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Development of an intuitionistic fuzzy ranking model for nontraditional machining processes

Mustafa Yurdakul¹ · Yusuf Tansel İç² · Kumru Didem Atalay²

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Abstract

Nontraditional machining processes (NTMPs) are capable of processing very small parts, producing intricate geometries, operating on very narrow machining areas and machining high strength materials. These capabilities lead to a very diverse and large application area for NTMPs. Such a diverse and large application area along with more than one hundred NTMPs requires development of systematic and comprehensive models to help manufacturing engineers in their NTMP selection decisions. Furthermore, fuzzy models instead of crisp ones are being used in the literature in recent years to represent preferences of decision makers more realistically. This study proposes intuitionistic and triangular fuzzy NTMP ranking models and compares their ranking results with the crisp ranking model. The comparisons show that there are statistically significant differences among all three ranking models' NTMP ranking results.

Keywords Nontraditional machining approaches · Multi-criteria decision making (MCDM) · Intuitionistic fuzzy NTMP ranking model · Triangular fuzzy NTMP ranking model

1 Introduction

Nontraditional manufacturing term refers to a list of processes that remove material by using different methods such as electrical, thermal, mechanical, chemical energy or integration of these energy types without using a sharp cutting tool in the traditional sense (Groover 2008).

Nontraditional manufacturing processes (NTMPs) are characterized by a number of criteria which reflect their capabilities. The performance levels of NTMPs are evaluated and measured at various criteria to determine their suitability and preference rankings in different types of application. However, limited process flexibilities of NTMPs, uncertainties regarding the capabilities of the NTMPs, multiple conflicting criteria and different cost elements of operating NTMPs make the comparison and

ranking of NTMPs a challenging task (Yurdakul and Cogun 2003). Furthermore, lack of enough experienced experts in the field of NTMPs makes the selection of the appropriate NTMPs a more difficult problem especially for the first time users of NTMPs. Therefore, there is a growing need for decision models to aid in making comparison and ranking decisions about NTMPs.

In the literature, various NTMP selection models are available. For example, Cogun (1993, 1994) developed a selection procedure for NTMPs by removing unsuitable NTMPs from consideration and listing the suitable NTMPs. The developed approach did not aim to rank the feasible NTMPs or select the most suitable one. Yurdakul and Cogun (2003) improved the selection approach proposed in Cogun (1993, 1994) not only by identifying the feasible NTMPs but also by obtaining a ranking among them. In their study, once the list of feasible NTMPs is obtained, a crisp (nonfuzzy) multi-criteria decision-making (MCDM) model is applied to rank the feasible NTM processes according to their suitability for a specific machining application.

In more recent MTMP ranking studies, Chakroborty and Dey (2007) developed a quality function deployment (QFD)-based expert system, Chakladar and Chakroborty (2008), Kul et al. (2014), and Yurdakul and Ic (2019)

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employed a combination of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Analytic Hierarchy Process (AHP) methods, Shadu and Chakraborty (2007) employed Data Envelopment Analysis, Das and Chakraborty (2011) proposed an Analytic Network Process (ANP)-based approach, and Chandrasselan et al. (2011) designed a web-based knowledge base system. Prasad and Chakraborty (2016) proposed a decision support model developed in Visual Basic to select the most appropriate NTMP for a specific combination of work material and shape feature. On the other hand, Sáenz et al. (2015) presented a fuzzy rule-based expert system for the comparison and ranking among nontraditional sheet metal cutting processes.

In the NTMP ranking literature, MCDM approaches are the most common ones used in developing models. MCDM refers to making decisions in the presence of multiple criteria and at least two alternative solutions (Tabucanon 1988; Karpak and Zionts 1987). Decision criteria must contain at least one conflicting criterion. The study presented in this study is based on such a multi-criteria ranking model developed in Yurdakul and Cogun (2003). Yurdakul and Cogun (2003)'s model uses AHP and TOPSIS in assigning pair-wise importance judgment values, converts them to criteria weights and ranks NTMP alternatives. The reason of the usage of AHP and TOPSIS approaches is their simplicity, ability to represent the ranking decision in a hierarchical structure and 'to yield an indisputable preference order (Chen and Hwang 1992).' In the model developed in Yurdakul and Cogun (2003), traditional formulations of AHP and TOPSIS approaches in which human's judgments were represented as crisp (nonfuzzy) numbers are used. However, in recent years, it is recommended in the literature to use fuzzy numbers instead of crisp ones when decision makers are unable to assign exact numerical values to their judgments (Duran and Aguilo 2008; Jafarian and Vahdat 2012; İç et al. 2013; Torfi et al. 2011). For example, it is noted in Jafarian and Vahdat (2012) related with the usage of AHP that 'fuzzy-AHP should be more appropriate and effective than conventional AHP in real practice where an uncertain pair-wise comparison environment exists.' This paper aims to incorporate fuzziness into the NTMP ranking model developed in Yurdakul and Cogun (2003) by using triangular and intuitionistic fuzzy MCDM models and illustrate the importance of representation of fuzziness in NTMP ranking decisions.

The proposed model first determines the feasible NTMPs in the first stage from the database that contains data belonging to the most common fourteen nontraditional manufacturing processes whose names are provided in Table 1. Since there is no change in the determination of the feasible NTMPs in the paper in comparison with Yurdakul and Cogun (2003), the reader is referred to

Table 1 The most common fourteen nontraditional machining processes

No	Code	Nontraditional manufacturing technologies
1	AJM	Abrasive Jet Machining
2	WJM	Water Jet Machining
3	AWJM	Abrasive Water Jet Machining
4	USM	Ultrasonic Abrasive Machining
5	RUM	Rotary Ultrasonic Machining
6	ECM	Electrochemical Machining
7	ECG	Electrochemical Grinding
8	ECH	Electrochemical Honing
9	CHM	Chemical Machining
10	EBM	Electron Beam Machining
11	EDM	Electric Discharge Machining
12	WEDM	Electric Discharge Wire Cutting
13	LBM	Laser Beam Machining
14	PAM	Plasma Beam (Arc) Machining

Yurdakul and Cogun (2003) for details and explanation of the determination of the feasible NTMPs. Once the feasible NTMPs are obtained, they can be ranked using the triangular and intuitionistic fuzzy MCDM models provided in this paper.

2 Development of fuzzy ranking models and illustration of their application in a case study

Ranking of the feasible NTMPs is performed using five independent ranking criteria and their sub-criteria that are arranged in a three-level hierarchical structure (Fig. 1). Fuzzy ranking models require the user to fill four pair-wise comparison matrices so that the local weights of the criteria (quality, material removal rate, process capability, material, and cost) and their sub-criteria in the hierarchy can be calculated.

In this study, a case study from Yurdakul and Cogun (2003) is used to illustrate the application of fuzzy models in ranking of the feasible NTMPs. In the case study, an automobile manufacturer wants to apply a NTMP to drill three lubrication holes in an internal gear used in transmissions. The lubrication holes have a diameter of 1.52 mm and length of 7.6 mm. The taper on the hole is considered critical, and it should not exceed 0.05 mm/mm. The gear is made of hot-forged powder steel 4662 alloy hardened to HRC 60. Other details of the case study can be found in Yurdakul and Cogun (2003).

Since the real contribution of the paper to the NTMP ranking literature is development and usage of an intuitionistic fuzzy ranking model, the development and application of the triangular fuzzy ranking model is

