IJPAS | INTERNATIONAL JOURNAL PURE AND APPLIED SCIENCES
ISSN: 2149-0910
Vol. 12(1); 306-315 (2026)
DOI: 10.29132/ijpas.1902675

Research Article

Multiplicative homothetic motion defined by a planar curve

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ARTICLE INFO

Keywords:
multiplicative homothetic motion;
pole points;
multiplicative speeds;
multiplicative accelerations

Dates:
Received: 4 Mar 2026
Accepted: 20 May 2026

ABSTRACT

In this study, multiplicative homothetic motion is defined in the two-dimensional multiplicative homothetic Euclidean space E_*^2 through the components of a curve. It is shown that at each time t, this motion has a $(q-1)$th-order center of acceleration. In addition, it is proven that the centers of acceleration are independent of the motion parameter in the special case where the translation vector of the motion is proportional to the planar curve.

1. Introduction

Grossman and Katz presented multiplicative calculus which is also named Non-Newtonian calculus. The two fundamental operations in multiplicative calculus are known as the multiplicative derivative and the multiplicative integral. For various forms of Non-Newtonian calculus and their applications, one may consult the works of Grossman and Katz [1], Stanley [2], Campbell [3], Grossman [4,5], and Jane Grossman [6,7]. Bashirov and coleaguess developed a detailed and mathematically rigorous framework for multiplicative calculus [8]. The generalization of multiplicative calculus to complex-valued functions is discussed in references [9–13]. Çakmak and Başar [14] investigated and described matrix transformations on sequence spaces within the framework of multiplicative calculus. K. Boruah and B. Hazarika [15] introduced the geometric real number line within the framework of a geometric coordinate system. K. Boruah and B. Hazarika [16] established the geometric trigonometric ratios and clarified their connections with the classical trigonometric functions. Gürefe [17] introduced several notions within the framework of multiplicative arithmetic, including multiplicative vector spaces, multiplicative inner product spaces, and multiplicative matrices, and also discussed their fundamental properties. In his research [18, 26–28], H. Es described multiplicative motion across various metrics and provided several related results and theorems. S. Nurkan, K. İ. Gürgil, and M. K. Karacan [23] presented the vector properties within the framework of geometric calculus. S. Aslan, M. Bekar, and Y. Yaylı [24] introduced the concepts of geometric 3-space and multiplicative quaternions. In their studies [28, 29], A. Has and B. Yılmaz introduced the concepts of multiplicative analytic geometry. Y. Yaylı [30] introduced the concepts of planer homothetic motions.

2. Preliminaries

Definition 1. The derivative of the function f at x in the classical sense is given by the limit expression $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Replace the difference $f(x+h)-f(x)$ by the ratio $f(x+h)/f(x)$ and the division by h by the raising to the reciprocal power $1/h$ in this formula. The multiplicative derivative of f at x is defined as

$$f^{\circ}(x) = \lim_{h\to 0} \left(\frac{f(x+h)}{f(x)}\right)^{\frac{1}{h}}.$$

In [8] it is shown that if f is a positive function on an open subset A of the real line $\mathbb{R}$ and its derivative $f'(x)$ exists, then its multiplicative derivative $f^{\circ}(x)$ also exists and they are related as

$$f^{\circ}(x) = e^{(lnf(x))'}.$$

Moreover, if $n-$th derivative $f^{(n)}(x)$ exists, then nth multiplicative derivative $f^{\circ(n)}(x)$ also exists and it can be given as

$$f^{\circ(n)}(x) = e^{(lnf(x))^{(n)}},\ n = 0, 1,\ldots \text{ [8-10, 17, 18, 25-27].}$$

Definition 2. Geometric and classical trigonometric functions are connected through exponential mappings, where

$$sing\ \theta = sin_*\theta = e^{sin\,\theta},$$
$$cosg\ \theta = cos_*\theta = e^{cos\,\theta},$$
$$tang\ \theta = tan_*\theta = e^{\tan\theta} = sin_*\theta /_* cos_*\theta \quad \text{[16, 25, 28-29].}$$

Definition 3. We define a multiplicative matrix as a matrix whose components are elements of $\mathbb{R}_*$. We use the notation $\mathbb{M}_{*\text{mxn}}$ to represent the set of all multiplicative mxn matrices [24].

