consequently obey the non-central chi-squared density of $2D$ degrees of freedom with individual component variances of $\frac{\sigma_c^2}{2}$; i.e. $\chi_{2D}^{2,nc}\left(\frac{\sigma_c^2}{2}\right)$, the analytic form of which is given by:

$$f_G^{(ns)}(r) = \frac{1}{\sigma_c^2}\left(\frac{r}{s^2}\right)^{\frac{D-1}{2}} \exp\left(-\frac{s^2+r}{2\sigma_c^2}\right) I_{D-1}\left(\frac{2\sqrt{r}s}{\sigma_c^2}\right) \tag{4}$$

for $r \geq 0$, and the mean and the variance of the norm-squared of the circularly-symmetric stochastic CIR under unity path-power D-path GWSSUS fading model is further given by:

$$\mu_G^{(ns)} = D,\ \sigma_G^{2,(ns)} = D(2\sigma_c^2 - \sigma_c^4) \tag{5}$$

For the channel-symbols of users, we assume that the channel-symbols are independent random variables over symbol index and users, and the channel-symbols of kth user has equal symbol energy $E_s^{(k)}$. Such a model is applicable for circular linear modulation schemes such as M-PSK, as well as a coded system employing large block-length Gaussian random encoding with symbol signal-to-noise ratio average power constraint [1]. Thus, based on the received signal model in (3) and further assuming that the noise PSD level is normalized to unity in watts/Hz; i.e. $N_0 = 1$, without loss of generality, the transmit symbol signal-to-noise ratio of kth user is defined as:

$$SNR_s^{(k)} = \frac{E_s^{(k)}}{N_0} = E_s^{(k)} \tag{6}$$

Furthermore, the average received symbol signal-to-noise ratio per path of kth user is thus given by:

$$SNR_s^{(k,av)} = E\left\{|h_{kd}|^2\right\}\frac{E_s^{(k)}}{N_0} = E_s^{(k)} = SNR_s^{(k)} \tag{7}$$

due to the assumptions of unity mean-squared value channel coefficients and unity noise power spectral density.

3. AVERAGE SIRs with DIVERSITY-RECEPTION LINEAR RECEIVERS

Within our finite-dimensional scenario, the two important parameters for the system dimensionality are the *load factor* in users/chip, $\beta = \frac{K}{L}$, as used in asymptotic approaches, and the *per-chip diversity order* or the *normalized delay spread*, $\alpha = \frac{D}{L} \simeq \frac{T_m}{T_{cs}} << 1$, which unitlessly measures the time-dispersivity of the channel with respect to the channel-symbol signalling period. Via these definitions, as the bandwidth of the system is increased through higher processing gains L, the number of users K and the multipath diversity order resolved D consequently grow to keep the load factor and the normalized delay spread constant. Hence, the fixed system operating point is defined by the system parameters load factor β, normalized delay spread α, diversity order D and symbol signal-to-noise ratio $SNR_s^{(k)}$.

3.1. Linear RAKE Receiver

The signal-to-interference ratio for a linear multiuser receiver $SIR_k^{(\mathrm{MUD})}$ is statistically identical over the set of users K. The soft-output conditional SIR for user k conditioned on the channel and the spreading sequences achievable by linear RAKE multiuser receiver, whose linear estimator matrix after chip-matched filtering by $\mathbf{S}^H$ is $\mathbf{M} = \mathbf{I}$ that is analogous to the conventional matched-filter receiver for AWGN channels resulting in no interference suppression or cancellation, and followed by optimal linear diversity combining method maximal-ratio-combining (MRC), that coherently combines the D diversity channels of each user in the maximal-SIR sense, is given by:

$$SIR_k^{(\mathrm{RAKE})} =_{\mathcal{D}} \frac{\mathbf{h}_k^H \mathbf{R}_{kk}^2 \mathbf{h}_k SNR_s^{(k)}}{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_{\acute{k}}^H \mathbf{R}_{k\acute{k}}^2 \mathbf{h}_{\acute{k}} SNR_s^{(k)} + P_{tot}^{(noise)}} \tag{8}$$