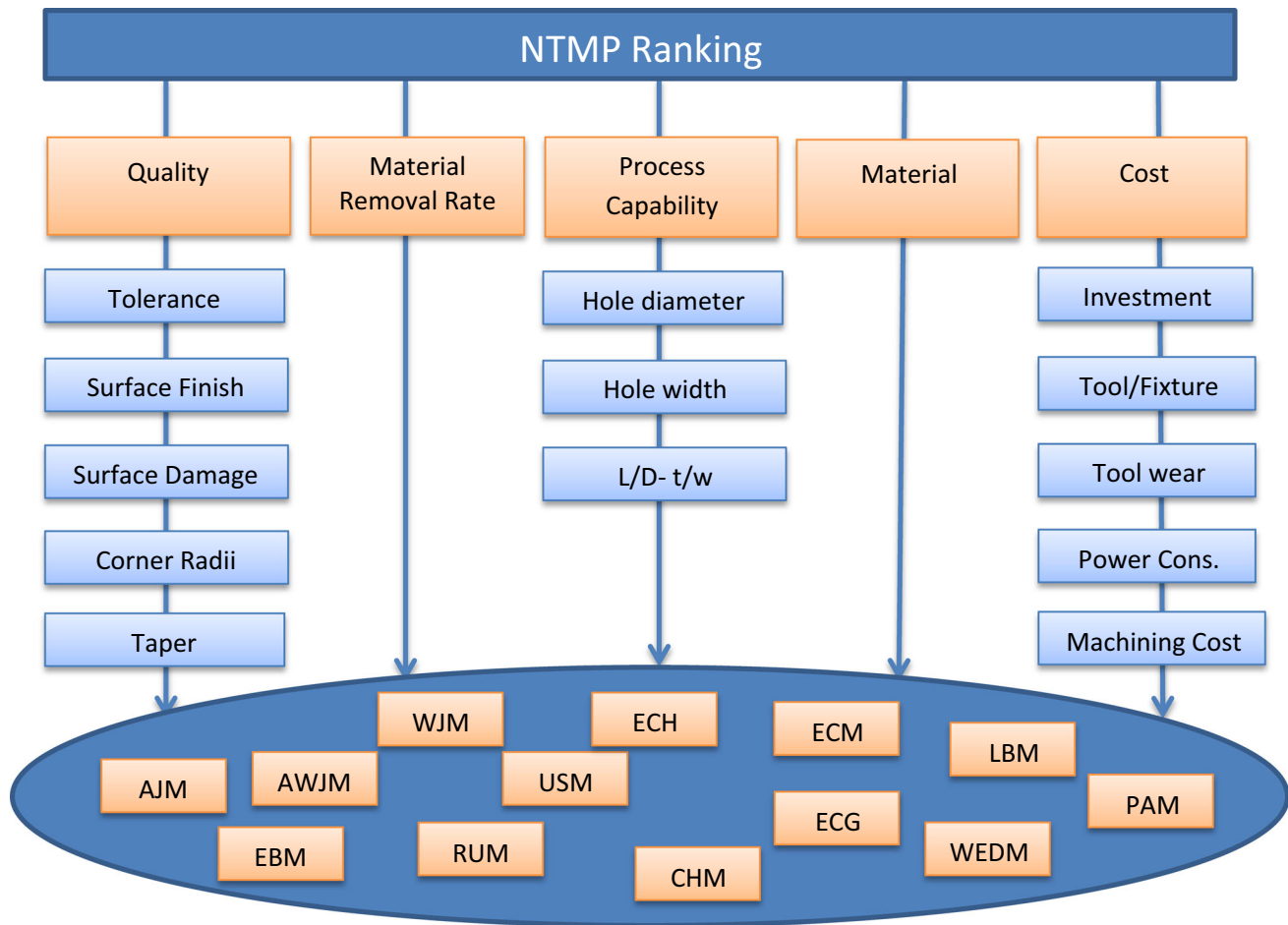


Fig. 1 Fuzzy ranking model hierarchy

presented in “Appendix”. The application of triangular fuzzy ranking model in “Appendix” shows that ECM is the highest ranked NTMP alternative and recommended for the case study.

2.1 Illustration of the application of intuitionistic fuzzy ranking model

Intuitionistic fuzzy sets (IFS), introduced by Atanassov (1986) as an extended version of fuzzy sets developed by Zadeh (1965), can deal with vagueness and uncertainty and reflect the degree of hesitancy of information in a decision-making process. Therefore, IFS can model uncertainty in more detail than conventional fuzzy sets.

The definition of an intuitionistic fuzzy set starts with its formulation as a fuzzy set A (Eq. 1) (Franceschini and Galetto 2001; Chen and Tsao 2008). In the formulation, X is a universe of discourse.

$$A = \{x, \mu_A(x) | x \in X\}, \quad (1)$$

where $\mu_A(x)$ is the membership function and indicates the degree of membership of element x to set A . The degree of

nonmembership is $\nu_A(x)$. IFS are characterized by two functions, a membership function and a nonmembership function. They seem to be useful when modeling many real-life cases such as negotiation processes. IFS are expressed as an extended version of fuzzy set A as follows.

$$A = \{x, \mu_A(x), \nu_A(x) | x \in X\}. \quad (2)$$

IFS A is defined by membership function $\mu_A : X \rightarrow [0, 1]$ and nonmembership function $\nu_A : X \rightarrow [0, 1]$, where $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.

Table 2 Conversion of 1–9 Scale intuitionistic fuzzy numbers

1–9 scale	IFNs
1	(0.10, 0.90, 0.00)
2	(0.10, 0.75, 0.15)
3	(0.25, 0.60, 0.15)
4	(0.40, 0.50, 0.10)
5	(0.50, 0.40, 0.10)
6	(0.60, 0.30, 0.10)
7	(0.70, 0.20, 0.10)
8	(0.80, 0.10, 0.10)
9	(0.90, 0.10, 0.00)

Table 3 Intuitionistic fuzzy numbers and their correspondence

Linguistic correspondence	Interval description	Interval	Intuitionistic fuzzy numbers
Very very low (VVL)	(0, minimum]	(0.00, 0.011]	(0.10, 0.90, 0.00)
Low (L)	(minimum, Q1]	(0.011, 0.023]	(0.25, 0.60, 0.15)
Medium (M)	(Q1, median]	(0.023, 0.053]	(0.50, 0.40, 0.10)
High (H)	(median, Q3]	(0.053, 0.082]	(0.70, 0.20, 0.10)
Very very high (VVH)	(Q3, maximum]	(0.082, 0.24]	(0.90, 0.10, 0.00)

For each IFS A in X , $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$, $\forall x \in X$ where $\pi_A(x)$ is defined as the degree of nondeterminacy or hesitation of x to A , $0 \leq \pi_A(x) \leq 1$ (Franceschini and Galetto 2001). If the value of $\pi_A(x)$ increases, the degree of accuracy of information related to x decreases.

An intuitionistic fuzzy number (IFN) is indicated as $\tilde{A} = (\mu_{\tilde{A}}, v_{\tilde{A}}, \pi_{\tilde{A}})$. Let $\tilde{A} = (\mu_{\tilde{A}}, v_{\tilde{A}}, \pi_{\tilde{A}})$ and $\tilde{B} = (\mu_{\tilde{B}}, v_{\tilde{B}}, \pi_{\tilde{B}})$ be two IFNs, $\lambda \in [0, 1]$ and some basic operations are given as follows (Atanassov 1986).

$$\tilde{A} \oplus \tilde{B} = ((\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}} \times \mu_{\tilde{B}}), v_{\tilde{A}} \times v_{\tilde{B}}), \quad (3)$$

$$\tilde{A} \otimes \tilde{B} = (\mu_{\tilde{A}} \times \mu_{\tilde{B}}, (v_{\tilde{A}} + v_{\tilde{B}} - v_{\tilde{A}} \times v_{\tilde{B}})), \quad (4)$$

$$\lambda \tilde{A} = (1 - (1 - \mu_{\tilde{A}})^\lambda, v_{\tilde{A}}^\lambda); \quad \lambda > 0, \quad (5)$$

$$\tilde{A}^\lambda = (\mu_{\tilde{A}}^\lambda, 1 - (1 - v_{\tilde{A}})^\lambda); \quad \lambda > 0. \quad (6)$$

Application of intuitionistic fuzzy ranking model starts with construction of intuitionistic fuzzy decision matrix.

Step 1 Construction of the intuitionistic fuzzy decision matrix

Let R be an intuitionistic fuzzy decision matrix. R is constructed by using Tables 2, 3. In Tables 2, 3, $A = \{A_1, A_2, \dots, A_m\}$ is a set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ represents a set of criteria. Then, intuitionistic fuzzy decision matrix can be defined as Eq. (7).

$$R = \begin{bmatrix} (\mu_{A_1}(C_1), v_{A_1}(C_1), \pi_{A_1}(C_1)) & (\mu_{A_1}(C_2), v_{A_1}(C_2), \pi_{A_1}(C_2)) & \dots & (\mu_{A_1}(C_n), v_{A_1}(C_n), \pi_{A_1}(C_n)) \\ (\mu_{A_2}(C_1), v_{A_2}(C_1), \pi_{A_2}(C_1)) & (\mu_{A_2}(C_2), v_{A_2}(C_2), \pi_{A_2}(C_2)) & \dots & (\mu_{A_2}(C_n), v_{A_2}(C_n), \pi_{A_2}(C_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_n}(C_1), v_{A_n}(C_1), \pi_{A_n}(C_1)) & (\mu_{A_n}(C_2), v_{A_n}(C_2), \pi_{A_n}(C_2)) & \dots & (\mu_{A_n}(C_n), v_{A_n}(C_n), \pi_{A_n}(C_n)) \end{bmatrix} \quad (7)$$

For the case problem, the intuitionistic fuzzy decision matrix (Table 5) is obtained from Table 4. Table 4 is converted to Table 5. In conversion of Table 4, Tables 2 and 3 are used for continuous criteria, such as ‘tolerance’ and ‘surface roughness,’ and discrete (or integer) criteria, such as ‘cost’ and ‘material steel,’ respectively.