Definition 4. Let $K = [k_{ij}] \in \mathbb{M}_{*\text{mxn}}$ and $N = [n_{ij}] \in \mathbb{M}_{*\text{mxn}}$ be multiplicative matrices and $\mu \in \mathbb{R}_*$. We now define the following

$$K \pm_* N = [k_{ij} \pm_* n_{ij}]$$

and

$$\mu \cdot_* N = [\mu \cdot_* n_{ij}] \text{ [17, 18, 23, 25-27].}$$

Definition 5. The $I_* \in \mathbb{M}_{*\text{nxn}}$, given by

$$I_* = \begin{bmatrix} e & 1 & \ldots & 1 \\ 1 & e & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & e \end{bmatrix}_{\text{nxn}}$$

Where, multiplicative zero $e^0 = 1$ and geometric identity $e^1 = e$ [17, 18, 22, 23, 24, 25-27].

Definition 6. Let M= $[m_{ij}] \in \mathbb{M}_{*\text{r}\times\text{n}}$ and R= $[r_{ij}] \in \mathbb{M}_{*\text{n}\times\text{m}}$.

The multiplicative product of the matrices M and R is defined as follows.

$$M \cdot_* R = [m_{k1} \cdot_* r_{1l} +_* m_{k2} \cdot_* r_{2l} +_* \ldots +_* m_{kn} \cdot_* r_{nl}] \text{ [17, 18, 23, 24, 25-27].}$$

Definition 7. Let

$$K = \begin{bmatrix} e^{k_{11}} & e^{k_{12}} \\ e^{k_{21}} & e^{k_{22}} \end{bmatrix} \in \mathbb{M}_{*2\text{x}2}.$$

and

$$det_*K = e^{k_{11}k_{22}-k_{21}k_{12}} \neq 1 \text{ [17, 18, 23, 24, 25-27].}$$

Next, we define the multiplicative inverse of the matrix K, denoted by K^{-1*}, as follows.

$$K^{-1*} = (e/_* det_* K).{}_* \begin{bmatrix} e^{k_{22}} & \frac{1}{e^{k_{12}}} \\ \frac{1}{e^{k_{21}}} & e^{k_{11}} \end{bmatrix} \text{ [24].}$$

Definition 8. $K = [k_{ij}] \in \mathbb{M}_{*\text{nxn}}$ is called multiplicatively orthogonal if it satisfies the condition

$$K ._* K^T = K^T ._* K = I_* \text{ [24].}$$

2.1 Geometric real numbers

The set of geometric real numbers can be expressed as

$\mathbb{R}_* = \{\exp(\mathrm{p}) = \mathrm{e}^{\mathrm{p}} : \mathrm{p} \in \mathbb{R}\}$.

In the set $\mathbb{R}_*$, the operations of addition and multiplication are defined as follows.

Addition

$$\mathrm{e}^{\mathrm{p}} +_* \mathrm{e}^{\mathrm{q}} = \mathrm{e}^{\mathrm{p}+\mathrm{q}}$$

multiplication

$$\mathrm{e}^{\mathrm{p}} \cdot_* \mathrm{e}^{\mathrm{q}} = \mathrm{e}^{\mathrm{pq}}$$

for every $\mathrm{e}^{\mathrm{p}}, \mathrm{e}^{\mathrm{q}} \in \mathbb{R}_*$. The set $\mathbb{R}_*$ with the geometric addition $+_*$ and geometric multiplication $.{}_*$ operators (i.e., $(\mathbb{R}_*, +_*, .{}_*)$ is a field with geometric zero $\mathrm{e}^0 = 1$ and geometric identity $\mathrm{e}^1 = e$. The relations between the basic geometric operations and ordinary arithmetic operations can be given for all $w, z \in \mathbb{R}_*$ as

$$\mathrm{w} +_* \mathrm{z} = wz, \quad \mathrm{w} -_* \mathrm{z} = \frac{w}{z},$$

