



Prediction of critical submergence in a two-layer-stratified fluid flow

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Abstract

This study examines the critical submergence of a pipe intake located in a two layer-stratified flow. The critical submergence for a vertically downward flowing pipe intake in a two-layer stratified fluid field is defined as the vertical distance of the interface of the fluid layers to the intake level for which the upper fluid just enters the intake. The available potential flow solution-based theory developed for the critical submergence of an intake located in a single fluid flow (with no stratification) is utilized to predict the critical submergence for an intake located in a two-layer-stratified flow. It is shown that whole boundary blockage effects may be disregarded, and an imaginary complete critical spherical sink surface (CSSS) can be used for the prediction of the critical submergence. The results of several available experimental studies indicate that the velocity at the imaginary complete CSSS is a constant for a given geometrical and flow conditions and it can be obtained by conducting a few experiments. Since this approach does not require computation of blockages caused by the flow boundaries, it is easy to use in practice. This theory can also be applied to the intakes located in general geometrical and flow conditions [for both two-layer-stratified flows and single layer flows (without stratification)]. Theoretical results obtained from this study match very well with the available experimental data.

Keywords Critical submergence · Intake · Stratified flow · Vortex

List of symbols

a	A constant	N_T	Kolf number
A_s	Net working area of the CSSS	Q	Intake discharge
A_c	Surface area of the imaginary complete CSSS (whole boundary blockages are disregarded)	Re	Intake Reynolds number
b_1, b_2	The clearances of the intake center to the left and right side walls of the channel, respectively	R	Correlation coefficient
c	Clearance of the level of the intake to the bottom boundary	S	Distance between the level of the intake and interface of the layered fluids
D	Internal diameter of the intake pipe	S_c	Critical value of S
F	Froude number of the pipe intake	U_∞	Average velocity of the incoming developed cross-flow
F_s	Densimetric Froude number of the pipe intake	V	Average velocity at the intake
g	Acceleration of gravity	V_c	Radial velocity distributed uniformly over the imaginary complete CSSS
g'	Reduced acceleration of gravity	V_s	Radial velocity at the CSSS
h	Thickness of the fluid in upper layer	W	Intake Weber number
K	A constant	μ_1	Dynamic viscosity of fluid in lower layer
l	Clearance of the intake center to the dead-end wall	μ_2	Dynamic viscosity of fluid in upper layer
		Γ	Imposed circulation
		ρ_1	Density of the lower layer fluid
		ρ_2	Density of the upper layer fluid
		σ	Surface tension between the lower and upper layer fluids
		ν_1	Kinematic viscosity of the fluid I

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1 Introduction

The withdrawal of water from a stratified environment by intake structures is frequently met in various engineering applications. Especially in power plants it is important to draw the water from bottom layer which is cooler and denser than the upper layer. Stratification may be due to thermal processes or concentration or density differences (e.g., fresh and salt water) between two layers. Depending on various parameters relating to flow and geometrical conditions, flow from either or both layers may be withdrawn by the intake structures. Therefore, it is important to find a criterion reflecting the relation between the selective withdrawal conditions and the influencing parameters.

Several types of intake structures are used in a two-layer-stratified flow such as; submerged vertical or horizontal pipe intake with or without a cap or a collar, skimmer walls, and rectangular caissons with slots. Parameters relating to a typical vertical circular intake in a two-layer-stratified fluid field are presented in Fig. 1.

In Fig. 1, D is the internal diameter of the intake; Q is the intake discharge; V is the average velocity at the intake; c is the clearance of the intake level to the bottom boundary; ρ_1 and ρ_2 are the densities of lower and upper layers fluids, accordingly; S is the submergence of the

intake (the vertical distance between the level of the intake center and interface of the layered fluids); S_c is the critical value of S at which the fluid II is just entrained by the intake; b_1 and b_2 are the clearances of the intake center to the left and right side walls of the channel, respectively; and l is the clearance of the intake center to the dead-end wall (if there exists so).