Step 2 Determination of the intuitionistic fuzzy weights of criteria

The set of importance weights of the criteria, $\tilde{W} = [w_1, w_2, \dots, w_n]$, can be calculated using Eq. 8. The importance weights calculated for the case problem are provided in Table 6.

$$\tilde{W} = \left[(\mu_{w_1}(C_1), v_{w_1}(C_1), \pi_{w_1}(C_1)), (\mu_{w_2}(C_2), v_{w_2}(C_2), \pi_{w_2}(C_2)), \dots, \right. \\ \left. (\mu_{w_n}(C_n), v_{w_n}(C_n), \pi_{w_n}(C_n)) \right]. \quad (8)$$

Step 3 Construct aggregated weighted intuitionistic fuzzy decision matrix

The aggregated intuitionistic fuzzy decision matrix is calculated by using Eqs. (9), (10). For the case problem, the calculated aggregated weighted intuitionistic fuzzy decision matrix is provided in Table 7.

$$R' = R \otimes \tilde{W}, \quad (9)$$

$$R' = \begin{bmatrix} (\mu_{A_1 w_1}(C_1), v_{A_1 w_1}(C_1), \pi_{A_1 w_1}(C_1)) & (\mu_{A_1 w_2}(C_2), v_{A_1 w_2}(C_2), \pi_{A_1 w_2}(C_2)) & \dots & (\mu_{A_1 w_n}(C_n), v_{A_1 w_n}(C_n), \pi_{A_1 w_n}(C_n)) \\ (\mu_{A_2 w_1}(C_1), v_{A_2 w_1}(C_1), \pi_{A_2 w_1}(C_1)) & (\mu_{A_2 w_2}(C_2), v_{A_2 w_2}(C_2), \pi_{A_2 w_2}(C_2)) & \dots & (\mu_{A_2 w_n}(C_n), v_{A_2 w_n}(C_n), \pi_{A_2 w_n}(C_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_n w_1}(C_1), v_{A_n w_1}(C_1), \pi_{A_n w_1}(C_1)) & (\mu_{A_n w_2}(C_2), v_{A_n w_2}(C_2), \pi_{A_n w_2}(C_2)) & \dots & (\mu_{A_n w_n}(C_n), v_{A_n w_n}(C_n), \pi_{A_n w_n}(C_n)) \end{bmatrix}. \quad (10)$$

Table 4 Decision matrix (Yurdakul and Coğun 2003)

Criteria	Quality																	
Sub-criteria	<i>T</i> (mm)			SF (Ra)			SD			CR			TP					
AJM	0.030	0.050	0.120	0.250	0.600	1.250	0.020	0.025	0.030	0.150	0.200	0.250	0.004	0.005	0.006			
AWJM	0.026	0.040	0.060	0.200	0.300	0.400	0.020	0.025	0.030	0.400	0.500	0.600	0.003	0.003	0.004			
RUSM	0.008	0.012	0.022	0.250	0.500	0.750	0.025	0.025	0.030	0.070	0.080	0.090	0.003	0.004	0.005			
ECM	0.025	0.050	0.100	0.200	1.000	1.500	0.044	0.005	0.006	0.150	0.200	0.250	0.001	0.002	0.003			
EDM	0.010	0.025	0.100	0.300	2.000	3.000	0.010	0.020	0.030	0.300	0.400	0.500	0.001	0.002	0.003			
EBM	0.010	0.020	0.030	1.000	3.000	4.000	0.025	0.025	0.030	0.150	0.250	0.350	0.015	0.020	0.025			
LBM	0.010	0.020	0.030	0.400	1.000	1.500	0.050	0.100	0.150	0.400	0.500	0.600	0.40	0.050	0.060			
Criteria	Cost																	
Sub-criteria	IC		<i>T/F</i>		TW		PW		MC									
AJM	1	2	3	2	3	4	5	6	7	1	2	3	3	4	5			
AWJM	3	4	5	2	3	4	5	6	7	1	3	4	2	3	4			
RUSM	3	4	5	6	7	8	6	7	8	2	3	4	5	6	7			
ECM	8	9	10	7	8	9	2	3	4	8	9	10	6	7	8			
EDM	7	8	9	8	9	10	8	9	10	2	3	4	2	3	4			
EBM	8	9	10	1	2	3	1	2	3	2	3	4	7	8	9			
LBM	7	8	9	1	2	3	1	2	3	1	2	3	7	8	9			
Criteria	Process capability																	
Sub-criteria	HD		HW		<i>L/D-t/w</i>													
AJM	0.140		0.150		0.160		0.090		0.100		0.150		8.000		10.000		12.000	
AWJM	0.140		0.150		0.160		0.140		0.150		0.160		25.000		30.000		35.000	
RUSM	0.040		0.050		0.060		0.040		0.050		0.060		160.000		180.000		200.000	
ECM	0.400		0.500		0.600		0.090		0.100		0.110		25.000		30.000		35.000	
EDM	0.140		0.150		0.160		0.040		0.050		0.060		25.000		30.000		35.000	
EBM	0.035		0.040		0.100		0.035		0.040		0.100		12.000		15.000		18.000	
LBM	0.040		0.050		0.060		0.090		0.100		0.110		17.000		20.000		23.000	
Criteria	Material removal rate mm ³ /min																	
Sub-criteria																		
AJM	20.000					50.000					200.000							
AWJM	300.000					600.000					2000.000							
RUSM	400.000					800.000					2400.000							
ECM	500.000					2000.00					14,000.000							
EDM	100.000					800.00					1300.00							
EBM	0.300					2.000					6.000							
LBM	0.100					2.000					5.000							
Criteria	Material steel																	
Sub-criteria																		
AJM	5					6					7							
AWJM	5					6					7							
RUSM	5					6					7							
ECM	8					9					10							
EDM	8					9					10							
EBM	5					6					7							
LBM	5					6					7							

Table 5 Intuitionistic fuzzy decision matrix for the case problem

A_i	Quality															
	$C_1(T)$				$C_2(SF)$				$C_3(SD)$				$C_4(CR)$			
	$\mu_{A_i}(C_1)$		$\pi_{A_i}(C_1)$		$\mu_{A_i}(C_4)$		$\pi_{A_i}(C_2)$		$\mu_{A_i}(C_3)$		$\pi_{A_i}(C_3)$		$\mu_{A_i}(C_4)$		$\pi_{A_i}(C_4)$	
	$\nu_{A_i}(C_1)$	$\nu_{A_i}(C_1)$	$\nu_{A_i}(C_1)$	$\nu_{A_i}(C_1)$	$\mu_{A_i}(C_4)$	$\mu_{A_i}(C_4)$	$\pi_{A_i}(C_2)$	$\pi_{A_i}(C_2)$	$\mu_{A_i}(C_3)$	$\mu_{A_i}(C_3)$	$\pi_{A_i}(C_3)$	$\pi_{A_i}(C_3)$	$\mu_{A_i}(C_4)$	$\mu_{A_i}(C_4)$	$\pi_{A_i}(C_4)$	$\pi_{A_i}(C_4)$
AIM	0.70	0.20	0.10	0.10	0.50	0.40	0.10	0.10	0.50	0.40	0.10	0.10	0.25	0.60	0.15	0.10
AWJM	0.70	0.20	0.10	0.10	0.10	0.90	0.00	0.00	0.50	0.40	0.10	0.10	0.70	0.20	0.10	0.15
RUSM	0.1	0.9	0.00	0.00	0.25	0.60	0.15	0.15	0.50	0.40	0.10	0.10	0.10	0.90	0.00	0.15
ECM	0.70	0.20	0.10	0.10	0.50	0.40	0.10	0.10	0.10	0.90	0.00	0.00	0.25	0.60	0.15	0.00
EDM	0.50	0.40	0.10	0.10	0.70	0.20	0.10	0.10	0.25	0.60	0.15	0.15	0.70	0.20	0.10	0.00
EBM	0.25	0.60	0.15	0.15	0.90	0.10	0.00	0.00	0.50	0.40	0.10	0.10	0.50	0.40	0.10	0.10
LBM	0.25	0.60	0.15	0.15	0.50	0.40	0.10	0.10	0.70	0.20	0.10	0.10	0.70	0.20	0.10	0.10
Cost																
A_i	$C_7(t/f)$								$C_8(tw)$							
	$\mu_{A_i}(C_6)$				$\mu_{A_i}(C_7)$				$\mu_{A_i}(C_8)$				$\mu_{A_i}(C_9)$			
	$\nu_{A_i}(C_6)$		$\pi_{A_i}(C_6)$		$\nu_{A_i}(C_7)$		$\pi_{A_i}(C_7)$		$\nu_{A_i}(C_8)$		$\pi_{A_i}(C_8)$		$\nu_{A_i}(C_9)$		$\pi_{A_i}(C_9)$	
	$\nu_{A_i}(C_6)$	$\nu_{A_i}(C_6)$	$\nu_{A_i}(C_6)$	$\nu_{A_i}(C_6)$	$\mu_{A_i}(C_7)$	$\mu_{A_i}(C_7)$	$\pi_{A_i}(C_7)$	$\pi_{A_i}(C_7)$	$\mu_{A_i}(C_8)$	$\mu_{A_i}(C_8)$	$\pi_{A_i}(C_8)$	$\pi_{A_i}(C_8)$	$\mu_{A_i}(C_9)$	$\mu_{A_i}(C_9)$	$\pi_{A_i}(C_9)$	$\pi_{A_i}(C_9)$
AIM	0.10	0.75	0.15	0.15	0.25	0.60	0.15	0.15	0.60	0.30	0.10	0.10	0.10	0.75	0.15	0.10
AWJM	0.25	0.60	0.15	0.15	0.25	0.60	0.15	0.15	0.60	0.30	0.10	0.10	0.25	0.60	0.15	0.15
RUSM	0.25	0.60	0.15	0.15	0.70	0.20	0.10	0.10	0.70	0.20	0.10	0.10	0.25	0.60	0.15	0.10
ECM	0.90	0.10	0.00	0.00	0.80	0.10	0.10	0.10	0.25	0.60	0.15	0.15	0.90	0.10	0.00	0.10
EDM	0.80	0.10	0.10	0.10	0.90	0.10	0.00	0.00	0.90	0.10	0.00	0.00	0.25	0.60	0.15	0.15
EBM	0.90	0.10	0.00	0.00	0.10	0.75	0.15	0.15	0.10	0.75	0.15	0.15	0.25	0.60	0.10	0.10
LBM	0.80	0.10	0.10	0.10	0.10	0.75	0.15	0.15	0.10	0.75	0.15	0.15	0.10	0.75	0.15	0.10
Process capability																
A_i	$C_{11}(T)$								$C_{12}(SF)$							
	$\mu_{A_i}(C_{11})$				$\mu_{A_i}(C_{11})$				$\mu_{A_i}(C_{12})$				$\mu_{A_i}(C_{13})$			
	$\nu_{A_i}(C_{11})$		$\pi_{A_i}(C_{11})$		$\nu_{A_i}(C_{11})$		$\pi_{A_i}(C_{11})$		$\nu_{A_i}(C_{12})$		$\pi_{A_i}(C_{12})$		$\nu_{A_i}(C_{13})$		$\pi_{A_i}(C_{13})$	
	$\nu_{A_i}(C_{11})$	$\nu_{A_i}(C_{11})$	$\nu_{A_i}(C_{11})$	$\nu_{A_i}(C_{11})$	$\mu_{A_i}(C_{11})$	$\mu_{A_i}(C_{11})$	$\pi_{A_i}(C_{11})$	$\pi_{A_i}(C_{11})$	$\mu_{A_i}(C_{12})$	$\mu_{A_i}(C_{12})$	$\pi_{A_i}(C_{12})$	$\pi_{A_i}(C_{12})$	$\mu_{A_i}(C_{13})$	$\mu_{A_i}(C_{13})$	$\pi_{A_i}(C_{13})$	$\pi_{A_i}(C_{13})$
AIM	0.50	0.40	0.10	0.10	0.50	0.40	0.10	0.10	0.50	0.40	0.10	0.10	0.90	0.90	0.00	0.00
AWJM	0.50	0.40	0.10	0.10	0.70	0.20	0.10	0.10	0.70	0.20	0.10	0.10	0.40	0.40	0.10	0.10
RUSM	0.25	0.60	0.15	0.15	0.25	0.60	0.15	0.15	0.25	0.60	0.15	0.15	0.20	0.20	0.10	0.10
ECM	0.70	0.20	0.10	0.10	0.50	0.40	0.10	0.10	0.50	0.40	0.10	0.10	0.40	0.40	0.10	0.10
EDM	0.50	0.40	0.10	0.10	0.25	0.60	0.15	0.15	0.25	0.60	0.15	0.15	0.40	0.40	0.10	0.10
EBM	0.10	0.90	0.00	0.00	0.10	0.90	0.00	0.00	0.10	0.90	0.00	0.00	0.60	0.60	0.15	0.15
LBM	0.25	0.60	0.15	0.15	0.50	0.40	0.10	0.10	0.50	0.40	0.10	0.10	0.40	0.40	0.10	0.10