$$\mathrm{w} .{}_* \mathrm{z} = w^{lnz} = z^{lnw}, \quad w/_* z = w^{(\frac{1}{lnz})},$$

$$\sqrt[*]{w} = e^{\sqrt{lnw}}, \quad w^{-1*} = e^{(\frac{1}{lnw})},$$

Hence, we assign the values

$$\mathrm{e}^{\mathrm{p}} +_* \mathrm{e}^{\mathrm{q}} = \mathrm{e}^{\mathrm{p}+\mathrm{q}}, \qquad \mathrm{e}^{\mathrm{p}} \cdot_* \mathrm{e}^{\mathrm{q}} = \mathrm{e}^{\mathrm{pq}},$$

$$\mathrm{e}^{\mathrm{p}} -_* \mathrm{e}^{\mathrm{q}} = \mathrm{e}^{\mathrm{p}-\mathrm{q}}, \qquad \mathrm{e}^{\mathrm{p}} /_* \mathrm{e}^{\mathrm{q}} = \mathrm{e}^{\frac{\mathrm{p}}{\mathrm{q}}}, \qquad \sqrt[*]{e^{\mathrm{p}}} = e^{\sqrt{\mathrm{p}}}$$

Positive and negative geometric real numbers are defined, respectively, as follows.

$$\mathbb{R}_*^+ = \{w \in \mathbb{R}_* : w > 1\}$$

and

$$\mathbb{R}_*^- = \{w \in \mathbb{R}_* : 0 < w < 1\}.$$

Further information on geometric real numbers is available in [11-15,19-22,24].

2.2. Geometric 2-Space

$$\mathbb{R}_*^2 = \{\hat{t} = (\mathrm{e}^{t_1}, \mathrm{e}^{t_2}) : \mathrm{e}^{t_1}, \mathrm{e}^{t_2} \in \mathbb{R}_*\}.$$

The operations of addition and scalar multiplication in $\mathbb{R}_*^2$ are defined, respectively, as

follows.

$$\hat{t}+_*\hat{s} = (e^{t_1},e^{t_2})+_*(e^{s_1},e^{s_2})$$

$$= (e^{t_1}+_*e^{s_1},e^{t_2}+_*e^{s_2})$$

$$= (e^{t_1+s_1},e^{t_2+s_2})$$

and

$$e^{\mu}._*\hat{t} = e^{\mu}._*(e^{t_1},e^{t_2})$$
$$= (e^{\mu}._*e^{t_1},e^{\mu}._*e^{t_2})$$
$$= (e^{\mu t_1},e^{\mu t_2})$$

where $e^{\mu} \in \mathbb{R}_*$ and $\hat{t},\hat{s} \in \mathbb{R}_*^2$, the standard basis for $\mathbb{R}_*^2$ is the set $\{\hat{e}_1 = (e,1),\hat{e}_2 = (1,e)\}$. Each element $\hat{t}$ of $\mathbb{R}_*^2$ is called a vector in $\mathbb{R}_*^2$ and we will call it as geometric vector in $\mathbb{R}_*^2$. The origin of $\mathbb{R}_*^2$ is

$$\hat{O} = (e^0,e^0) = (1,1).$$

The geometric inner (or scalar) product in $\mathbb{R}_*^2$ is defined as

$$\langle\hat{t},\hat{s}\rangle_* = \langle(e^{t_1},e^{t_2}),(e^{s_1},e^{s_2})\rangle_*$$
$$= (e^{t_1}._*e^{s_1})+_*(e^{t_2}._*e^{s_2})$$
$$= e^{t_1s_1+t_2s_2}.$$

The geometric norm in $\mathbb{R}_*^2$ is defined as

$$\|\hat{s}\|_* = \sqrt[*]{\langle\hat{s},\hat{s}\rangle_*} = e^{\sqrt{(s_1)^2+(s_2)^2}}.$$

A geometric vector $\hat{s}$ is unit if $\|\hat{s}\|_* = e$.