In the literature, selective withdrawal problems have been investigated experimentally or analytically and researchers have found different equations relating to the critical submergence of a pipe intake located in stratified flows [1–6]. Their equations are in a similar form that is as follows.

$$\frac{S_c}{D} = KF_s^a, \quad (1)$$

where $F_s = V/(g'D)^{0.5}$ = densimetric Froude number of the pipe intake, $g' = g(\rho_1 - \rho_2)/\rho_1$ = reduced acceleration of gravity (g is the gravitational acceleration), and K and a are constants depending on the flow and geometrical conditions for which they are obtained. Lubin and Springer [1] experimentally investigated the critical submergence of an intake pipe located centrally in a cylindrical tank with $c = 0$ and obtained K and a as 0.626 and 0.2, respectively. Goldring [2] studied the effect of intake cap in a stratified cross flow on the critical submergence. Sharp and Parchure [3, 4] showed that if 1% mixing of two stratified layers is allowed at the intake, critical intake discharge can be increased up to 40% with comparison to incipient draw-down condition. They also found the magnitudes K and a as 0.58 and 0.4 (if $c = 0$), and 0.53 and 0.4 (if $c \neq 0$), respectively. Hocking [5] studied the withdrawal from two-layer fluid through a line sink located at the bottom corner of a rectangular channel. Davidian and Glover [6] found K and a as 0.42 and 0.25 for vertically upward flowing intake. Shamma and Zhu [7] experimentally investigated the effect of a temperature-control curtain on selective withdrawal in a two-layer reservoir. Islam and Zhu [8] compared the results of CFD solver and the theory they developed in the case of a point sink (circular orifice intake) sited in a two-layer stratified flow.

Methods used in aforementioned previous studies are only valid for the conditions for which they are obtained. In most of the previous studies effects of flow boundaries (channel side walls, dead-end wall or channel bottom) are not investigated. However, in the present study following points are considered to address the gaps in the literature.

- A potential flow solution-based theory is applied to the circular intake(s) located in two-layer-stratified fluid flow to obtain the critical submergence.
- The method considered in this study to find the critical submergence can be applied to the intakes located in

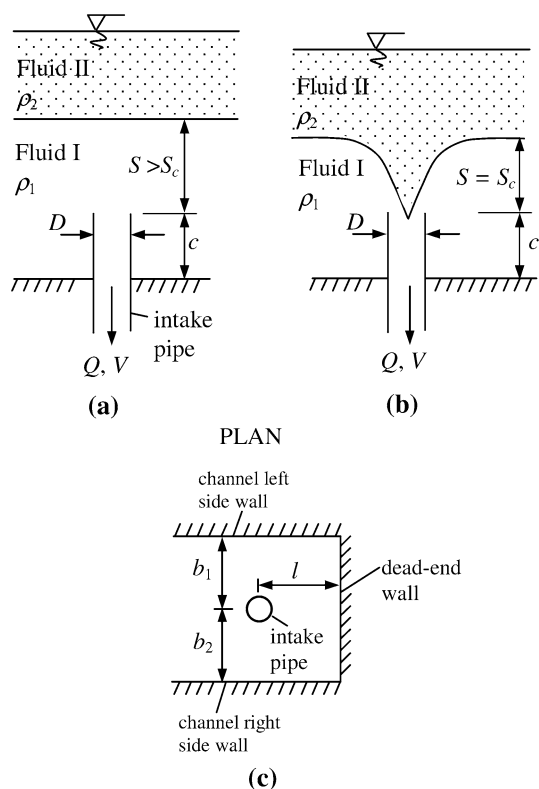


Fig. 1 General definitions of a pipe intake in a stratified fluid medium **a** submergence, **b** critical submergence and **c** plan view of the intake in a dead-end channel

general flow conditions (cross flow, still-water reservoir, etc.). This method can also be applied to the intakes located at various positions in a channel or a reservoir (different c/D , l/D , b_1/D , b_2/D). Theoretical results obtained from this study match very well with the available experimental data.

- In the present study, effects of different flow boundaries (e.g., channel bottom and dead-end wall) and effects of stratification on the critical submergence are discussed by making a comparison with no stratification case.