Table 5 continued

A_i	Material removal rate		Material steel	
	$C_{14}(\text{MRR})$		$C_{15}(\text{MS})$	
	$\mu_{A_i}(C_{14})$	$\nu_{A_i}(C_{14})$	$\mu_{A_i}(C_{15})$	$\pi_{A_i}(C_{15})$
AIM	0.25	0.60	0.60	0.30
AWJM	0.25	0.60	0.60	0.30
RUSM	0.50	0.40	0.60	0.30
ECM	0.70	0.20	0.90	0.10
EDM	0.50	0.40	0.90	0.10
EBM	0.10	0.90	0.60	0.30
LBM	0.10	0.90	0.60	0.30

Step 4 Determine intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution

Let J_B be a set of benefits criteria and J_C be a set of cost criteria. Intuitionistic fuzzy positive ideal solution A^* and intuitionistic fuzzy negative ideal solution A^- are obtained as

$$\begin{aligned} A^* &= (\mu_{A^*w}(C_j), \nu_{A^*w}(C_j), \pi_{A^*w}(C_j)) \quad \text{and} \\ A^- &= (\mu_{A^-w}(C_j), \nu_{A^-w}(C_j), \pi_{A^-w}(C_j)) \end{aligned} \quad (11)$$

(see Table 8), where

$$\mu_{A^*w}(C_j) = \left(\max_i \mu_{A_iw}(C_j) \mid j \in J_B \right), \left(\min_i \mu_{A_iw}(C_j) \mid j \in J_C \right), \quad (12)$$

$$\nu_{A^*w}(C_j) = \left(\min_i \nu_{A_iw}(C_j) \mid j \in J_B \right), \left(\max_i \nu_{A_iw}(C_j) \mid j \in J_C \right), \quad (13)$$

$$(\mu_{A^-w}(C_j) = \left(\min_i \mu_{A_iw}(C_j) \mid j \in J_B \right), \left(\max_i \mu_{A_iw}(C_j) \mid j \in J_C \right), \quad (14)$$

$$\nu_{A^-w}(C_j) = \left(\max_i \nu_{A_iw}(C_j) \mid j \in J_B \right), \left(\min_i \nu_{A_iw}(C_j) \mid j \in J_C \right), \quad (15)$$

$$\pi_{A^*w}(C_j) = 1 - \mu_{A^*w}(C_j) - \nu_{A^*w}(C_j), \quad (16)$$

$$\pi_{A^-w}(C_j) = 1 - \mu_{A^-w}(C_j) - \nu_{A^-w}(C_j). \quad (17)$$

Step 5 Calculate the alternatives' distances from intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution

The alternatives' distances from the intuitionistic fuzzy positive ideal solution (S_i^*) and the intuitionistic fuzzy negative ideal solution (S_i^-) on intuitionistic fuzzy sets can be calculated according to Hamming distance proposed by Atanassov (1986) by using Eqs. (18) and (19), respectively.

$$\begin{aligned} S_i^* &= \frac{1}{2} \sum_{j=1}^n [|\mu_{A_iw}(C_j) - \mu_{A^*w}(C_j)| + |\nu_{A_iw}(C_j) \\ &\quad - \nu_{A^*w}(C_j)| + |\pi_{A_iw}(C_j) - \pi_{A^*w}(C_j)|], \end{aligned} \quad (18)$$

$$\begin{aligned} S_i^- &= \frac{1}{2} \sum_{j=1}^n [|\mu_{A_iw}(C_j) - \mu_{A^-w}(C_j)| + |\nu_{A_iw}(C_j) \\ &\quad - \nu_{A^-w}(C_j)| + |\pi_{A_iw}(C_j) - \pi_{A^-w}(C_j)|], \end{aligned} \quad (19)$$

$$i = 1, 2, \dots, m.$$

Step 6 Calculate the relative closeness coefficients (C_i^*) of alternatives

Table 6 Intuitionistic fuzzy weights of criteria ($\tilde{W}$)

Criteria	Weights ($\tilde{W}$)		
	$\mu_{w_j}(C_j)$	$\nu_{w_j}(C_j)$	$\pi_{w_j}(C_j)$
$C_1(T)$	0.70	0.20	0.10
$C_2(SF)$	0.70	0.20	0.10
$C_3(SD)$	0.50	0.40	0.10
$C_4(CR)$	0.25	0.60	0.15
$C_5(TP)$	0.50	0.40	0.10
$C_6(IC)$	0.90	0.10	0.00
$C_7(T/F)$	0.70	0.20	0.10
$C_8(TW)$	0.70	0.20	0.10
$C_9(PW)$	0.50	0.40	0.10
$C_{10}(MC)$	0.25	0.60	0.15
$C_{11}(HD)$	0.50	0.40	0.10
$C_{12}(HW)$	0.25	0.60	0.15
$C_{13}(L/D-t/w)$	0.10	0.90	0.00
$C_{14}(MRR)$	0.90	0.10	0.00
$C_{15}(MS)$	0.90	0.10	0.00

Finally, relative closeness coefficients (ranking scores) can be calculated by using Eq. (20). The calculated alternatives' ranking scores are presented in Table 9. Among the seven alternatives, RUSM is the highest ranked alternative and recommended for the application described.

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \quad (20)$$

where $0 \leq C_i^* \leq 1$. Alternatives are ranked and selected according to descending order of C_i^* s.

For the case problem, the ranking scores and results of the proposed triangular and intuitionistic fuzzy ranking models and the crisp ranking model presented in Yurdakul and Cogun (2003) (crisp model) are presented in Figs. 2, 3. Analysis of the figures shows that all three models result in significantly different ranking scores and ranking results for each alternative.

The significance of the differences among the ranking results can be measured statistically by using Spearman's rank correlation test. Spearman's rank correlation test is explained and applied in the following section.

2.2 Spearman rank correlation test

Assume K is the number of alternatives to be compared and the alternatives are ranked according to two separate variables (X and Y). If the rankings are given as $X_1, X_2, \dots, X_K$ and $Y_1, Y_2, \dots, Y_K$, a measure of rank correlation can be used to obtain the relation between the X 's and

Y 's. It can be said that the correlation between the $X_1, X_2, \dots, X_K$ and $Y_1, Y_2, \dots, Y_K$ would be perfect if and only if $X_i = Y_i$ for all i 's. Therefore, it would seem logical to define the difference between X_i and Y_i as an indication of the disparity between two sets of rankings (Siegel and Castellan 1956):

$$d_i = X_i - Y_i \quad (21)$$

If $x = X - \bar{X}$ where $\bar{X}$ is the mean of the scores on the X variable, and if $y = Y - \bar{Y}$, then a general expression for a correlation coefficient is:

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} = r_s. \quad (22)$$

Now, when the X 's and Y 's are rankings, $r = r_s$ and the sum of the K integers $1, 2, \dots, K$ is $\sum X = \frac{K(K+1)}{2}$ and the sum of their squares can be computed by $\sum X^2 = \frac{K(K+1)(2K+1)}{6}$.