3. The Homothetic Motion Defined By A Planar Curve

Let $\hat{r} : I\mathbb{C}\mathbb{R}_* \longrightarrow E_*^2,\ \hat{r}(t) = (e^{r_1(t)},e^{r_2(t)})$

be curve with unit speed which does not pass through the origin. Consider the matrix

$$B = \begin{bmatrix} e^{r_1(t)} & e^{-r_2(t)} \\ e^{r_2(t)} & e^{r_1(t)} \end{bmatrix} \qquad (3.1)$$

$$h: I\mathbb{C}\mathbb{R}_* \longrightarrow E_*^2,\ h(t) = \|\hat{r}(t)\|_* = e^{\sqrt{{r_1}^2+{r_2}^2}} \qquad (3.2)$$

we can write

$$B(t) = e^{h(t)}._*\begin{bmatrix} e^{r_1(t)}/_*e^{h(t)} & e^{-r_2(t)}/_*e^{h(t)} \\ e^{r_2(t)}/_*e^{h(t)} & e^{r_1(t)}/_*e^{h(t)} \end{bmatrix}$$

$$B(t) = e^{h(t)}._*A(t) \qquad (3.3)$$

Theorem 3.1. The matrix A in expression (3.3) is a geometric real orthogonal matrix.

Theorem 3.2. Let B° represent the multiplicative derivative of the matrix B with respect to the parameter t. Then, B is a geometric real orthogonal matrix.

Proof. Let $B^{\circ} = \begin{bmatrix} e^{r_1'} & e^{-r_2'} \\ e^{r_2'} & e^{r_1'} \end{bmatrix}$.

Then

$$B^{\circ} ._* (B^{\circ})^T = (B^{\circ})^T ._* B^{\circ} = I_*$$

Thus, the theorem is fully established.

4. One-Parameter Multiplicative Motion

Let the fixed and moving multiplicative planes be denoted by $\mathcal{R}'$ and $\mathcal{R}$, respectively. Thus, the motion of the moving line with respect to the fixed line is represented by $\mathcal{R}/_{\mathcal{R}'}$. For all t $C(t) \in R_*^2$,

The one-parameter motion is defined in the form of

$$\begin{bmatrix} \hat{Y} \\ e \end{bmatrix} = \begin{bmatrix} B & C \\ 1 & e \end{bmatrix} ._* \begin{bmatrix} \hat{X} \\ e \end{bmatrix} \qquad (4.1)$$

or

$$\hat{Y} = B ._* \hat{X} +_* C \qquad (4.2)$$

Here, the vectors $\hat{Y}$ and $\hat{X}$ correspond to the position vectors of an arbitrary point in the fixed and moving multiplicative spaces, respectively. In what follows, we take as given that

$$B^{\circ}(t) \neq 1 \text{ and } C(t) \neq 1.$$

Theorem 4.1. Equation (4.1) defines a regular motion that does not depend on h.

Proof.

$$det_* B^{\circ} = \begin{vmatrix} e^{r_1'} & e^{-r_2'} \\ e^{r_2'} & e^{r_1'} \end{vmatrix}_*$$

$$= e^{r_1'} ._* e^{r_1'} -_* e^{r_2'} ._* e^{-r_2'}$$

$$= e^{r_1' r_1'} -_* e^{r_2'(-r_2')}$$

$$= e^{(r_1')^2} -_* e^{-(r_2')^2}$$

$$= e^{(r_1')^2 + (r_2')^2}$$

$$= e^1$$

$$= e.$$

Since $det_* B^{\circ} = e$, the motion is regular, and det $det_* B$ is independent of h.

5. The Pole Curves of The Motion

Taking the derivative of $\hat{Y} = B ._* \hat{X} +_* C$ with respect to t in the multiplicative sense yields $\hat{Y}^{\circ} = B^{\circ} ._* \hat{X} +_* C^{\circ} +_* B ._* \hat{X}^{\circ}$ (5.1)

Here, the quantity $B^{\circ} ._* \hat{X} +_* C^{\circ}$ is referred to as the sliding velocity of the motion and is denoted by $\hat{V}_f$. Points satisfying the condition

$$\hat{V}_f = 1$$

are called pole points. Let us now examine the pole point in the moving plane. $\hat{V}_f = 1$

$$\hat{\mathbb{V}}_f = B^{\circ} \cdot_* \hat{X} +_* C^{\circ} = 1 \qquad (5.2)$$