2 Dimensional analysis

Influencing parameters of critical submergence (S_c) of a pipe intake located in a two-layer-stratified fluid medium subjected to a cross-flow can be written as follows.

$$S_c = f(\rho_1, \rho_1 - \rho_2, \mu_1, \mu_2, \sigma, h, g, V, D, c, b_1, b_2, l, \Gamma, U_\infty). \quad (2)$$

In which, μ_1 and μ_2 are the dynamic viscosities of fluids in lower and upper layers, respectively; σ is the surface tension between lower and upper layer fluids; h is the thickness of the fluid in upper layer; Γ is the imposed circulation; and U_∞ is the average velocity of the incoming developed cross-flow.

Dimensional analysis of the variables in Eq. (2) yields

$$S_c/D = f(Re, F_s, W, N_\Gamma, V/U_\infty, c/D, h/D, b_1/D, b_2/D, l/D), \quad (3)$$

where, $Re = VD/\nu_1$ = intake Reynolds number ($\nu_1 = \mu_1/\rho_1$ = kinematic viscosity of the fluid I); $W = \rho_1 DV^2/\sigma$ = intake Weber number; $N_\Gamma = \Gamma/(VD)$ = Kolf number.

Daggett and Keulegan [9] indicated that the effect of Re may be neglected if $Re > 5.9 \times 10^4$. Jain et al. [10] showed that surface tension is not effective on S_c for $W > 120$. Re and W in this study are larger than these limit values, therefore, the effects of surface tension and viscosity may be neglected. Effect of the thickness of the upper layer fluid (h) is also negligible [3]. If there is no imposed circulation N_Γ can be dropped out from Eq. (3). After these simplifications Eq. (3) becomes

$$S_c/D = f(F_s, V/U_\infty, c/D, b_1/D, b_2/D, l/D). \quad (4)$$

If c , b_1 , b_2 , and l are larger than or equal to S_c ; c/D , b_1/D , b_2/D , l/D may also be non-effective on the critical submergence and may be dropped out from the Eq. (4).

3 Theoretical background

Yıldırım and Kocabaş [11] showed that critical submergence of an intake pipe located in a single layer (no stratification) cross-flow can be predicted by a potential flow solution-based theory in which the point sink flow (circular intake flow) and uniform flow are superimposed. They observed that the upper boundary of the Rankine body almost matches with the water surface and the critical submergence is equal to the radius of imaginary critical spherical sink surface (CSSS) having the identical center and discharge as the actual intake, and S_c can be computed from the integral continuity equation that is as follows.

$$Q = A_s V_s. \quad (5)$$

In which A_s is the net working surface area of the CSSS (net area of CSSS after subtracting loss of area due to solid flow boundaries from the complete spherical sink surface), and V_s is the radial velocity at the CSSS. Yıldırım and Kocabaş [12] proved that V_s is a constant for the given flow and geometrical conditions. Taştan and Yıldırım [13] and Taştan [14] indicated that an imaginary complete critical sink surface (neglecting all kinds of boundary blockages) can also be used for the prediction of critical submergence of a pipe intake as presented in Eq. (6).

$$Q = A_c V_c, \quad (6)$$

V_c is the radial velocity distributed uniformly over the imaginary complete critical spherical sink surface which has the area of $A_c = 4\pi S_c^2$ and it covers whole characteristics of flow and geometrical conditions in which the intake is located. In this study, Eq. (6) is used instead of Eq. (5) since the evaluation of boundary blockages may become tedious. But similar analysis may be done for Eq. (5). Note that if there is a cross flow, V_c is also related to U_∞ in Eq. (4). Yıldırım and Kocabaş [11] showed that for a pipe intake located in a uniform single layer cross flow without a boundary-blockage effect, V_c is equal to $U_\infty/2$.

Since most of the theoretical studies relating to selective withdrawal assumed irrotational flow, potential flow solution-based approach suggested by Yıldırım and Kocabaş [11, 12] and Taştan and Yıldırım [13] for the prediction of critical submergence of intakes located in a single layer flow may also be applicable for a pipe intake withdrawing fluid from a two-layer-stratified flow. Furthermore, the shape of the interface between the two layers is very similar to the upper boundary of the Rankine body as illustrated in Fig. 3 of [15] and Fig. 5 of [8]. Therefore, the concept of imaginary complete CSSS [or Eq. (6)] should work well for a two-layer-stratified flow.

