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Effects of a temperature cycle on a rotating shrink fit with FGM-hub

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Subject of the investigation is a rotating shrink fit with hollow cylindrical inclusion and functionally graded hub in a state of generalized plane strain. Elevated temperature during operation may have both a transient and – in case of partial plastification – a permanent influence on the interface pressure. These effects are exemplified by means of a shrink fit with steel/aluminum FGM-hub.

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1 Introduction

For a shrink fit with given geometry and state of friction at the interface solely the interface pressure is decisive for reliable operation. Hence, the stress distributions not only at standstill and at rotation but also during (transient) heating have been subject of many investigations, see, e.g., [1-3]. A temperature cycle may cause not only a transient variation – in general, a reduction – of the interface pressure but also a permanent one in case that plastification occurs. Whereas this was studied comprehensively for devices with homogeneous inclusion and hub, respectively, this is however not the case for shrink fits with component parts of functionally graded material (FGM). Indeed, using a hub with radially decreasing density like, e.g., one of a steel/aluminum FGM [4] facilitates both significantly better performance at rotation and substantial weight-reduction [5,6] which is particularly pronounced for a wide hub. It is the aim of the present contribution to show the effects of a temperature cycle on a shrink fit with such an FGM-hub; specifically, a device with hollow inclusion and large axial length of the hub is considered, so that a treatment as (generalized) plane strain problem is appropriate (Fig. 1a).

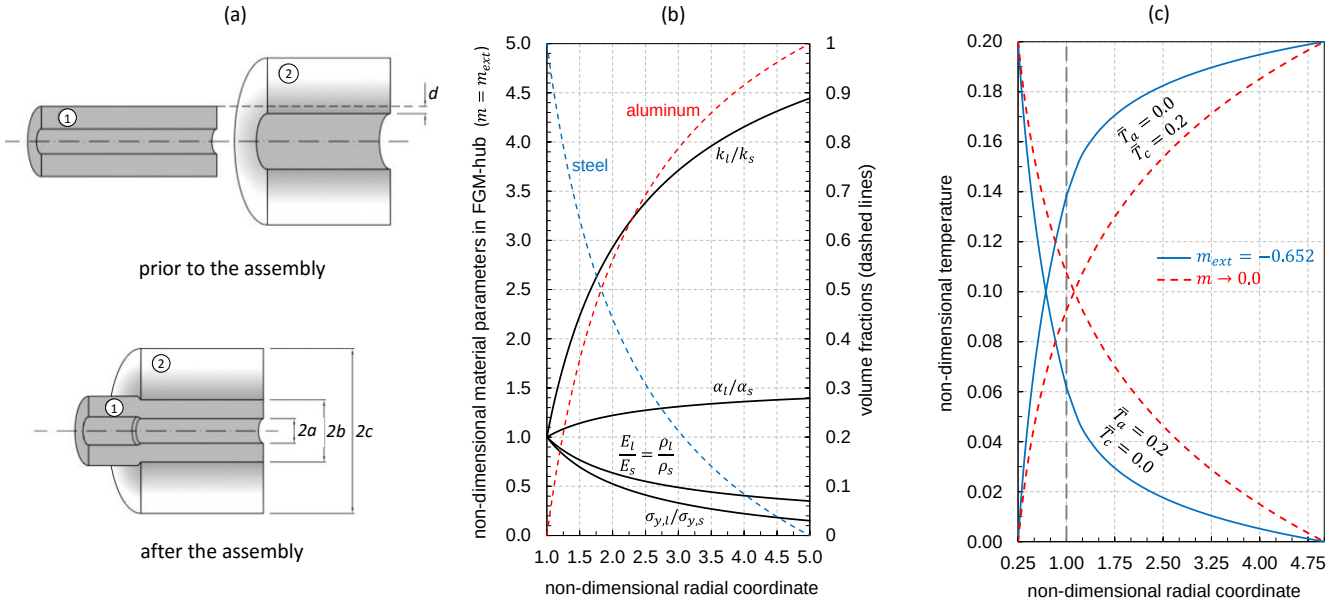


Fig. 1: **a** Sketch of the shrink fit with the denomination of the radii (① homogeneous inclusion, ② FGM-hub; d means the interference); **b** Parameter ratios (solid lines) and volume fractions (dashed lines) vs. non-dimensional radius $\overline{r}$; **c** Examples of temperature distributions in a shrink fit with FGM-hub and homogeneous hub ($m \to 0$), respectively; $k_s = 45\ W/mK$

2 Characteristics of the shrink fit and effects of a temperature cycle

The material of the inclusion is homogeneous (denoted by s), and the hub is presumed to be maximum graded from material s at the interface to pure material l at its outer surface. Postulating the basic grading law for Young's modulus as $E(r) =$

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$E_s\,(r/b)^m$ facilitates an analytical treatment of the problem; the dependence of the other physical parameters on the radius then follows from the rule of mixture. For the considered maximum grading the exponent is $m = m_{ext} = \ln(E_l/E_s)\,/\ln(c/b)$. As the numerical example is based on a steel/aluminum FGM-hub, the parameter ratios are $k_l/k_s = 4.44$, $E_l/E_s = \rho_l/\rho_s = 0.35$, $\alpha_l/\alpha_s = 1.40$, $\sigma_{y,l}/\sigma_{y,s} = 0.15$, and Poisson's ratio $\nu = 0.3$ for both constituents (k_j means thermal conductivity, ρ_j density, α_j the coefficient of thermal expansion, and $\sigma_{y,j}$ the uniaxial yield limit); moreover, the material is presupposed to obey Tresca's yield criterion. Figure 1b shows the variation of the above parameters and the volume fractions of both constituents vs. the non-dimensional radius $\overline{r} = r/b$ for an FGM-hub with $c/b = 5$.