The pole point $\hat{X}$ can be obtained from equation (5.2) and is given by

$$\hat{X} = e^{-1} \cdot_* \left(\mathrm{B}^{\circ}\right)^{\mathrm{T}} \cdot_* (\mathrm{C}^{\circ})$$

$$= e^{-1} \cdot_* \begin{bmatrix} e^{r_1'} & e^{r_2'} \\ e^{-r_2'} & e^{r_1'} \end{bmatrix}_* \cdot_* \begin{bmatrix} e^{c_1'} \\ e^{c_2'} \end{bmatrix}_*$$

$$= e^{-1} \cdot_* \begin{bmatrix} e^{r_1'} \cdot_* e^{c_1'} +_* e^{r_2'} \cdot_* e^{c_2'} \\ e^{-r_2'} \cdot_* e^{c_1'} +_* e' \cdot_* e^{c_2'} \end{bmatrix}_*$$

$$= e^{-1} \cdot_* \begin{bmatrix} e^{c_1' r_1' + c_2' r_2'} \\ e^{-c_1' r_2' + c_2' r_1'} \end{bmatrix}_*$$

$$= \begin{bmatrix} e^{-c_1' r_1' - c_2' r_2'} \\ e^{c_1' r_2' - c_2' r_1'} \end{bmatrix}. \qquad (5.3)$$

The pole point of the motion is

$$\hat{\mathrm{q}}_0 = e^{-1} \cdot_* (\mathrm{B}^{\circ})^{\mathrm{T}} \cdot_* (\mathrm{C}^{\circ})$$

$$\hat{\mathrm{q}}_0 = \begin{bmatrix} e^{p_1} \\ e^{p_2} \end{bmatrix}_* = \begin{bmatrix} e^{-c_1' r_1' - c_2' r_2'} \\ e^{c_1' r_2' - c_2' r_1'} \end{bmatrix}. \qquad 5.4)$$

Using equation (5.4), we can write

$$\left.\begin{aligned} e^{p_1} &= e^{-c_1' r_1' - c_2' r_2'} \\ e^{p_2} &= e^{c_1' r_2' - c_2' r_1'} \end{aligned}\right\} \qquad (5.5)$$

From equation (5.5), we also obtain

$$\left.\begin{aligned} e^{c_1'} &= e^{r_2' p_2 - r_1' p_1} \\ e^{c_2'} &= e^{-r_1' p_2 - r_2' p_1} \end{aligned}\right\} \qquad (5.6)$$

We parameterize the shifting vector $\hat{V}_f$ by the coordinates e^{p_1}, e^{p_2} associated with the pole point $\hat{\mathrm{p}}_0 = (e^{p_1}, e^{p_2})$ in $\mathcal{R}$.

For an arbitrary point $\hat{X} \in \mathcal{R}$, substituting the expression for C° provided in (5.6) into the following relation yields

$$\hat{\mathbb{V}}_f = B^{\circ} \cdot_* \hat{X} +_* C^{\circ}, \quad \hat{X} = (e^{x_1}, e^{x_2})$$

$$\hat{\mathbb{V}}_f = \left\{(e^{x_1} -_* e^{p_1}) \cdot_* e^{r_1'} +_* (e^{p_2} -_* e^{x_2}) \cdot_* e^{r_2'}, (e^{x_2} -_* e^{p_2}) \cdot_* e^{r_1'} +_* (e^{x_1} -_* e^{p_1}) \cdot_* e^{r_2'}\right\}$$

On the other hand

$$\widehat{PX} = (e^{x_1} -_* e^{p_1}, e^{x_2} -_* e^{p_2}).$$

Through straightforward calculations, the norms of the vectors $\hat{V}_f$ and $\widehat{PX}$ are obtained as

$$\left\| \hat{V}_f \right\|_* = \sqrt[*]{\langle \hat{V}_f, \hat{V}_f \rangle_*} = e^{\sqrt{(x_1 - p_1)^2 + (x_2 - p_2)^2}}$$

and

$$\left\| \widehat{PX} \right\|_* = \sqrt[*]{\langle \widehat{PX}, \widehat{PX} \rangle_*} = e^{\sqrt{(x_1 - p_1)^2 + (x_2 - p_2)^2}}.$$