4 Results and discussion

In a similar manner explained in the studies by Yıldırım and Kocabaş [11], Kocabaş and Yıldırım [16], considering Eq. (6) for the case of a two-layer-stratified fluid field, the relation between the intake discharge and the area of imaginary complete CSSS may be described by a linear line whose slope should be equal to V_c . In Fig. 2, experimental results from several studies relating to the critical submergence of a pipe intake located in various geometrical and flow conditions are presented. Figure 2 indicates a linear relationship between Q and A_c . R in Fig. 2 is the correlation coefficient. Details of the experimental conditions are summarized in Table 1.

As it is seen from Fig. 2, the variation of Q with A_c for a pipe intake located in a two-layer-stratified fluid field is linear regardless of the given flow and geometrical conditions, and V_c is a constant. Therefore, V_c can be found by conducting a few experiments. Values of V_c are given in Fig. 2. Note that the magnitude of V_c varies with the flow and geometrical conditions for which they are obtained. This shows that for each flow condition and geometry, the critical submergence has to be determined as a case study. It is not possible to obtain a single general graph for the prediction of S_c/D .

Comparison of the experimental and theoretical results relating to S_c/D in conjunction with the magnitudes of V_c obtained from Fig. 2 is presented in Fig. 3. The agreement between measured and computed critical submergences is good (average relative error is about $\pm 2\%$, note that this value may be little higher for different flow and geometrical conditions).

4.1 Effect of stratification on critical submergence

In Fig. 4, theoretical results relating to S_c and Q in the studies by Yıldırım and Kocabaş [11] and Goldring [2] are compared to illustrate the effect of stratification on critical submergence of an intake pipe located in a cross-flow. In the case of a single layer flow V_c becomes larger than that in the case of a two-layer-stratified flow. The main reasons for this are the pressure at the surface (interface) of the lower layer, which is larger than the atmospheric pressure, friction and surface tension effects at the interface of the two fluids. Due to the high pressure caused by the upper layer fluid, pressure drop becomes smaller at the summit point of the CSSS compared to that in the case without stratification. Furthermore, friction effects at the interface of the two fluids cause a decrement in the V_c at the upper parts of the lower layer fluid. Therefore, velocities at the upper region of the CSSS decrease and movement of the tip

of the depression to the intake becomes more difficult. As V_c decreases surface area of the CSSS must increase to satisfy the continuity. Hence, for the case of a two-layer stratified flow, S_c required to entrain the upper layer fluid to the intake is larger than the S_c required to entrain the air to the intake in the case of a single layer flow as shown in Fig. 4.

4.2 Effect of flow boundaries on S_c or V_c

In this section, the effect of flow boundaries on critical submergence is investigated for the pipe intake located in both single fluid flow and a two-layer-stratified fluid flow. A solid flow boundary may have two different types of effects on the magnitude of critical submergence, these are: (a) boundary-blockage and (b) boundary-friction. If CSSS is cut by a solid boundary, a loss in surface area of CSSS, and hence a loss in intake discharge occurs. This effect is called “blockage effect” of the boundary. If the distance between the intake and solid boundary is larger than the critical submergence, CSSS is not cut by the boundary, resulting in no “blockage” effect.

According to potential flow solution if there exist a boundary-blockage effect, the critical submergence should increase to compensate the loss of the surface area in CSSS caused by the solid boundary and satisfy the continuity. Figure 5a indicates that as the clearance between the channel bottom and intake level decreases the critical submergence increases since the boundary-blockage (or loss in the surface area of the CSSS) increases. But the same principle does not work from the boundary friction effect point of view when the distance between the dead-end wall and the intake center decreases. The critical submergence decreases when the intake center approaches the dead-end wall (Fig. 5b). The reason for this is as follows. Occurrences of vortices originating from either air–fluid interface or fluid–fluid interface are very sensitive to the disturbances at the interface. A boundary close the free surface or interface of the layered fluids above the intake center highly reduces the development of surface depression due to no-slip boundary condition on the solid boundary surface. This effect, called “boundary-friction”, generally dominates the boundary-blockage depending on the clearance of the intake center to the boundary cutting or close to the free surface or interface of the layered fluids [18].