Therefore,

$$\begin{aligned} \sum x^2 &= \sum (X - \bar{X})^2 = \sum X^2 - \frac{(\sum X)^2}{K} \quad \text{and} \\ \sum x^2 &= \frac{K(K+1)(2K+1)}{6} - \frac{\left(\frac{K(K+1)}{2}\right)^2}{K} = \frac{K^3 - K}{12} \end{aligned} \quad (23)$$

Similarly,

$$\sum y^2 = \frac{K^3 - K}{12}. \quad (24)$$

On the other hand,

$$d = x - y, \quad (25)$$

$$\begin{aligned} d^2 &= (x - y)^2 = x^2 - 2xy + y^2 \quad \text{and} \\ \sum d^2 &= \sum x^2 + \sum y^2 - 2 \sum xy. \end{aligned} \quad (26)$$

In Eq. (25), d represents the ranking difference of the feasible NTMP. Then, correlation coefficient can be rewritten as follows:

$$\begin{aligned} r_s &= \frac{\sum x^2 + \sum y^2 - \sum d^2}{2\sqrt{\sum x^2 \sum y^2}} = \frac{\frac{K^3-K}{12} + \frac{K^3-K}{12} - \sum d^2}{2\sqrt{\left(\frac{K^3-K}{12}\right)\left(\frac{K^3-K}{12}\right)}} \\ &= 1 - \frac{\sum d^2}{\frac{K^3-K}{6}} = 1 - \frac{6 \sum d^2}{K^3 - K}. \end{aligned} \quad (27)$$

Finally, in Spearman's test, the similarity of the rankings can be evaluated with two separate test statistics, r_s and Z , which are calculated with Eqs. (27), (28) (Parkan and Wu 1999; Glasser and Winter 1961);

$$Z = r_s \sqrt{K - 1}. \quad (28)$$

The test statistic r_s is a relative measure that varies from $+1$, implying a perfect positive relationship between the

Table 7 Aggregated weighted intuitionistic fuzzy decision matrix

A_i	Quality													
	$C_1(T)$		$C_2(SF)$		$C_3(SD)$		$C_4(CR)$		$C_5(TP)$		$C_6(IC)$		$C_7(T/F)$	
	$\mu_{A_i}(C_1)$	$\nu_{A_i}(C_1)$	$\pi_{A_i}(C_1)$	$\mu_{A_i}(C_4)$	$\nu_{A_i}(C_2)$	$\pi_{A_i}(C_2)$	$\mu_{A_i}(C_3)$	$\nu_{A_i}(C_3)$	$\pi_{A_i}(C_3)$	$\mu_{A_i}(C_4)$	$\nu_{A_i}(C_4)$	$\pi_{A_i}(C_4)$	$\mu_{A_i}(C_5)$	$\nu_{A_i}(C_5)$
AIM	0.49	0.36	0.15	0.35	0.52	0.13	0.25	0.64	0.11	0.0625	0.84	0.0975	0.25	0.64
AWJM	0.49	0.36	0.15	0.07	0.92	0.01	0.25	0.64	0.11	0.175	0.68	0.145	0.125	0.76
RUSM	0.07	0.92	0.01	0.175	0.68	0.145	0.25	0.64	0.11	0.025	0.96	0.015	0.125	0.76
ECM	0.49	0.36	0.15	0.35	0.52	0.13	0.05	0.94	0.01	0.0625	0.84	0.0975	0.05	0.94
EDM	0.35	0.52	0.13	0.49	0.36	0.15	0.125	0.76	0.115	0.175	0.68	0.145	0.05	0.94
EBM	0.175	0.68	0.145	0.63	0.28	0.09	0.25	0.64	0.11	0.125	0.76	0.115	0.25	0.64
LBM	0.175	0.68	0.145	0.35	0.52	0.13	0.35	0.52	0.13	0.175	0.68	0.145	0.35	0.52
Cost														
A_i	$C_6(IC)$		$C_7(T/F)$		$C_8(TW)$		$C_9(PW)$		$C_{10}(MC)$		$C_{11}(T)$		$C_{12}(SF)$	
	$\mu_{A_i}(C_6)$	$\nu_{A_i}(C_6)$	$\pi_{A_i}(C_6)$	$\mu_{A_i}(C_7)$	$\nu_{A_i}(C_7)$	$\pi_{A_i}(C_7)$	$\mu_{A_i}(C_8)$	$\nu_{A_i}(C_8)$	$\pi_{A_i}(C_8)$	$\mu_{A_i}(C_9)$	$\nu_{A_i}(C_9)$	$\pi_{A_i}(C_9)$	$\mu_{A_i}(C_{10})$	$\nu_{A_i}(C_{10})$
	$\mu_{A_i}(C_{11})$	$\nu_{A_i}(C_{11})$	$\pi_{A_i}(C_{11})$	$\mu_{A_i}(C_{12})$	$\nu_{A_i}(C_{12})$	$\pi_{A_i}(C_{12})$	$\mu_{A_i}(C_{13})$	$\nu_{A_i}(C_{13})$	$\pi_{A_i}(C_{13})$	$\mu_{A_i}(C_{14})$	$\nu_{A_i}(C_{14})$	$\pi_{A_i}(C_{14})$	$\mu_{A_i}(C_{15})$	$\nu_{A_i}(C_{15})$
AIM	0.07	0.8	0.13	0.175	0.68	0.145	0.3	0.58	0.12	0.025	0.9	0.075	0.2	0.7
AWJM	0.175	0.68	0.145	0.175	0.68	0.145	0.3	0.58	0.12	0.0625	0.84	0.0975	0.125	0.76
RUSM	0.175	0.68	0.145	0.49	0.36	0.15	0.35	0.52	0.13	0.0625	0.84	0.0975	0.3	0.58
ECM	0.63	0.28	0.09	0.56	0.28	0.16	0.125	0.76	0.115	0.225	0.64	0.135	0.35	0.52
EDM	0.56	0.28	0.16	0.63	0.28	0.09	0.45	0.46	0.09	0.0625	0.84	0.0975	0.125	0.76
EBM	0.63	0.28	0.09	0.07	0.8	0.13	0.05	0.85	0.1	0.0625	0.84	0.0975	0.4	0.46
LBM	0.56	0.28	0.16	0.07	0.8	0.13	0.05	0.85	0.1	0.025	0.9	0.075	0.4	0.46
Process capability														
A_i	$C_{11}(T)$		$C_{12}(SF)$		$C_{13}(SD)$		$C_{14}(MRR)$		$C_{15}(MS)$		$C_{16}(MRR)$		$C_{17}(MS)$	
	$\mu_{A_i}(C_{11})$	$\nu_{A_i}(C_{11})$	$\pi_{A_i}(C_{11})$	$\mu_{A_i}(C_{12})$	$\nu_{A_i}(C_{12})$	$\pi_{A_i}(C_{12})$	$\mu_{A_i}(C_{13})$	$\nu_{A_i}(C_{13})$	$\pi_{A_i}(C_{13})$	$\mu_{A_i}(C_{14})$	$\nu_{A_i}(C_{14})$	$\pi_{A_i}(C_{14})$	$\mu_{A_i}(C_{15})$	$\nu_{A_i}(C_{15})$
	$\mu_{A_i}(C_{16})$	$\nu_{A_i}(C_{16})$	$\pi_{A_i}(C_{16})$	$\mu_{A_i}(C_{17})$	$\nu_{A_i}(C_{17})$	$\pi_{A_i}(C_{17})$	$\mu_{A_i}(C_{18})$	$\nu_{A_i}(C_{18})$	$\pi_{A_i}(C_{18})$	$\mu_{A_i}(C_{19})$	$\nu_{A_i}(C_{19})$	$\pi_{A_i}(C_{19})$	$\mu_{A_i}(C_{20})$	$\nu_{A_i}(C_{20})$
AIM	0.35	0.52	0.13	0.35	0.52	0.13	0.05	0.94	0.01	0.175	0.68	0.145	0.42	0.44
AWJM	0.35	0.52	0.13	0.49	0.36	0.15	0.25	0.64	0.11	0.175	0.68	0.145	0.42	0.44
RUSM	0.175	0.68	0.145	0.175	0.68	0.145	0.35	0.52	0.13	0.35	0.52	0.13	0.42	0.44
ECM	0.49	0.36	0.15	0.35	0.52	0.13	0.25	0.64	0.11	0.49	0.36	0.15	0.63	0.28
EDM	0.35	0.52	0.13	0.175	0.68	0.145	0.25	0.64	0.11	0.35	0.52	0.13	0.63	0.28
EBM	0.07	0.92	0.01	0.07	0.92	0.01	0.125	0.76	0.115	0.07	0.92	0.01	0.42	0.44
LBM	0.175	0.68	0.145	0.35	0.52	0.13	0.25	0.64	0.11	0.07	0.92	0.01	0.42	0.44