Consequently, we can state the following theorem:

Theorem 5.1. The length of the shifting vector $\widehat{V_f}$ in the motion does not depend on h, and we have:

$$\left\|\widehat{V_f}\right\|_* = \left\|\widehat{PX}\right\|_*.$$

Theorem 5.2. Let K denote the tangent vector of the curve $\hat{r}(t)$. Then, the orthogonality condition $\langle \widehat{V_f}, \widehat{K} \rangle_* = 1$ holds if and only if $e^{x_1} = e^{p_1}$.

Proof. $\langle \widehat{V_f}, \widehat{K} \rangle_* =$

$$\langle ((e^{x_1} -_* e^{p_1})._* e^{r_1}{}' +_* (e^{p_2} -_* e^{x_2})._* e^{r_2}{}', (e^{x_2} -_* e^{p_2})._* e^{r_1}{}' +_* (e^{x_1} -_* e^{p_1})._* e^{r_2}{}'), (e^{r_1}{}', e^{r_2}{}') \rangle_*$$

$$= (e^{x_1} -_* e^{p_1})._* e^{(r_1{}')^2} +_* (e^{p_2} -_* e^{x_2})._* e^{r_1{}' r_2{}'} +_* (e^{x_2} -_* e^{p_2})._* e^{r_1{}' r_2{}'} +_* (e^{x_1} -_* e^{p_1})._* e^{(r_2{}')^2}$$

$$= (e^{x_1} -_* e^{p_1})._* e^{(r_1{}')^2} +_* (e^{x_1} -_* e^{p_1})._* e^{(r_2{}')^2}$$

$$= ((e^{x_1} -_* e^{p_1}) +_* (e^{x_1} -_* e^{p_1}))._* (e^{(r_1{}')^2} +_* e^{(r_2{}')^2})$$

$$= ((e^{x_1} -_* e^{p_1})).._* (e^{(r_1{}')^2 + (r_2{}')^2})$$

$$= (e^{x_1} -_* e^{p_1}).._* e = 1 = e^0 e^{x_1 - p_1} = e^0$$

So

$$e^{x_1} = e^{p_1}.$$

Now, let us consider a special case

Let $C(t) = e^{\mu}._* \hat{r}(t)$ in Eq. (4.2), where $e^{\mu} \in \mathbb{R}_*, e^{\mu} \neq 1$. Hence, from (4.2) we obtain

$$\hat{Y} = B._* \hat{X} +_* e^{\mu}._* \hat{r}(t) \qquad (5.7)$$

The pole point of the motion in $\mathcal{R}$ is determined by solving (5.7).

$$B^{\circ}._* \hat{X} +_* e^{\mu}._* (\hat{r}(t))^{\circ} = 1$$

The solution leads to

$$\hat{X} = (e^{p_1}, e^{p_2}) = (e^{-\mu}, e^0).$$

Within the fixed space $\mathcal{R}'$, this point is represented by

$$\hat{Y} = B._* \hat{X} +_* C$$

so

$$\hat{Y} = (1,1).$$

Theorem 5.3. If, in the homothetic motion in $\mathcal{R}$ given by (4.2), the function $C(t)$ is replaced by $\hat{r}(t)$, then the pole points of the motion become independent of the motion parameter.

6. The Acceleration Center of Order (q-1)-th Multiplicative Motion

The points at which the multiplicative shifting acceleration of order q becomes zero are defined as the (q−1)th-order centers of acceleration. These points solve the equation

$$B^{(q)}._* X +_* C^{(q)} = 1 \qquad (6.1)$$

When the curve a is regular of order q, then $det_*(B^{\circ})^{(q)} = (e^{r_1})^{(q)^2} +_* (e^{r_2})^{(q)^2} \neq 1$

Under this condition, equation (6.1) has a single solution expressed as

$$X = \left(B^{(q)}\right)^{-1^*} ._* (-_* C)^{(q)}.$$

Theorem 6.1. If the curve $\hat{r}(t)$ of the motion in E_*^2, defined by (4.2), is a regular curve of order q, then there exists a unique acceleration center of order (q−1)-th.