in terms of power by power ratio, where conditional total power of desired signal components $P_{tot}^{(desired)} = \mathbf{h}_k^H \mathbf{R}_{kk}^2 \mathbf{h}_k SNR_s^{(k)}$, conditional total power of multiaccess interference terms $P_{tot}^{(MAI)} = \sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_{\acute{k}}^H \mathbf{R}_{k\acute{k}}^2 \mathbf{h}_{\acute{k}} SNR_s^{(k)}$, total power of noise component $P_{tot}^{(noise)} = \mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})}\right|^2\right\}$, $(=_{\mathcal{D}})$ is the distribution-equivalence operator and $\mathcal{E}\{.\}$ is the expectation operator. The total power of the zero-mean noise component in the denominator of (8) for $N_0 = 1$ can further be expanded via eigenvalue decomposition (EVD) as:

$$\begin{aligned} P_{tot}^{(noise)} &= \mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})}\right|^2\right\} = \\ &\mathcal{E}\left\{\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})} \left(\mathbf{w}_k^{(\mathrm{RAKE})}\right)^H \mathbf{h}_k\right\} = \\ &\mathcal{E}\left\{\mathbf{h}_k^H \mathbf{R}_{kk} \mathbf{h}_k\right\} = \mathcal{E}\left\{\mathbf{h}_k^H \mathbf{U}_{kk}^H \mathbf{D}_{kk} \mathbf{U}_{kk} \mathbf{h}_k\right\} \end{aligned} \tag{9}$$

where $\mathbf{D}_{kk} = diag\{\lambda_{kk1}, \lambda_{kk2}, ..., \lambda_{kkD}\}$ is the diagonal eigenvalues matrix of $\mathbf{R}_{kk}$ and $\mathbf{U}_{kk}$ is the unitary eigenvectors matrix of $\mathbf{R}_{kk}$. Since $\mathbf{U}_{kk}$ is unitary - $\mathbf{U}_{kk}^H = \mathbf{U}_{kk}^{-1}$ - and if we denote the transformed channel coefficients vector by $\acute{\mathbf{h}}_k = \mathbf{U}_{kk} \mathbf{h}_k$:

$$\acute{\mathbf{h}}_k^H \acute{\mathbf{h}}_k =_{\mathcal{D}} \mathbf{h}_k^H \mathbf{h}_k \tag{10}$$

Due to this distribution-equivalence property, the following expression from (10) is valid:

$$\mathcal{E}\left\{\mathbf{h}_k^H \mathbf{U}_{kk}^H \mathbf{D}_{kk} \mathbf{U}_{kk} \mathbf{h}_k\right\} = \tag{11}$$

$$\mathcal{E}\left\{\acute{\mathbf{h}}_k^H \mathbf{D}_{kk} \acute{\mathbf{h}}_k\right\} = \mathcal{E}\left\{\sum_{i=1}^{D} \lambda_{kki} \left|h_{ki}\right|^2\right\} \tag{12}$$

As it can be deducted, for the statistical characterization of the SIR with multiuser receivers, the finite-dimensional eigenvalue densities of the Hermitian diagonal $\mathbf{R}_{kk}$ and the off-diagonal $\mathbf{R}_{k\acute{k}}$ submatrices of the Hermitian crosscorrelation matrix $\mathbf{R}$ have to be accurately characterized. The *cumulative magnitude-ordered eigenvalue distribution function* for an $M \times M$ random matrix $\mathbf{A}$:

$$F_{\mathbf{A}}^{(O)} = \Pr\{\frac{1}{M} \sum_{i=1}^{M} \delta(\lambda - \lambda_i^{(O)}(\mathbf{A})) \leq \lambda_t\} \tag{13}$$

defines the probability for the percentage of magnitude-ordered eigenvalues of $\mathbf{A}$ being below a specific threshold λ_t, and the density of the corresponding *eigenvalue-counting* or the *average-trace* rv λ - *eigenvalue density* - is:

$$\frac{dF_{\mathbf{A}}^{(\pi)}}{d\lambda} = f_{\mathbf{A}}(\lambda) \leftrightarrow \lambda = \frac{1}{M} \sum_{i=1}^{M} \lambda_i^{(\pi)}(\mathbf{A}) = \frac{1}{M} Tr\{\mathbf{A}\} \tag{14}$$

where $F_{\mathbf{A}}^{(\pi)} = \frac{1}{K!} F_{\mathbf{A}}^{(O)}$ is the *cumulative magnitude-unordered eigenvalue distribution function* obtained by averaging $F_{\mathbf{A}}^{(O)}$ over all possible $K!$ permutations. Via this definition of the eigenvalue density, we also have the following important *averaging lemma:*