Table 8 Intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution

Criteria	A^+			A^-		
	$\mu_{A^+w}(C_j)$	$\nu_{A^+w}(C_j)$	$\pi_{A^+w}(C_j)$	$\mu_{A^-w}(C_j)$	$\nu_{A^-w}(C_j)$	$\pi_{A^-w}(C_j)$
$C_1(T)$	0.07	0.92	0.01	0.49	0.36	0.15
$C_2(SF)$	0.07	0.92	0.01	0.63	0.28	0.09
$C_3(SD)$	0.05	0.94	0.01	0.35	0.52	0.13
$C_4(CR)$	0.025	0.96	0.015	0.175	0.68	0.145
$C_5(TP)$	0.05	0.94	0.01	0.35	0.52	0.13
$C_6(IC)$	0.07	0.8	0.13	0.63	0.28	0.09
$C_7(T/F)$	0.07	0.8	0.13	0.63	0.28	0.09
$C_8(TW)$	0.05	0.85	0.10	0.45	0.46	0.09
$C_9(PW)$	0.025	0.90	0.075	0.225	0.64	0.135
$C_{10}(MC)$	0.125	0.76	0.115	0.40	0.46	0.14
$C_{11}(T)$	0.07	0.92	0.01	0.49	0.36	0.15
$C_{12}(SF)$	0.07	0.92	0.01	0.49	0.36	0.15
$C_{13}(SD)$	0.05	0.94	0.01	0.35	0.52	0.13
$C_{14}(MRR)$	0.49	0.36	0.15	0.07	0.92	0.01
$C_{15}(MS)$	0.63	0.28	0.09	0.42	0.44	0.14

Table 9 Ranking scores for the alternative NTMPs

	S_i^+	S_i^-	C_i^*
AJM	3.920	3.325	0.459
AWJM	3.825	3.08	0.446
RUSM	3.145	3.71	0.541
ECM	4.155	2.785	0.401
EDM	4.205	2.74	0.395
EBM	3.815	3.16	0.453
LBM	4.435	2.49	0.36

two sets of rankings, to -1 , implying a perfect negative relationship between the two sets of rankings. The closer r_s is to $+$ or -1 , the stronger the relationship

between rankings (Parkan and Wu 1999). On the other hand, the test statistic Z is compared with a pre-determined level of significance α value. Generally, 1.645 which is the value that corresponds to the level of significance of $\alpha = 0.05$ is taken as the pre-determined value. Then, if the calculated Z value exceeds 1.645, it can be concluded that there is evidence of a positive relationship between the two sets of rankings (Parkan and Wu 1999). Otherwise, the two rankings can be accepted as dissimilar.

The results of the application of the Spearman rank correlation test for the case problem are provided in Table 10. In the last three rows of Table 10, the calculated Z values are given as 2.1, 1.050, and 0.787, respectively.

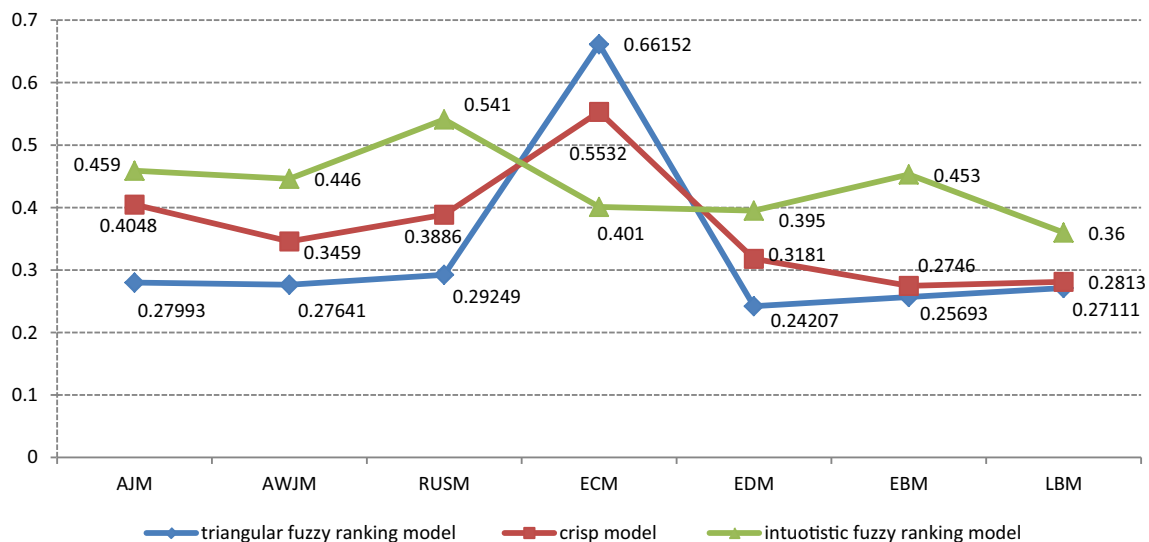


Fig. 2 Illustration of the ranking scores for the illustrative case study

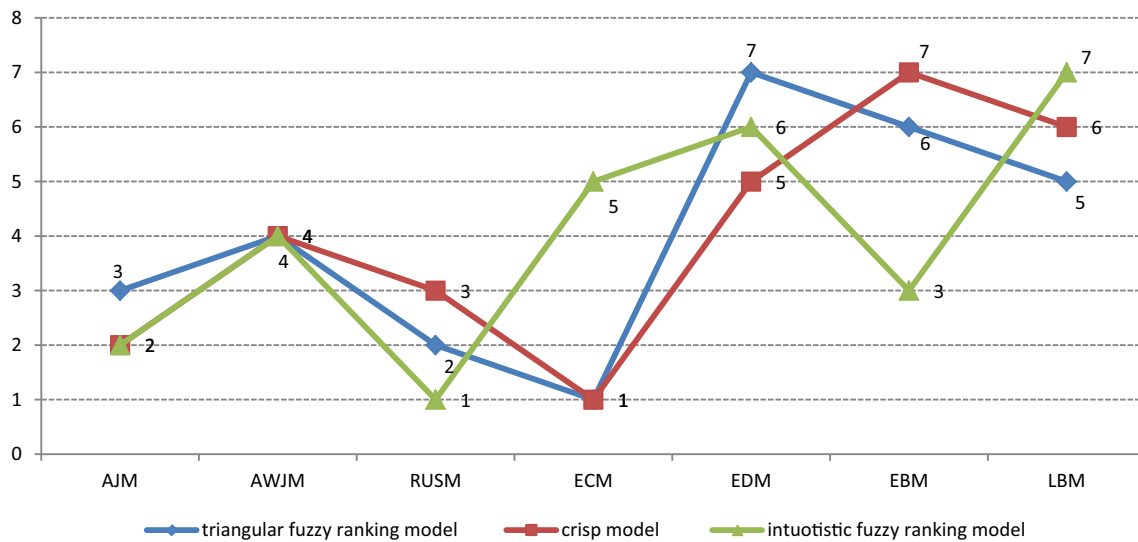


Fig. 3 Illustration of the ranking results for the illustrative case study

Table 10 Comparison of the ranking results for the illustrative case study

NTMP	Triangular fuzzy ranking model (I)	Ranking	The crisp model in Yurdakul and Cogun (2003) (II)	Ranking	Intuitionistic fuzzy ranking model (III)	Ranking	Spearman's rank correlation test		
							Ranking differences		
							I-II	I-III	II-III
AJM	0.27993	3	0.4048	2	0.459	2	1	1	0
AWJM	0.27641	4	0.3459	4	0.446	4	0	0	0
RUSM	0.29249	2	0.3886	3	0.541	1	- 1	1	2
ECM	0.66152	1	0.5532	1	0.401	5	0	- 4	- 4
EDM	0.24207	7	0.3181	5	0.395	6	2	1	- 1
EBM	0.25693	6	0.2746	7	0.453	3	- 1	3	4
LBM	0.27111	5	0.2813	6	0.36	7	- 1	- 2	- 1
r_s							0.857	0.429	0.321
Z							2.100	1.050	0.787

The calculated r_s values in the last two columns are not close to +1, and the calculated Z values are lower than 1.645. Both test statistics show that the rankings obtained by the intuitionistic fuzzy ranking model are significantly dissimilar from the rankings obtained with the other models.

3 Conclusions

It can be concluded that using the intuitionistic fuzzy ranking model instead of the crisp (nonfuzzy) and triangular fuzzy model in the NTMP ranking decision represents decision makers' preferences and relative importance values more accurately and provides more realistic ranking

results. The comparison of the rankings between the crisp and fuzzy models shows a statistically significant difference in the ranking orders. The authors recommend the preference of the intuitionistic fuzzy ranking model over the crisp and triangular fuzzy ones especially in situations where vagueness and imprecision exist in NTMP ranking decisions for NTMP applications.

4 Future scope

Further studies can use different uncertainty modeling techniques for process parameters, such as fuzzy approximate solution methods, i.e., kernel Hilbert space method (Abu Arqub et al. 2016, 2017; Abu Arqub 2017) for

modeling and solving the NTMP ranking problems. Their results can be statistically compared with the proposed fuzzy ranking models provided in this paper to determine whether the usage of the more complex and advanced fuzzy modeling approaches are justified.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Human and animal participants This article does not contain any studies with human participants or animals performed by any of the authors.

Appendix

Development and application of the triangular fuzzy ranking model (Chang 1996; Chen and Hwang 1992; Yurdakul and Ic 2005, 2009; Jafarian and Vahdat 2012):

Definition 1 If X is a collection of objects denoted generically by x , then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}. \quad (29)$$

where x denotes the element of the set X , $\mu_{\tilde{A}}(x)$ is called the membership function or the degree of membership of x in $\tilde{A}$ that maps X to the membership space $[0, 1]$.