Proof. For the given motion, the (q−1)th-order centers of acceleration are the points that fulfill equation (6.1). From equation (6.1), we can derive

$$B^{(q)} ._* X = -_* C^{(q)} \tag{6.2}$$

Equation (6.2) represents a multiplicative linear system, and the existence and uniqueness of its solution depend on the existence of the multiplicative inverse of $B^{(q)}$. Since

$$det_*(B^{\circ})^{(q)} = (e^{r_1})^{(q)^2} +_* (e^{r_2})^{(q)^2} \neq 1,$$

the multiplicative inverse $\left(B^{(q)}\right)^{-1^*}$ exists. Hence, we obtain

$$X = \left(B^{(q)}\right)^{-1^*} ._* (-_* C)^{(q)}.$$

This result shows that the acceleration center of order $(q-1)$ exists and is unique.

Theorem 6.2. When $C(t) = e^{\mu} ._* \hat{r}(t)$, all centers of acceleration coincide at every instant t, and thus they do not depend on the motion parameter.

Proof . For $C(t) = e^{\mu} ._* \hat{r}(t)$,

the acceleration center of order $(q-1)$ satisfy the equation

$$B^{(q)} ._* X +_* C^{(q)} = 1.$$

From this, the solution is given by

$$X = \left(B^{(q)}\right)^{-1^*} ._* (-_* C)^{(q)}. \tag{6.3}$$

By taking the multiplicative derivative of the expression $C(t) = e^{\mu} ._* \hat{r}(t)$ and substituting it into equation (6.3), followed by the necessary computations, the following result is obtained:

$$\hat{X} = (e^{p_1}, e^{p_2}) = (e^{-\mu}, e^{0}).$$

As a result, all centers of acceleration of the motion coincide at every instant and are therefore indpendent of the motion parameter.

4. Conclusion

In this study, a multiplicative homothetic motion is defined in the two-dimensional multiplicative Euclidean space E_*^2 through the components of a curve. The velocities of this motion and the relationships between them are determined. The accelerations are also obtained and the relationships between them are established. It is proven that, for every t, the motion has a (q−1)th-order center of acceleration. Furthermore, it is shown that when the translation vector is proportional to the planar curve, the accelerations are independent of the motion parameter.

Acknowledgment

The author expresses gratitude to the journal editors, the scientific publication board, and the secretariat for their support.

Conflict of Interest

The author declares that, to the best of their knowledge, there is no conflict of interest or shared interest with any institution, organization, or individual that could influence the review process of this article.

Author Contribution

This research was conducted collaboratively by the authors. The authors adhered strictly to ethical guidelines and principles of academic integrity in preparing this article.

Research and Publication Ethics Statement

The authors declare that, in preparing this study, they have complied with the journal's scientific, ethical, and citation standards.

References

[1] M.Grossman and R.Katz, *Non-Newtonian calculus*, Lee Press, Piegon Cove, Massachusetts, 1972.

[2] D. A.Stanley, "*Multiplicative calculus*", *Primus IX, 4*, 310, 1999.

[3] D.Campbell, "*Multiplicative calculus and student projects*", Department of Mathematical Sciences, United States Military Academy, West Point, NY,10996, USA.

[4] M.Grossman, *Bigeometric calculus: A system with a scale-free Derivative*, Archimedes Foundation, Massachusetts, 1983.

[5] J.Grossman, M. Grossman, " *An introduction to non-Newtonian calculus* ", Int. J. Math. Educ. Sci. Technol. 0(4), 525, 1979.

[6] J.Grossman, M Grossman and R. Katz, *The first systems of weighted differential and integral calculus*, University of Michigan, 1981.

[7] J.Grossman, *Meta-Calculus: Differential and Integral*, University of Michigan, 1981.

[8] A.E.Bashirov, E. M.Kurpınar and A.Ozyapici, " *Multiplicative calculus and its applications*", J. Math. Anal. Appl. 337, 36, 2008.