For slowly varying temperature at the inner (T_a) and/or outer (T_c) cylindrical surface of the shrink fit a calculation of the occurring temperature fields as quasi-stationary is reasonable and enables an analytical solution. Figure 1c provides examples of (non-dimensional) temperature fields (with $\overline{T} = E_s\alpha_s T/\sigma_{y,s}$). Then, the effects of various temperature distributions on the non-dimensional interface pressure $\overline{p} = -\sigma_r\,(b)\,/\sigma_{y,s}$ in a shrink fit with $a/b = 0.25$ and $c/b = 5$ can be seen from Fig. 2; at standstill an initial interface pressure of $\overline{p} = 0.35$ is presumed. For a non-dimensional angular speed $\Omega = \omega b\sqrt{\rho_s/\sigma_{y,s}} = 0.05$ (corresponding to an already rather high number of rpm in engineering [6]) and $m = m_{ext} = -0.652$ Fig. 2a shows $\overline{p}$ vs. combinations of surface temperatures $\overline{T}_a$ and $\overline{T}_c$, respectively; the dashed line represents spatially constant elevated temperatures. Note that – contrary to a homogeneous shrink fit – even homogeneous heating influences the stress distribution in case of an FGM-hub. Whereas heating from the outside reduces $\overline{p}$, in general, it is an interesting effect that raising the inner surface temperature while keeping the outside (at least almost) at ambient temperature may even increase the interface pressure. For comparison, in Fig. 2b the corresponding interface pressures in the shrink fit with FGM-hub and in one with homogeneous steel hub ($m \to 0$) are contrasted. Roughly speaking, for heated outer surface and moderate inner surface temperature the FGM-hub yields higher values of $\overline{p}$ (and the same holds true for moderate homogeneous heating), for other cases the homogeneous hub proves to be advantageous with respect to $\overline{p}$, in general. Nevertheless, the difference is small, and if also weight-reduction is an assessment criterion the FGM-hub may throughout be considered superior.

In the load range considered in Fig. 2, the shrink fit behaves elastically, and hence the stress distribution is not altered permanently by a temperature cycle. If however the temperature gradients and the angular speed are further increased, in the FGM-hub a plastic region with $\sigma_{Tresca}\,(r) = \sigma_y\,(r) = \sigma_\theta\,(r) - \sigma_r\,(r)$ may spread from its inner edge. In this case, for the above geometric and material parameter values but, e.g., $\overline{T}_a = 0.462$, $\overline{T}_c = 0$, and $\Omega = 0.15$ one finds a marginal permanent reduction of $\overline{p}$ by 0.14% after cooling and standing still, and also in this case the FGM-hub proves to be efficient.

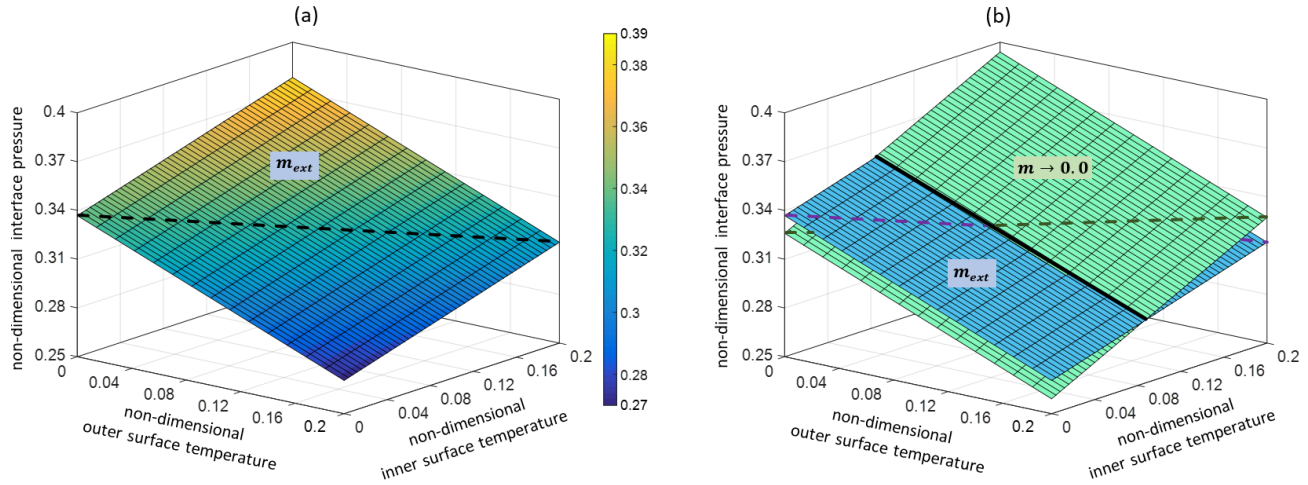


Fig. 2: Interface pressure in a shrink fit rotating at $\Omega = 0.05$ vs. surface temperatures for: **a** FGM-hub; **b** FGM-hub as compared to homogeneous hub

References

[1] H. Lippmann. Int. J. Plasticity **8**, 567-582 (1992).
[2] W. Mack and M. Plöchl. Int. J. Eng. Sci. **38**, 921-938 (2000).
[3] N. Antoni. Appl. Math. Model. **37**, 2352-2363 (2013).
[4] M.M. Nemat-Alla, M.H. Ata, M.R. Bayoumi, and W. Khair-Eldeen. Mater. Sci. Appl. **2**, 1708-1718 (2011).
[5] E. Arslan and W. Mack. Comp. Struct. **121**, 217-224 (2015).
[6] T. Apatay, E. Arslan, and W. Mack. Mech. Based Des. Struct. Mach. **47**, 339-355 (2019).

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