Experimental data of [19] in Fig. 5a, b is for the critical submergence of an intake located in a dead-end wall canal flow without stratification, and $F = V/(gD)^{0.5}$ = intake Froude number in a single layer flow. The effect of the clearance of the bottom boundary to the intake in a two-layered stratified flow on V_c is given in Fig. 5c (data in

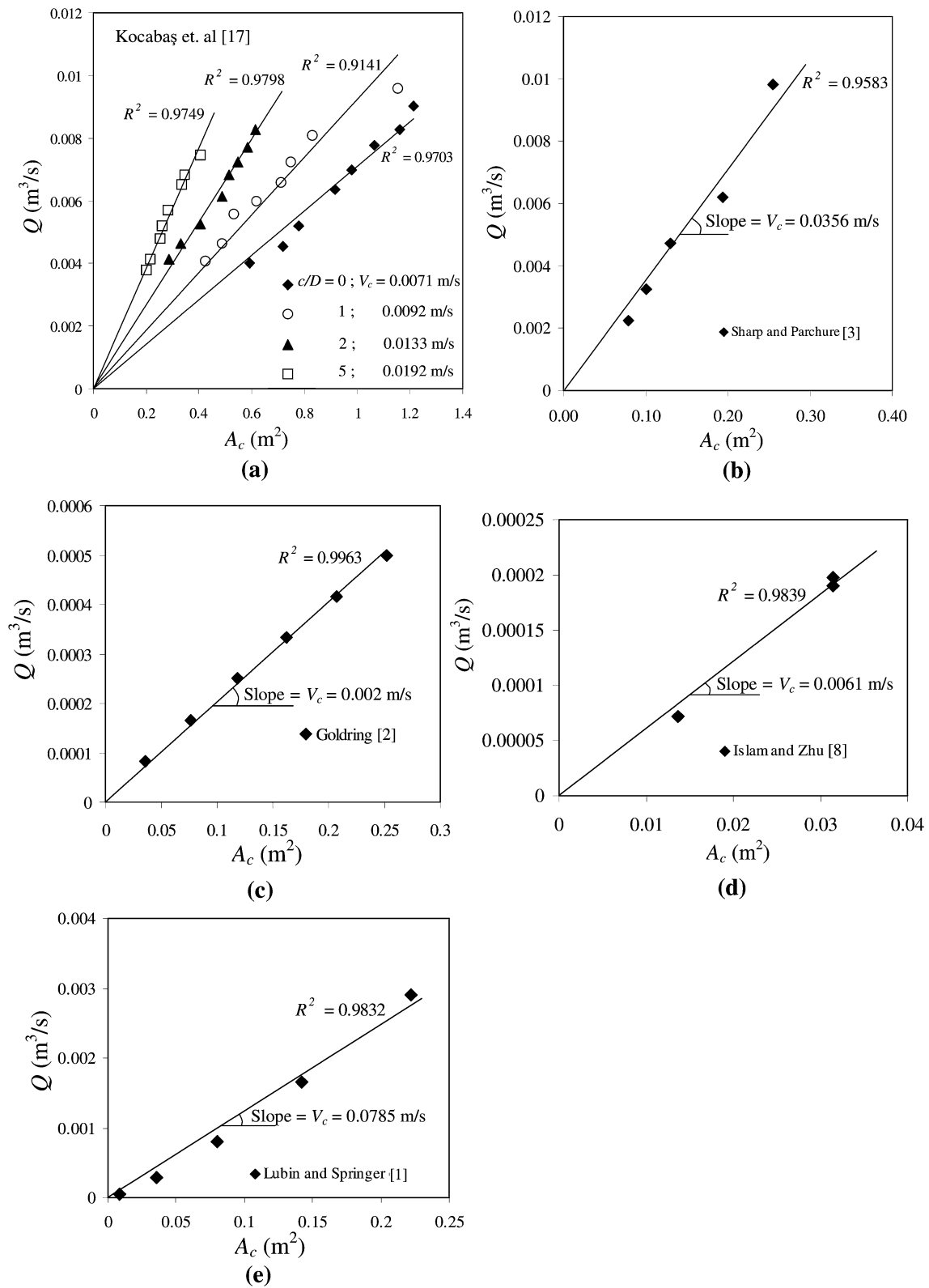


Fig. 2 Relations between Q and A_c : **a** Kocabaş et al. [17], **b** Sharp and Parchure [3], **c** Goldring [2], **d** Islam and Zhu [8] and **e** Lubin and Springer [1]