[9] A.E.Bashirov and M.Rıza, "On complex multiplicative differentiation", *TWMS J. App. Eng. Math*. 1, 85, 2011.

[10] A.E.Bashirov, E.Mısırlı,Y.Tandoğdu and A.Ozyapıcı,"On modeling with multiplicative differential equations", *Appl. Math. J. Chinese Univ*. 26(4), 425, 2011.

[11] S.Tekin and F.Başar, " *Certain sequence spaces over the non-Newtonian complex field* ", Abstr. Appl. Anal, Article ID 739319, 11 pages, 2013.

[12] C.Türkmen and F.Başar, " *Some basic results on the sets of sequences with geometric calculus*", Commun. Fac. Fci. Univ. Ank. Series A1. 61(2), 17, 2012.

[13] A.Uzer, " *Multiplicative type complex calculus as an alternative to the classical calculus*", Comput. Math. Appl. 60, 2725, 2010.

[14] A.F.Çakmak and F.Başar, "*Some sequence spaces and matrix transformations in multiplicative sense*", TWMS J. Pure Appl. Math. , 6(1), 27, 2015.

[15] K.Boruah and B.Hazarika, " *Some basic properties of G-calculus and its applications in numerical analysis* ", arXiv:1607.07749v1, 24 July 2016.

[16] K.Boruah and B.Hazarika, " *G- Calculus*", *TWMS J. App. Eng. Math*. 8, 94, 2018.

[17] Y.Gürefe, *Multiplicative differential equations and its applications*, Ph.D. in Department of Mathematics, 2013.

[18] H.Es, " *On The 1-Parameter Motions With Multiplicative Calculus* ", Journal of Science and Arts, 2(59), 2022.

[19] K.Boruah and B.Hazarika, "*Application of geometric calculus in numerical analysis and difference sequence spaces*", arXiv:1603.09479v1, 2016.

[20] A.F.Çakmak and F.Başar, " *On classical sequence spaces and non-Newtonian calculus* ", J. Inequal, Appl. Art., ID 932734, 12pages. 2012.

[21] E.Misirli and Y.Gurefe, " *Multiplicative Adams Bashforth–Moulton methods* ", Numer Algor, 57, 425, 2011.

[22] S.Aslan, M.Bekar and Y.Yaylı, "*Geometric 3-space and multiplicative quaternions*", International Journal 1 of Geometric Methods in Modern Physics, 20-9, 2023.

[23] S.Nurkan, K.İ. Gürgil and M.K. Karacan, " *Vector properties of geometric calculus* ", Math. Meth. Appl. Sci. 1–20, 2023.

[24] S.G.Georgiev and K.Zennir, *Multiplicative Differential Calculus* (1st ed.), Chapman and Hall/CRC, New York, 2022.

[25] H. Es, "*Plane Kinematics in Homothetic Multiplicative Calculus*", JUM, vol. 7, no. 1, pp. 37–47, Jan. 2024, doi: 10.33773/jum.1408506.

[26] H. Es, "*Plane Kinematics in Lorentzian Homothetic Multiplicative Calculus*", Journal of Universal Mathematics, 7(2), 64-74, (2024), https://doi.org/10.33773/jum.1411844

[27] H. Es , " *Geometric Interpretation Of Multiplicative Calculus In Lorentzian Space*", Journal of Science and Arts, Vol.24, Issue 3, pp. 613-626, 2024.

[28] A. Has and B. Yılmaz , " *On non-Newtonian Helices in Multiplicative Euclidean Space E^3*", Fundamentals of Contemporary Mathematical Sciences,Vol.6, Issue2, 196-217, 2025.

[29] A. Has and B. Yılmaz, "*A non-Newtonian Conics in Multiplicative Analytic Geometry*", Turkish Journal of Mathematics *48* (5): 976-994, (2024). *https://doi.org/10.55730/1300-0098.3554*.

[30] Y. Yaylı, "*On the Planer Homothetic Motions*", J. Inst. Sci. Techno. Gazi Üniv., vol. 4, no. 1, Jan. 1991.