Lemma 1.(Haagerup and Thornbjorsen, [4]): For an $M \times M$ random matrix $\mathbf{A}$ with eigenvalue density $f_{\mathbf{A}}(\lambda)$, let $g(x) : R \to R$ be a Borel function. Then,

$$\mathcal{E}\left\{\frac{1}{M}\sum_{i=1}^{M} g\left(\lambda_i(\mathbf{A})\right)\right\} = \int_R g(\lambda) f_{\mathbf{A}}(\lambda) d\lambda \qquad (15)$$

Due to the normalized spherical complex-Gaussian random spreading sequences assumed where the real and imaginary parts of each chip is independently and identically distributed with variance $\frac{1}{2}$, the diagonal elements of the crosscorrelation matrix $\mathbf{R}$ are distributed according to the central chi-squared distribution $\chi^{2,c}_{2L}(1, \frac{1}{L})$ of $2L$ degrees of freedom with unity mean and variance $\frac{1}{L}$ due to the scaling by $\frac{1}{L}$. Furthermore, when the chip-synchronism is complete and the path delays are integer multiples of the chip period T_{cs}, the off-diagonal elements of $\mathbf{R}$ converge by Central Limit Theorem to the circularly-symmetric complex-Gaussian distribution $N(0, \frac{1}{L})$ with zero mean and variance $\frac{1}{L}$. Thus, by the average trace-rv definition, the eigenvalue-counting rv λ_{kk} of $\mathbf{R}_{kk}$ is distributed according to the central chi-squared $\chi^{2,c}_{2DL}$ distribution with $2DL$ degrees of freedom scaled by $\frac{1}{DL}$ having unity mean and variance $\frac{1}{DL}$:

$$f_{\mathbf{R}_{kk}}(\lambda_{kk}) = \frac{(DL)^{DL}}{\Gamma_c(DL)} \lambda_{kk}^{DL-1} \exp(-DL\lambda_{kk}) \qquad (16)$$

and moreover, using Lemma 1., the total power of the noise component by RAKE in (9) is given by:

$$P_{tot}^{(noise)} = \mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{RAKE})}\right|^2\right\} = \mathcal{E}\left\{\sum_{i=1}^{D} \lambda_{kki} \left|h_{ki}\right|^2\right\} = D^2 \qquad (17)$$

with the definition of the density of norm-squared pdf of CIR $f_G^{(ns)}(r)$ - for $\|\mathbf{h}_k\|^2$ - in (4). Furthermore, the fact that the crosscorrelation matrix under the considered spreading scenario tending to the identity matrix $\mathbf{R} \to \mathbf{I}$ as $L \to \infty$ validates the averaging out of MAI components asymptotically at the wideband limit.

For the statistical characterization of the multi-access interference terms with RAKE in (8) via the eigenvalue distribution of the off-diagonal Hermitian random submatrices $\mathbf{R}_{k\acute{k}}$ of $\mathbf{R}$, $\mathbf{R}_{k\acute{k}}$ can be further partitioned into two matrices as:

$$\mathbf{R}_{k\acute{k}} = \mathbf{A} + \mathrm{sqrt}(-1)\mathbf{B} \qquad (18)$$

where $\mathbf{B}$ is a diagonal matrix of independently and identically distributed $N(0, \frac{1}{2L})$ random variables having eigenvalue density $N(0, \frac{1}{2DL})$ and $\mathbf{A}$ is a proper Hermitian complex-Gaussian random matrix [4]. The eigenvalue density of proper Hermitian complex-Gaussian random matrices $\mathbf{SGRM}(n, \sigma^2_{WG})$ is identified as follows due to Wigner in [5]:

$$f_{WG(n,\sigma^2_{WG})}(\lambda) = \frac{1}{n\sigma_{WG}\sqrt{2}} \sum_{k=0}^{n-1} \left[\varphi_k\left(\frac{\lambda}{\sigma_{WG}\sqrt{2}}\right)\right]^2 \qquad (19)$$

where $\varphi_k(.)$ is the sequence of Hermite functions, $\varphi_k(x) = \frac{1}{(2^k k! \sqrt{\pi})^{\frac{1}{2}}} H_k(x) \exp(-\frac{x^2}{2})$, in terms of the sequence of Hermite polynomials $H_k(x) = (-1)^k \exp(x^2).(\frac{d^k}{dx^k} \exp(-x^2))$.