Table 1 Summary of the available experimental studies ($b_1/D > S_c/D$, $b_2/D > S_c/D$)

Study	Flow condition	Geometrical properties	Densities
Kocabaş et al. [17]	Still-water	$c/D = 0, 1, 2, 5$ $D = 6.9$ cm No dead-end wall	$\rho_1 = 1000$ kg/m ³ $\rho_2 = 920$ kg/m ³
Sharp and Parchure [3]	Still-water	$c/D = 0$ $D = 20.32$ cm No dead-end wall	$\rho_1 = 1004$ kg/m ³ $\rho_2 = 1021$ kg/m ³
Islam and Zhu [8]	Wide channel	$c/D > S_c/D$ $D = 0.4$ cm $l/D = 0$	$(\rho_1 - \rho_2)/\rho_1 = 0.002$
Goldring [2]	Cross flow $U_\infty = 1.67$ cm/s	$c/D = 0$ $D = 5.42$ cm No dead-end wall	$(\rho_1 - \rho_2)/\rho_1 = 0.00068$
Lubin and Springer [1]	Still-water	$c/D = 0$ $D = 1.27$ cm No dead-end wall	$\rho_1 = 1000$ kg/m ³ $\rho_2 = 870$ kg/m ³

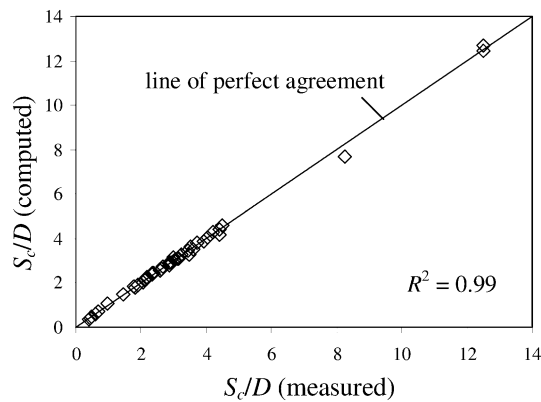
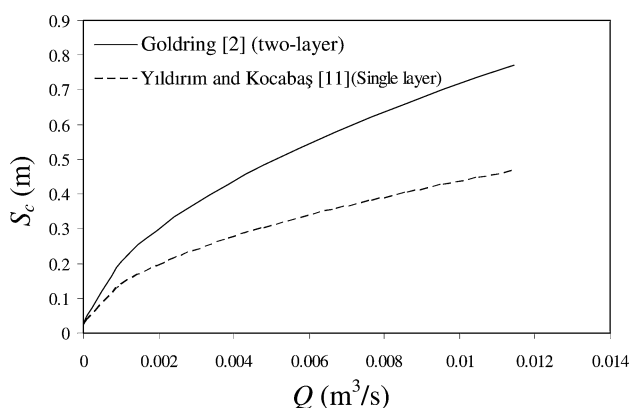
**Fig. 3** Comparison of experimental and theoretical critical submergences**Fig. 4** Comparison of critical submergence of an intake pipe in single layer and two-layer stratified cross-flow ($D = 5.42$ cm; $U_\infty = 1.67$ cm/s, $c/D = 0$, $b_1/D > S_c/D$, $b_2/D > S_c/D$, no dead-end wall)

Fig. 5c is from [17]). As c/D increases V_c increases, and hence S_c decreases.

5 Conclusions

The important results obtained from this study are as follows.

- The method introduced in this study may be utilized to find the critical submergence for intakes located in two-layer stratified flows. Furthermore, this method does not require the computation of blockages caused from the flow boundaries. Therefore, it is simple and easy to use in practice. Theoretical results obtained from this study are very close to the experimental results available in literature.
- It is shown that imaginary complete CSSS can be used and velocities at the CSSS can be found by conducting only few experiments. Since the velocity at the CSSS is determined from experiments it describes the whole characteristics of the geometrical and the flow conditions. Therefore, this method can be used for the intakes located in general geometrical and flow conditions [for both two-layer-stratified flows and single layer flows (without stratification)].
- Due to the high pressure caused by the upper layer fluid, pressure drop becomes smaller at the summit point of the CSSS compared to that in the case without stratification. In addition to that, because of the friction effects at the interface of the two fluids, critical submergence for the case of a two-layer stratified flow is larger than the critical submergence for an intake in the case of a single layer flow.