Definition 2 If $\sup_x \mu_{\tilde{A}}(x) = 1$, the fuzzy set $\tilde{A}$ is called normal.

Definition 3 The membership function $\mu_{\tilde{C}}(x)$ of the intersection $\tilde{C} = \tilde{A} \cap \tilde{B}$ is defined by:

$$\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X. \quad (30)$$

Definition 4 Suppose $\tilde{A}$ is a triangular fuzzy number (a^p, a^m, a^o) that is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - [(a^m - x)/(a^m - a^p)], & a^p \leq x \leq a^m \\ 1 - (x - a^m)/(a^o - a^m), & a^m \leq x \leq a^o \\ 0, & \text{otherwise} \end{cases} \quad (31)$$

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ ve $\tilde{B} = (b_1, b_2, b_3, b_4)$;

$$\tilde{A} \oplus \tilde{B} = [a^p + b^p, a^m + b^m, a^o + b^o], \quad (32)$$

$$\tilde{A} \ominus \tilde{B} = [a^p - b^o, a^m - b^m, a^o - b^p], \quad (33)$$

$$\tilde{A} \otimes \tilde{B} = [a^p b^p, a^m b^m, a^o b^o], \quad (34)$$

$$\tilde{A} \oslash \tilde{B} = [a^p/b^o, a^m/b^m, a^o/b^p]. \quad (35)$$

Definition 5 Given two triangular fuzzy numbers $\tilde{A}, \tilde{B}$, the truth value of the fuzzy number $\tilde{A}$ is greater than or equal to the fuzzy number $\tilde{B}$ expressed as in the following:

$$V(\tilde{A} \geq \tilde{B}) = \sup\{\min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\}\}. \quad (36)$$

And can be equivalently expressed as follows:

$$V(\tilde{A} \geq \tilde{B}) = \mu(d) = \begin{cases} 1 & \text{if } a^m \geq b^p \\ 0 & \text{if } a^p \geq b^o \\ \frac{b^p - a^o}{(a^m - a^o) - (b^m - b^o)} & \text{otherwise} \end{cases}, \quad (37)$$

where d is ordinate of the highest intersection point d between $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(y)$ (Fig. 4).

Let $A = [\tilde{a}_{ij}]_{n \times n}$ be a preference matrix that $a_{ij} = (a_{ij}^p, a_{ij}^m, a_{ij}^o)$ is a triangular fuzzy number from Table 11. Then triangular fuzzy pair-wise comparison matrix (Table 12) is constructed by using triangular fuzzy numbers given in Table 11.

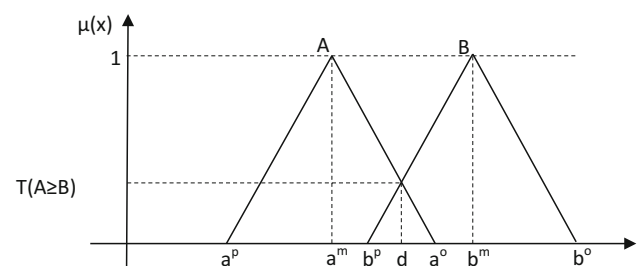


Fig. 4 The intersection between $\tilde{A}$ and $\tilde{B}$

Table 11 Fuzzy numbers and their linguistic correspondences

Linguistic correspondence (importance level)	Triangular fuzzy scale (a, b, c)	Reverse scale ($1/c, 1/b, 1/a$)
Equal	(1, 1, 1)	(1/1, 1/1, 1/1)
Medium	(1, 3, 5)	(1/5, 1/3, 1/1)
High	(3, 5, 7)	(1/7, 1/5, 1/3)
Very high	(5, 7, 9)	(1/9, 1/7, 1/5)
Essential	(7, 9, 9)	(1/9, 1/9, 1/7)

Table 12 An illustrative example for filling the four triangular fuzzy pair-wise comparison matrices

Quality	<i>T</i>	SF	SD	CR	TP
<i>T</i>	(1, 1, 1)	(1, 2.5, 4)	(1, 3, 5)	(2, 4, 6)	(1.5, 3.5, 5.5)
SF	(0.25, 0.4, 1)	(1, 1, 1)	(1, 3, 5)	(1.5, 3.5, 5.5)	(1, 3, 5)
SD	(0.2, 0.33, 1)	(0.2, 0.33, 1)	(1, 1, 1)	(1, 3, 5)	(1, 2, 3)
CR	(0.17, 0.25, 0.50)	(0.18, 0.29, 0.67)	(0.2, 0.33, 1)	(1, 1, 1)	(0.33, 0.5, 1)
TP	(0.18, 0.29, 0.67)	(0.2, 0.33, 0.67)	(0.33, 0.5, 1)	(1, 2, 3)	(1, 1, 1)
Cost	IC	<i>T/F</i>	TW	PW	MC
IC	(1, 1, 1)	(1, 3, 5)	(1, 3, 5)	(2, 4, 6)	(3, 5, 7)
<i>T/F</i>	(0.2, 0.33, 1)	(1, 1, 1)	(1, 2, 2)	(1, 3, 5)	(1.5, 3.5, 5.5)
TW	(0.2, 0.33, 1)	(0.5, 0.5, 1)	(1, 1, 1)	(1, 2, 3)	(1, 3, 5)
PW	(0.17, 0.25, 0.50)	(0.2, 0.33, 1)	(0.33, 0.5, 1)	(1, 1, 1)	(1, 3, 5)
MC	(0.14, 0.20, 0.33)	(0.18, 0.29, 0.67)	(0.2, 0.33, 1)	(0.2, 0.33, 1)	(1, 1, 1)
Process capability	HD		HW		L/D
HD	(1, 1, 1)		(1, 1.5, 2)		(1, 3, 5)
HW	(0.5, 0.67, 1)		(1, 1, 1)		(1, 3, 5)
<i>L/D</i>	(0.2, 0.33, 1)		(0.2, 0.33, 1)		(1, 1, 1)

Tolerance (*T*); surface finish (SF); surface damage (SD); corner radii (CR); taper (TP); investment cost (IC); tool/fixture (*T/F*); tool wear (TW); power consumption (PW); machining cost (MC); hole diameter (HD); hole width (HW); *L/D*–*t/w* ratio (*L/D*)

Step 1 The fuzzy synthetic range with respect to each (*i*th) criterion is calculated by using Eq. (38) and provided in Tables 13, 14.

$$\tilde{M}_i = \sum_{j=1}^n \tilde{a}_{ij} \otimes \left[\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1} \quad (38)$$

Step 2 The truth value of $\tilde{M}_i \geq \tilde{M}_j$ is defined by Eqs. (36) and (37). To compare $\tilde{M}_i$ and $\tilde{M}_j$, both the values of $V(\tilde{M}_i \geq \tilde{M}_j)$ and $V(\tilde{M}_i \leq \tilde{M}_j)$ are needed.

Step 3 The degree possibility for a convex fuzzy number $\tilde{M}_j$ to be greater than *n* convex fuzzy numbers $\tilde{M}_i$ ($i = 1, 2, \dots, n$; $i \neq j$) can be defined by

$$\begin{aligned} V(\tilde{M} \geq \tilde{M}_1, \tilde{M}_2, \dots, \tilde{M}_n) &= V[(\tilde{M} \geq \tilde{M}_1) \text{ and } (\tilde{M} \geq \tilde{M}_2) \\ &\quad \text{and } (\tilde{M} \geq \tilde{M}_n)], \\ &= \min V(\tilde{M} \geq \tilde{M}_i), \quad i = 1, 2, \dots, n \end{aligned} \quad (39)$$

Assume that,

$$d'(A_i) = \min V(\tilde{S}_i \geq \tilde{S}_n), \quad \text{for } n = 1, 2, \dots, m; \quad i \neq n. \quad (40)$$

Then the weight factor is given by

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_n))^T. \quad (41)$$

where A_i ($i = 1, 2, \dots, n$) is the *n*th criteria.

Table 13 Triangular fuzzy pair-wise comparison matrix for the five ranking criteria

	Q	C	MR	PC	MI	$\sum_{j=1}^n \tilde{a}_{ij}$
Quality (Q)	(1, 1, 1)	(0.2, 0.33, 1)	(0.33, 0.5, 1)	(1, 3, 5)	(1, 2, 2)	(3.53, 6.83, 10.0)
Cost (C)	(1, 3, 5)	(1, 1, 1)	(1, 3, 5)	(3, 5, 7)	(1, 3, 5)	(7, 15, 23)
Material removal rate (MRR)	(1, 2, 3)	(0.2, 0.33, 1)	(1, 1, 1)	(1.5, 3.5, 5.5)	(1, 2, 2)	(4.7, 8.83, 12.5)
Process capability (PC)	(0.2, 0.33, 1)	(0.14, 0.2, 0.33)	(0.18, 0.29, 0.67)	(1, 1, 1)	(0.2, 0.33, 1)	(1.72, 2.15, 4)
Material (M)	(0.5, 0.5, 1)	(0.2, 0.33, 1)	(0.5, 0.5, 1)	(1, 3, 5)	(1, 1, 1)	(3.2, 5.33, 9)
Total						(20.16, 38.15, 58.5)

$$\sum_{j=1}^n \tilde{a}_{ij} = (1, 1, 1) + (0.2, 0.33, 1) + \dots + (1, 2, 2) = (3.53, 6.83, 10)$$

$$\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} = (3.53, 6.83, 10) + (7, 15, 23) + \dots + (3.2, 5.33, 9) = (20.16, 38.15, 58.5)$$