It is a general assumption for analysis with RAKE receivers that the total MAI components are distributed normally and to use the corresponding average total power in SIR expressions which is in general termed the Gaussian assumption [6]. This further requires us to determine the mean-squared value of the eigenvalue-counting rv of a proper Hermitian complex-Gaussian random matrix $\mathbf{A}$, since by eigenvalue decomposition and averaging lemma, the average total power of the MAI components is given by:

$$\begin{aligned} P_{tot}^{(MAI,av)} &= \mathcal{E}\left\{\sum_{\acute{k}=1,\acute{k}\neq k}^{K} \mathbf{h}_k^H \mathbf{R}^2_{k\acute{k}} \mathbf{h}_{\acute{k}} SNR_s^{(k)}\right\} \\ &= \sum_{\acute{k}=1,\acute{k}}^{K} DSNR_s^{(k)} \int_0^\infty \int_{-\infty}^\infty \lambda^2_{k\acute{k}} r f_{\mathbf{R}_{k\acute{k}}}(\lambda_{k\acute{k}}) \\ &\quad \times f_G^{(ns)}(r) d\lambda_{k\acute{k}} dr \end{aligned} \qquad (20)$$

The mean value of the eigenvalue-counting rv of an $n \times n$ proper Hermitian complex-Gaussian random matrix can be calculated as 0 [7]. Similarly, the mean-squared value being equivalent to the variance is given by $n\sigma^2_{WG} = \frac{D}{L}$ in our case that can be derived via Harer-Zagier recursion defined as follows:

Theorem 1. (Harer and Zagier, Section 4, Proposition 1,[8]): Let $\mathbf{A}$ be an element of $\mathbf{SGRM}(n, 1)$ and define:

$$C(p, n) = \mathcal{E}\left\{Tr_n[\mathbf{A}^{2n}]\right\}, p \in N_0 \qquad (21)$$

Then $C(0, n) = n, C(1, n) = n^2$, and for fixed n in N, the numbers $C(p, n)$ satisfy the recursion formula:

$$C(p+1, n) = n.\frac{4p+2}{p+2}.C(p, n) + \frac{p.(4p^2-1)}{p+2} C(p-1, n) \qquad (22)$$

for $p \geq 1$.

Letting $K - 1 \simeq K$ as $K \to \infty$ and following from (20), the average total power of MAI components is given by:

$$P_{tot}^{(MAI,av)} = \beta(D^2 + \frac{1}{2}) SNR_s^{(k)} \qquad (23)$$

If we further expand the total conditional power of the desired signal components in the numerator of (8) using eigenvalue decomposition and the averaging lemma reversely as:

$$P_{tot}^{(desired)} = \mathbf{h}_k^H \mathbf{R}_{kk}^2 \mathbf{h}_k SNR_s^{(k)} =$$

$$\sum_{i=1}^{D} \lambda_{kki}^2 \left|h_{ki}\right|^2 SNR_s^{(k)} =_{\mathcal{D}} D\lambda_{kk}^2 . \left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)} \quad (24)$$

since $\acute{\mathbf{h}}_k \ ^H \acute{\mathbf{h}}_k =_{\mathcal{D}} \mathbf{h}_k^H \mathbf{h}_k$ for $\acute{\mathbf{h}}_k = \mathbf{U}_{kk} \mathbf{h}_k$ and unitary $\mathbf{U}_{kk}$. Following, the SIR of kth user with RAKE receiver under Gaussian assumption is distributed as:

$$SIR_k^{(\mathrm{RAKE,GA})} =_{\mathcal{D}} \frac{D\lambda_{kk}^2 . \left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)}}{\beta(D^2 + \frac{1}{2}) SNR_s^{(k)} + D^2} \quad (25)$$