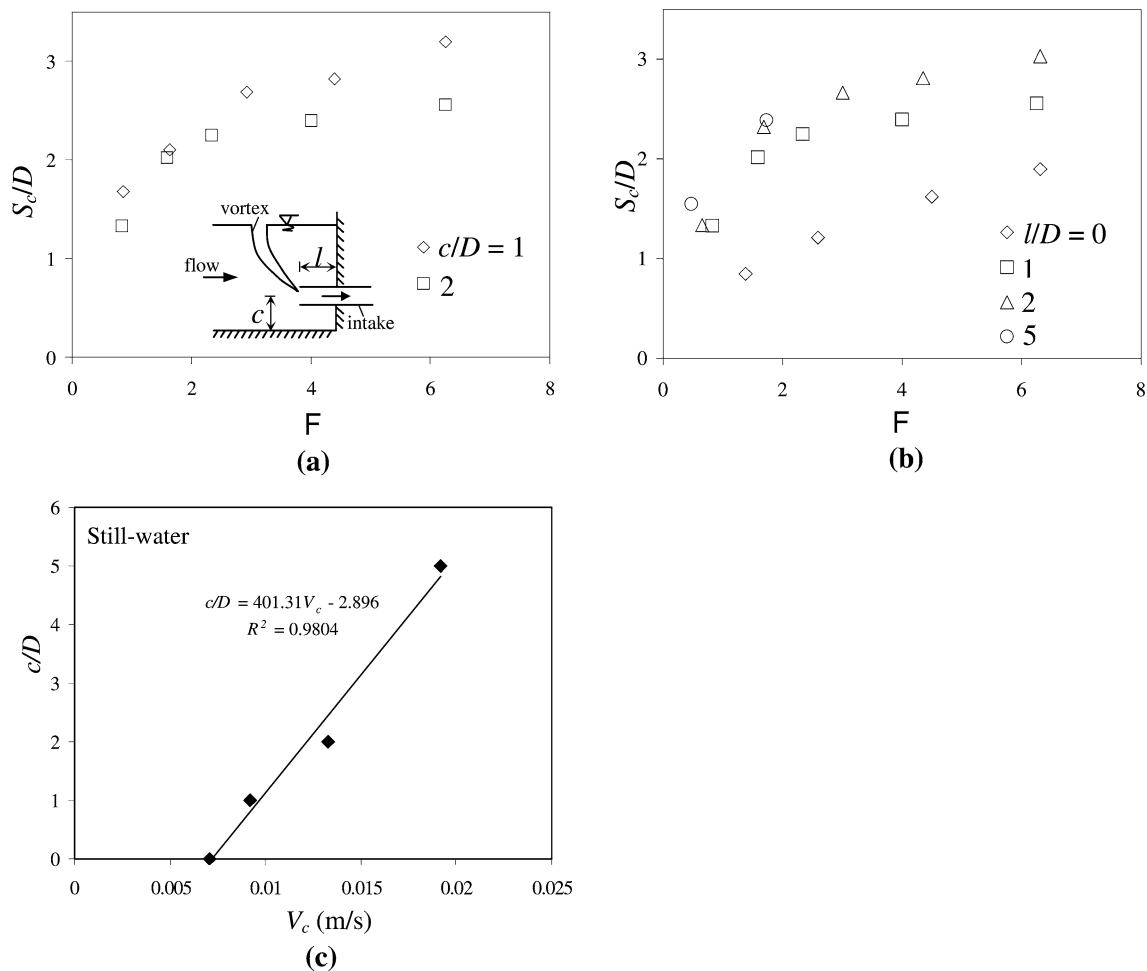


Fig. 5 For a single layer flow; **a** effect of clearance of bottom boundary ($l/D = 1$ and $b_1/D, b_2/D > S_c/D$); **b** effect of clearance of dead-end wall ($c/D = 2$ and $b_1/D, b_2/D > S_c/D$) for two-layered-stratified flow; **c** variation of c/D with V_c

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