Table 14 Outcomes of the application of step 1

	$\sum_{j=1}^n \tilde{a}_{ij}$	$\left[\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1}$ [1/(20.16,38.15,58.5)]	$\tilde{M}_i = \sum_{j=1}^n \tilde{a}_{ij} \otimes \left[\sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1}$
Quality (Q)	(3.53,6.83,10.0)	(0.017,0.026,0.050)	(0.060,0.179,0.496)
Cost (C)	(7,15,23)	(0.017,0.026,0.050)	(0.120,0.393,1.141)
Material removal rate (MRR)	(4.7,8.83,12.5)	(0.017,0.026,0.050)	(0.080,0.232,0.620)
Process capability (PC)	(1.72,2.15,4)	(0.017,0.026,0.050)	(0.029,0.056,0.198)
Material (M)	(3.2,5.33,9)	(0.017,0.026,0.050)	(0.055,0.140,0.446)
Total	(20.16,38.15,58.5)		

$$\tilde{M}_1 = (3.53, 6.83, 10) \times (20.16, 38.15, 58.5)^{-1} = (3.53, 6.83, 10) \times (0.017, 0.026, 0.05) = (0.06, 0.179, 0.496)$$

Table 15 Application of step 2 and step 3

Criterion	$V(S_i \geq S_n)$	$D'(A_1) = \text{Min}(S_i)$
Quality (Q)	$V(S_1 \geq S_2) = 0.64$ $V(S_1 \geq S_3) = 0.89$ $V(S_1 \geq S_4) = 1.36$ $V(S_1 \geq S_5) = 1.01$	0.64

$$V(\tilde{M}_1 \geq \tilde{M}_2) = [(0.120-0.496)] - [(0.179-0.496) - (0.393-0.120)] = 0.64 \text{ (see Table 5, Last Column, row 2 and 3)}$$

Then, we have $d'(A_1) = V(S_1 \geq S_2, S_3, S_4, S_5) = \min(0.64, 0.89, 1.36, 1.10) = 0.64$

The application and outcomes of steps 2 and 3 are provided in Tables 15 and 16.

Step 4 The normalized weight vectors via normalization are given by

$$W = (d(A_1), d(A_2), \dots, d(A_n))^T, \quad (42)$$

where W is a crisp number.

The application of step 4 provides local and global weights of the sub-criteria for triangular fuzzy ranking model. The local and global weights of the criteria and sub-criteria are provided in Table 17.

Table 17 Calculated weights of the sub-criteria for triangular fuzzy ranking model

Criteria	Local weight	Sub-criteria	Local weight	Global weight
Q	0.20	T	0.30	0.060 ^a
		SF	0.27	0.055
		SD	0.21	0.043
		CR	0.08	0.016
		TP	0.14	0.028
C	0.32	IC	0.32	0.104
		T/F	0.26	0.082
		TW	0.21	0.066
		PWC	0.16	0.053
		MC	0.05	0.012
PC	0.06	HD	0.42	0.025
		HW	0.39	0.023
		L/D - t/w	0.19	0.011
MR	0.24	MR	1.00	0.240
M	0.18	M	1.00	0.179

$$^a 0.060 = (0.20 \times 0.30)$$

Once the global weights of the sub-criteria [w_j ; $j = 1, 2, \dots$ number of sub-criteria (n)] and performance scores of alternatives at sub-criteria [x_{ij} ; $i = 1, 2, \dots$ number

Table 16 Calculated weights of the five ranking criteria

Criteria	Q	C	PC	MR	M
$V(S_i \geq S_n)$	$V(S_1 \geq S_2) = 0.64$ $V(S_1 \geq S_3) = 0.89$ $V(S_1 \geq S_4) = 1.00$ $V(S_1 \geq S_5) = 1.00$	$V(S_2 \geq S_1) = 1$ $V(S_2 \geq S_3) = 1$ $V(S_2 \geq S_4) = 1$ $V(S_2 \geq S_5) = 1$	$V(S_3 \geq S_1) = 0.53$ $V(S_3 \geq S_2) = 0.19$ $V(S_3 \geq S_4) = 0.40$ $V(S_3 \geq S_5) = 0.63$	$V(S_4 \geq S_1) = 1.00$ $V(S_4 \geq S_2) = 0.76$ $V(S_4 \geq S_3) = 1.00$ $V(S_4 \geq S_5) = 1.00$	$V(S_5 \geq S_1) = 0.91$ $V(S_5 \geq S_2) = 0.56$ $V(S_5 \geq S_3) = 0.80$ $V(S_5 \geq S_4) = 1.00$
$d(A)$	0.64	$d(A_2) = 1$	$d(A_3) = 0.19$	$d(A_4) = 0.76$	$d(A_5) = 0.56$
W	0.20	$W_2 = 0.3$	$W_3 = 0.06$	$W_4 = 0.24$	$W_5 = 0.18$

$$W^* = (0.64, 1.00, 0.19, 0.76, 0.56)^T$$

$$\text{Therefore, } W_1 = 0.64 / (0.64 + 1 + 0.76 + 0.19 + 0.56) = 0.20$$

of alternative NTMPs (m), $j = 1, 2, \dots$ number of sub-criteria] are calculated, the triangular fuzzy decision matrix can be obtained and the rankings of the NTMP alternatives can be calculated using the steps 5–11:

Step 5 A triangular fuzzy decision matrix ($\tilde{D}$) is built by taking the triangular fuzzy performance values of feasible NTMPs at criteria (Table 4). In $\tilde{D}$, which is shown in Eq. (43), $\tilde{x}_{ij}$ denotes the triangular fuzzy performance value of NTMPs m ($m = 1, 2, \dots$, number of feasible NTMPs (m)) at attribute i ($i = 1, 2, \dots$ number of criteria (n)).

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} \dots & \tilde{x}_{mn} \end{bmatrix} \quad (43)$$

Step 6 Calculate normalized ratings by the vector normalization:

$$\tilde{r}_i = \frac{\tilde{x}_i}{\sqrt{\sum_{i=1}^m \tilde{x}_i^2}}$$

$$i = 1, \dots, m \text{ and } j = 1, \dots, n. \quad (44)$$

Step 7 Calculate triangular fuzzy weights: The triangular fuzzy weighted normalized value $\tilde{v}_i$ is calculated by Eq. (45).

$$\tilde{v}_i = \tilde{w}_j \tilde{r}_i \quad i = 1, \dots, m; \quad j = 1, \dots, n. \quad (45)$$

where $\tilde{w}_j$ is the weight of the j th criterion

Step 8 Defuzzification: triangular fuzzy performance values are converted to crisp values by calculating the means of the triangular fuzzy numbers. Given a triangular fuzzy number $\tilde{A}$, its generalized mean value is calculated using Eq. 45. When $\tilde{A}$ is a triangular fuzzy number (a_1, a_2, a_3), Eq. (46) can be simplified into Eq. (47) (Cheng and Hwang 1992):

$$\bar{x}(\tilde{A}) = \frac{\int x \mu_{\tilde{A}}(x) dx}{\int \mu_{\tilde{A}}(x) dx}, \quad (46)$$

$$\bar{x}\tilde{A} = \frac{(a_1 + a_2 + a_3)}{3} \quad (47)$$

The triangular fuzzy weighted normalized values ($\tilde{v}_i$) provided in Eq. (45) are defuzzified using Eq. (47) and the crisp weighted normalized values (v_i) are obtained.

Step 9 Identify positive ideal and negative ideal solutions: The A^* and A are defined in terms of the weighted normalized values, as shown in Eqs. (48) and (49):

Table 18 Calculation of the rankings of the NTMPs for triangular fuzzy ranking model

NTMP	D^+	D^-	C^*	Ranking
AJM	0.260	0.101	0.27993	3
AWJM	0.228	0.087	0.27641	4
RUSM	0.222	0.092	0.29249	2
ECM	0.130	0.254	0.66152	1
EDM	0.249	0.079	0.24207	7
EBM	0.270	0.093	0.25693	6
LBM	0.268	0.100	0.27111	5

$$A^* = \left\{ (\max_i v_i | i \in I), (\min_i v_i | i \in I') \right\}, \quad (48)$$

$$A^- = \left\{ (\min_i v_i | i \in I), (\max_i v_i | i \in I') \right\}, \quad (49)$$

where I is the set of benefit type measures and I' is the set of cost type attribute.

Step 10 Calculation of separation measures: The separation between scenarios can be measured by the n -dimensional Euclidean distance. The separation of each scenario from the positive ideal solution, A^* , and the negative ideal solution, A , is then given by Eqs. (50) and (51), respectively:

$$D_i^+ = \sqrt{\sum_{i=1}^n (v_i - v_i^*)^2}, \quad (50)$$

$$D_i^- = \sqrt{\sum_{i=1}^n (v_i - v_i^-)^2}, \quad (51)$$

where v_i^* is the positive-ideal value for i th attribute, and v_i^- is the negative-ideal value for the i th attribute.

Step 11 Calculation of the ranking scores:

$$C_i^* = \frac{D_i^-}{D_i^- + D_i^+} \quad i = 1, \dots, m. \quad (52)$$

Application of steps 5–11 of triangular fuzzy ranking model provides the ranking scores and rankings of the alternative NTMPs. The results are provided in Table 18.

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