where $\left\|\mathbf{h}_k\right\|^2$ is the norm-squared of the channel filter for desired user with density given in (4). Hence, the mean SIR with RAKE receiver under Gaussian approximation is given by:

$$\mathcal{E}\left\{SIR_k^{(\mathrm{RAKE,GA})}\right\} = \frac{(D^2 + \alpha) SNR_s^{(k)}}{\beta(D^2 + \frac{1}{2}) SNR_s^{(k)} + D^2} \quad (26)$$

since $\mathcal{E}\left\{\lambda_{kk}^2\right\} = 1 + \frac{1}{DL}$ and $\mathcal{E}\left\{\left\|\mathbf{h}_k\right\|^2\right\} = D$.

3.2. Linear Decorrelating Interference Supression

Linear decorrelating interference suppression multiuser receiver [2,3] applies the linear estimator matrix $\mathbf{M}^{(\mathrm{Dec})} = \mathbf{R}^{-1}$ after chip-matched filtering by $\mathbf{S}^H$, resulting in complete multi-access interference suppression via zero-forcing sense equalization. The soft-output conditional random SIR of user kth achievable by decorrelating multiuser detector followed by maximal-ratio-combining is given by:

$$SIR_k^{(\mathrm{Dec})} =_{\mathcal{D}} \frac{D\mathbf{h}_k^H \mathbf{h}_k SNR_s^{(k)}}{\mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{Dec})}\right|^2\right\}} \quad (27)$$

The total power of the noise term $\mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{Dec})}\right|^2\right\}$ can be further expanded via eigenvalue decomposition and averaging lemma as:

$$\mathcal{E}\left\{\left|\mathbf{h}_k^H \mathbf{w}_k^{(\mathrm{Dec})}\right|^2\right\} = \mathcal{E}\{\mathbf{h}_k^H \mathbf{R}_{kk}^{-1} \mathbf{h}_k\} =$$

$$\mathcal{E}\left\{\sum_{i=1}^{D} \lambda_{kki}^{-1} \left|\breve{h}_{ki}\right|^2\right\} =$$

$$D^2 \frac{\left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}\right)}}{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha})} \frac{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha} - 1)}{\left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha} - 1\right)}} \quad (28)$$

since the covariance matrix of $\mathbf{w}_k^{(\mathrm{Dec})}$ is $\mathcal{E}\left\{\mathbf{w}_k^{(\mathrm{Dec})} \left(\mathbf{w}_k^{(\mathrm{Dec})}\right)^H\right\} = \mathbf{R}_{kk}^{-1}$, $\int_0^\infty t^{n-1} \exp(-at) dt =$

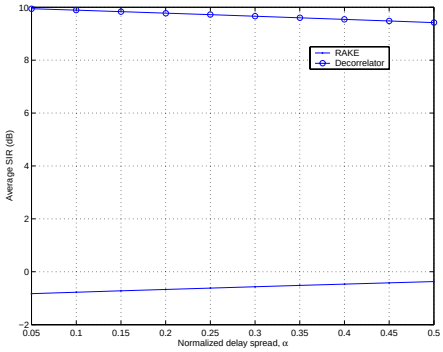


Figure 1: Average SIRs versus normalized delay spread α at β=1, D=2 and SNR_s=10 dB.

$\frac{\Gamma_{\mathrm{c}}(n)}{a^{n+1}}$ and $\breve{\mathbf{h}}_k \ ^H \breve{\mathbf{h}}_k =_{\mathcal{D}} \mathbf{h}_k^H \mathbf{h}_k$ for $\breve{\mathbf{h}}_k = \mathbf{U}_{kk}^{-H} \mathbf{h}_k$ and unitary $\mathbf{U}_{kk}$. Hence, the SIR of kth user with linear decorrelating interference suppression multiuser receiver followed by MRC is derived to be distributed as:

$$SIR_k^{(\mathrm{Dec})} =_{\mathcal{D}} \frac{\Gamma_{\mathrm{c}}(\frac{D^2}{\alpha}) \left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha} - 1\right)} \left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)}}{D \left(\frac{D^2}{\alpha}\right)^{\left(\frac{D^2}{\alpha}\right)} \Gamma_{\mathrm{c}}(\frac{D^2}{\alpha} - 1)} \quad (29)$$

that is a scaled noncentral chi-squared rv. Simplifying the above expression (29):

$$SIR_k^{(\mathrm{Dec})} =_{\mathcal{D}} \frac{(\frac{D^2}{\alpha} - 1) \left\|\mathbf{h}_k\right\|^2 SNR_s^{(k)}}{D \left(\frac{D^2}{\alpha}\right)} \quad (30)$$

Following, the mean of the SIR with decorrelating receiver and MRC is given by:

$$\mathcal{E}\left\{SIR_k^{(\mathrm{Dec})}\right\} = \frac{(\frac{D^2}{\alpha} - 1) SNR_s^{(k)}}{\left(\frac{D^2}{\alpha}\right)} \quad (31)$$

since $\mathcal{E}\left\{\left\|\mathbf{h}_k\right\|^2\right\} = D$.

4. NUMERICAL RESULTS AND DISCUSSION

We illustrate average SIRs versus normalized delay spread α at $\beta = 1, D = 2$ and $SNR_s^{(k)} = 10$ dB in Fig. 1. The average SIR of decorrelator strictly decreases with α from non-fading AWGN bound $SNR_s^{(k)}$ since increased delay spread is detrimental for zero-forcing type linear equalization.

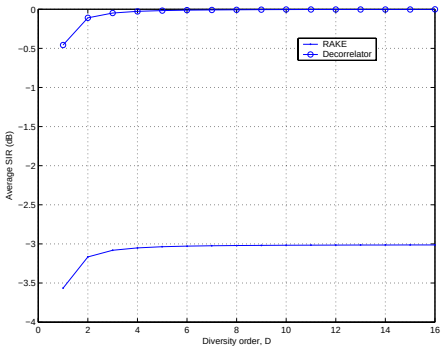


Figure 2: Average SIRs versus diversity order D at β=1, α=0.1 and SNR$_s$=0 dB

We further provide average SIRs versus diversity order D in Fig. 2. Average SIRs of both multiuser receivers strictly increases with diversity order converging to the bounds:

$$\lim_{D\to\infty} \mathcal{E}\left\{SIR_k^{(\mathrm{RAKE,GA})}\right\} = \frac{SNR_s^{(k)}}{\beta SNR_s^{(k)}+1} \qquad (32)$$

$$\lim_{D\to\infty} \mathcal{E}\left\{SIR_k^{(\mathrm{Dec})}\right\} = SNR_s^{(k)} \qquad (33)$$

as $D \to \infty$. It also further turns out that it is not judicious to increase transmit $SNR_s^{(k)}$ with RAKE receivers at high diversity orders resolved. Most importantly, the average SIRs converging to certain limits in Fig. 2 present the validation of the predictions of previous large-system results on the deterministic SIRs with multiuser receivers at wideband limits.

In Fig. 3, the average SIRs are presented versus $SNR_s^{(k)}$. Average SIR of decorrelator strictly increases in log-linear fashion while that of RAKE saturates and converges to:

$$\lim_{\mathrm{SNR}_s\to\infty} \mathcal{E}\left\{SIR_k^{(\mathrm{RAKE,GA})}\right\} = \frac{1}{\beta} \qquad (34)$$

5. CONCLUSIONS

In this paper, we analyzed the SIRs achievable with linear multiuser receivers over GWSSUS time-variant fading uplink DS-CDMA channels based on finite-dimensional generalized random spreading performance evaluation methodology. The analysis of the derived average SIR expressions elucidates the performance expectations independent of the specific choice and assignment of spreading sequences.

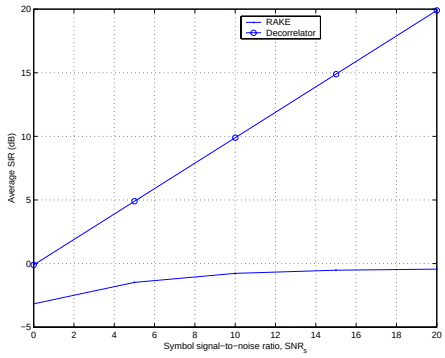


Figure 3: Average SIRs versus SNR$_s$ at $\beta = 1$, D=2 and $\alpha = 0.1$.

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